

Professor: J.D. Wilson Time available: 80 mins Value: 20%

Instructions: Closed book exam. Please record your answers in the exam booklet. Pertinent data and diagrams are at the back, and should be read before answering any questions.

Multi-choice (20 x $\frac{1}{2}\%$ \rightarrow 10%)

1. The circuit shown in Figure (1) is known as a
 - (a) lowpass RC filter
 - (b) highpass RC filter
 - (c) Wheatstone bridge
 - (d) Half-bridge or voltage divider ✓✓
 - (e) Error-detector

2. In a rigorous interpretation of Figure (1), the internal resistance of the voltage supply should be considered to have been
 - (a) ignored
 - (b) subtracted from the supply voltage V
 - (c) added to the output voltage V_o
 - (d) lumped with R_1
 - (e) lumped with R_2 ✓✓

3. Referring to Figure (1), if $V_o = 2$ V and $R_1 = R_2 = 7.5$ K Ω , then the supply voltage V is _____ volts
 - (a) 7.5
 - (b) 7.5/2
 - (c) 4 ✓✓
 - (d) 2
 - (e) 1

4. Referring to Figure (1), if $V_o = 2$ V and $R_1 = R_2 = 1$ K Ω , then the current through R_1 is _____ amps
- (a) 0
 - (b) 0.5
 - (c) 0.5×10^{-3}
 - (d) 1×10^{-3}
 - (e) 2×10^{-3} ✓✓
5. Referring to Figure (1), if $V = 12$ V and $R_1 = 5$ K Ω and $R_2 = 15$ K Ω , then the output voltage V_o is _____ volts
- (a) 12
 - (b) 8
 - (c) 4
 - (d) 3 ✓✓
 - (e) 1
6. If the output from Figure (1) is to be taken to a 12 bit datalogger with Full Scale Range ± 5 volts, then the smallest detectable change in output voltage δV_o is _____ volts
- (a) 1.22×10^{-3}
 - (b) 2.44×10^{-3} ✓✓
 - (c) 1
 - (d) 1.22
 - (e) 2.44
7. If in Figure (1) $R_2 = 10$ K Ω but the current through R_2 is zero, then
- (a) $V_o = V$
 - (b) $R_1 = \infty$
 - (c) $V_o = 0$
 - (d) $R_1 = 0$
 - (e) both (a) and (b) are true ✓✓

8. If in Figure (2) $\tau = RC = 10^{-3}$ sec, then the half-power frequency f_o is
- (a) 1000 Hz
 - (b) 1000 KHz
 - (c) 159 Hz ✓✓
 - (d) 159 KHz
 - (e) none of the above
9. If the frequency of the input sine wave in Figure (2) is $f \gg f_o$ then the output amplitude V_o is
- (a) V_s
 - (b) $V_s/\sqrt{2}$
 - (c) 1
 - (d) 1/2
 - (e) ≈ 0 ✓✓
10. If the frequency of the input sine wave in Figure (2) is $f \equiv f_o$ then the output amplitude V_o is
- (a) V_s
 - (b) $V_s/2$
 - (c) $V_s/\sqrt{2}$ ✓✓
 - (d) 1/2
 - (e) ≈ 0
11. With reference to Figure (3), the condition that $\frac{R_1}{R_1+R_2} = \frac{R_3}{R_3+R_4}$ has the result that
- (a) the bridge is in balance
 - (b) $V^- = V^+$
 - (c) the error voltage vanishes
 - (d) $R_1R_4 = R_2R_3$
 - (e) all of the above ✓✓

12. Suppose a data-logger displays a number N representing the voltage $V^+ - V^-$ across its two input terminals, and that it can be assumed that the logger is “linear,” ie., that $N = \alpha(V^+ - V^-) + \beta$. Furthermore, suppose the Full Scale Range (FSR) of the logger is $FSR = \pm 10$ volts. If we measure a reading N^+ when $V^+ - V^- = 10.0$ volts, and a reading N^- when $V^+ - V^- = -10.0$ volts, then the quantity $(N^+ - N^-)/20.0$ is
- the “offset” of the logger, β
 - zero
 - variable
 - the sensitivity, α ✓✓
 - none of the above
13. Consideration of the energy balance of an ordinary (and dry) thermometer leads to the conclusion that the “system output”, ie. the thermometer temperature T , responds to *several* environmental inputs, including air temperature T_a , air motion (eg. wind speed U), and the radiation environment as characterized by incoming solar radiation ($K \downarrow$), etc. Thus in general the measured temperature $T = T(T_a, U, K \downarrow, \dots)$. However a steady-state response to T_a alone, ie. a response $T = T(T_a)$ at steady state, is assured
- since this is a first-order, linear system
 - only if the radiation exchange term Q^* can be eliminated ✓✓
 - only if the time constant is short
 - only if the time constant is long
 - only if the thermometer is held in still air ($U = 0$)
14. Which of the following does not apply to the thermocouple
- floating voltage source
 - internal resistance $R_s = 0$ ✓✓
 - responds *linearly* to temperature *difference*
 - difficult to measure, microvolt (μV)-level signal
 - sensitivity N (units, $\mu V K^{-1}$) known as the “Seebeck coefficient”
15. A “floating differential voltage receiver” has two inputs labelled V^+, V^- . Which of the following statements is untrue
- the resistance from either terminal to powerline-ground is infinite
 - the resistances from the terminals to receiver common are large and equal
 - the resistance from one terminal to the other is small (ideally, zero) ✓✓
 - the resistance from one terminal to the other is large (ideally, infinite)
 - the common mode voltage relative to the receiver common is $(V^+ + V^-)/2$, ie. half the sum of the voltages applied at the terminals

