

Professor: J.D. WilsonValue: 10%

Instructions: All questions have equal value. Retain high precision in your calculations, but please round your final answers to three significant digits¹ and state the unit (examples: 20.1°C, 1.09 kg m⁻³, 391 W m⁻², 4.11 × 10² W m⁻²). Document your working tidily — there is potentially a one mark penalty for illegible or unintelligible working. Please drop off your assignment (labelled with your name and ID number) in class, or in the drop-off box outside Tory 3-40. A two mark penalty will be applied for late assignments received before noon Thursday 20 Oct. After that time, the late penalty will be five marks.

Task: Add together the *last five digits* of your student I.D. number, to form what we shall interpret as a ground-level temperature T_1 , as a latitude ϕ_1 , and as a surface shortwave reflectivity (albedo) α_1 expressed as a percentage, e.g.

$$\text{I.D. number 1198765} \rightarrow T_1 = 35^\circ\text{C}, \quad \phi_1 = 35^\circ, \quad \alpha_1 = 35\% \equiv 0.35.$$

In what follows, the subscript “1” denotes properties at level 1, which is ground level. Level 2 (denoted with subscript “2”) will be the 850 hPa level.

1. Assuming the emissivity of the ground surface is $\epsilon = 0.96$, compute the emitted longwave radiative flux density “ $L_1 \uparrow$ ” corresponding to your ground temperature T_1 .
2. Compute the ratio of this flux $L_1 \uparrow$ to the flux “ $L_{\text{ref}} \uparrow$ ” that would be emitted by the same surface if it had a temperature of 0°C (273.15 K).
3. Based on your $L_1 \uparrow$ but neglecting incoming (i.e. downward) longwave radiation ($L \downarrow$), compute the net radiation “ Q_{*1} ” at solar noon at the times of the equinox and express this as a fraction of the solar constant Q_{*1}/S_0 . (To compute $K \downarrow$ assume the solar beam is not subject to scattering or absorption by the atmosphere, i.e. is transmitted without attenuation so that $K \downarrow$ depends only on the latitude ϕ_1 .)
4. Your net radiative energy supply Q_{*1} would suffice to evaporate what depth d_1 of water over a period of one hour? (Take the density of liquid water to be $\rho_w = 1000 \text{ kg m}^{-3}$)
5. Assuming ground-level pressure is $P_1 = 925 \text{ hPa}$, compute the ground-level air density ρ_1 implied by this combination (P_1, T_1).
6. Assuming the specific humidity at ground is $q_1 = 0.002$, compute the vapour pressure e_1 .
7. Compute the mass of water m_1 in one cubic metre of air at ground level.

¹However if you are providing an answer in Kelvin units, it is appropriate to provide five significant figures, e.g. 278.75 K.

8. Adopt the hydrostatic law (given as data), evaluating the right hand side as $-\rho_1 g$, to compute the vertical distance Δz between the ground and the 850 hPa surface (i.e. the altitude where pressure is $P_2 = 850$ hPa). Note: here we are using the hydrostatic law in an approximate way, by placing a fixed constant value of $\rho (= \rho_1)$ on the right hand side.
9. Assuming the temperature variation from ground to the P_2 level follows the dry adiabatic lapse rate (DALR), compute the temperature T_2 (in Celcius units) at this level.
10. Again use the hydrostatic law, evaluating the right hand side as $-\rho_2 g$, to determine the pressure P_3 at a point that is 300 m above the 850 hPa surface.

Data

- $1 \text{ hPa} = 100 \text{ Pa}$, $T \text{ [K]} = T \text{ [}^\circ\text{C]} + 273.15$ (Note that a *change* of one degree Kelvin is the same as a *change* of one degree Celcius).

- $\frac{\Delta P}{\Delta z} = -\rho g$

The hydrostatic law. ΔP [Pascals], the change in pressure as one ascends a distance Δz [m]; ρ [kg m^{-3}] the air density; $g = 9.81$ [m s^{-2}] acceleration due to gravity.

- $P = \rho R T$

The ideal gas law. P [Pascals], pressure; ρ , [kg m^{-3}] the density; T [Kelvin], the temperature; and $R = 287$ [$\text{J kg}^{-1} \text{K}^{-1}$], the specific gas constant for air.

- $e = \rho_v R_v T$

The ideal gas law for water vapor. e [Pascals], the vapour pressure (i.e. partial pressure of water vapour); ρ_v , [kg m^{-3}] the absolute humidity (ie. vapor density); T [Kelvin], the temperature; and $R_v = 462$ [$\text{J kg}^{-1} \text{K}^{-1}$], the specific gas constant for water vapor.

- $q = \rho_v / \rho$

the specific humidity

- $L \uparrow = \epsilon \sigma T^4$

Stefan-Boltzmann law. $L \uparrow$ [W m^{-2}], the emitted longwave energy flux density; ϵ , the emissivity of the surface (dimensionless); $\sigma = 5.67 \times 10^{-8}$ [$\text{W m}^{-2} \text{K}^{-4}$], the Stefan-Boltzmann constant; T [K], the surface temperature.

- $\frac{\Delta T}{\Delta z} = -0.01$ [K m^{-1}]

The dry adiabatic lapse rate (DALR), i.e. for every one metre of ascent the temperature decreases by 0.01 degrees Kelvin.

- $L_v = 2.5 \times 10^6$ J kg^{-1}

The latent heat of vapourization of water

- $\theta = 90 - \phi + \phi_{sol.dec}$

The solar elevation θ at solar noon, at a location having latitude ϕ , at the time of year when solar declination is $\phi_{sol.dec}$. Latitude is taken as positive for both hemispheres; solar declination is negative if the subsolar point is in the opposite hemisphere.