16. If a potential drop V occurs across a resistance R , such that a current i flows, then the rate of power dissipation in the resistor (P) is
- (a) iR [volts]
 - (b) V^2/R [Joules]
 - (c) i^2R [Watts]
 - (d) V^2/R [Watts]
 - (e) both (c) and (d) are correct ✓✓
17. A lowpass filter has frequency-dependent power gain $G(f)$. If the input to this filter is a sinusoidal signal $x(t) = A_{in} \sin(2\pi ft)$, the output from the filter will be:
- (a) sinusoidal, but with the frequency doubled
 - (b) sinusoidal, but with the frequency halved
 - (c) sinusoidal, with the infinitely high frequency
 - (d) sinusoidal, with the infinitely low frequency
 - (e) sinusoidal, with the same frequency, and with amplitude $\sqrt{G}A_{in}$ ✓✓
18. A tank, of volume D^3 , is kept in a well-stirred condition by a powerful fan, and initially contains a pure gas “A.” At $t = 0$ it begins to be flushed by an inflow (volumetric flow rate Q [$\text{m}^3 \text{s}^{-1}$]) of pure gas “B,” that displaces (at equal rate) mixed gas through an outlet. The transition of the tank’s contents from “pure A” to “pure B” takes place with time constant
- (a) $(A - B)/Q$
 - (b) $A - B$
 - (c) D^3/Q ✓✓
 - (d) Q/D^3
 - (e) $A - BD^3/Q$
19. Given two identical thermistors R_{1T}, R_{2T} and two identical control resistors R_{1c}, R_{2c} , a differential temperature sensor could be constructed by placing _____ in the full bridge shown in Figure (4).
- (a) one thermistor in each of slots 1,2
 - (b) one thermistor in each of slots 3,4
 - (c) one thermistor in each of slots 1,3
 - (d) one thermistor in each of slots 2,4
 - (e) both (c) and (d) would work ✓✓

20. If the governing equation for a ψ -sensor is an o.d.e. of form

$$\frac{d\psi}{dt} = \frac{\psi_0 - \psi}{\tau} \quad (1)$$

where t is time and τ is a property of the sensor, then we may say

- (a) the sensor is a linear device
- (b) the sensor is a first-order system
- (c) the sensor has time constant τ
- (d) $\psi_0(t)$ is the input and $\psi(t)$ the response of the sensor
- (e) all of the above ✓✓

Short Answer (10 %)

Answer any **two** questions from this section. Give diagrams where appropriate to clarify your working, which should be shown. Justify any assumptions or simplifications you make.

1. Suppose a cyclist is riding at $U = 5 \text{ m s}^{-1}$ on a calm morning when the air temperature is $T_a = 2^\circ \text{ C}$. S/he has forgotten to wear gloves, and his/her hands are very cold due to convective heat loss. Treating the hand as a sphere of diameter $d = 8 \text{ cm}$, and assuming forced convection and that the skin surface temperature $T_s = 10^\circ \text{ C}$, compute the rate of loss of heat (J s^{-1}) from each hand. Compute the density ρ using the ideal gas law, assuming the pressure $P = 100 \text{ kPa}$.

Noting that core body temperature $T_c = 37^\circ \text{ C}$, draw a heat transfer “circuit” (for which driving forces are temperature differences and heat fluxes are moderated by transfer resistances) that could serve as a model for an assessment of how reasonable is the assumption that the outer surface of the hand has temperature $T_s = 10^\circ \text{ C}$.

2. Draw a tidy and complete circuit schematic representing a Wheatstone bridge (resistors R_1, R_2, R_3, R_4), that is driven by a grounded voltage source (no-load voltage V_s , internal resistance R_s), and whose error voltage ΔV is monitored by a balanced (ie. differential), grounded receiver (input resistances to ground R_{in}).
3. Using a diagram composed of the usual circuit symbols, explain the procedure by which, given a battery whose voltage is known to be exactly $V_s = 1.35 \text{ volts}$ and access to whatever tools and hookup wire you wished, you would perform a 3-point calibration check of a datalogger having full scale range $\pm 5 \text{ volts}$ (ie. determine the data-logger readings corresponding to 3 known voltages). You may neglect the internal resistance of the battery, since it will be negligible compared to the logger’s input resistance.

Data:

- Voltage resolution δV of an n-bit recorder with full scale range $\pm N$ is

$$\delta V = \frac{2N}{2^n - 1} \quad (2)$$

- $P = \rho R T$

The ideal gas law. P [Pascals], pressure; ρ , [$kg\ m^{-3}$] the density; T [Kelvin], the temperature; and $R = 287$ [$J\ kg^{-1}\ K^{-1}$], the specific gas constant for air).

- Kinematic viscosity of air: $\nu \approx 1.5 \times 10^{-5}$ [m^2s^{-1}]
- Thermal diffusivity¹ of air: $\kappa \equiv D_H \approx 2.1 \times 10^{-5}$ [m^2s^{-1}]
- Specific heat capacity of air at constant pressure: $c_p \approx 1000$ [$J\ kg^{-1}K^{-1}$]
- $C \frac{dT}{dt} = A (Q^* + Q_H + Q_E) + P$

Energy balance for a thermometer having bulk heat capacity C and surface area A . The Q 's are (left-to-right) the net radiative, sensible, and latent heat flux densities ($W\ m^{-2}$), and P is (any) internal heating.

- $\frac{dV_o}{dt} = \frac{V_s(t) - V_o}{\tau}$

Differential equation giving the relationship between the output $V_o(t)$ from a lowpass RC filter (time constant $\tau = RC$) and the input $V_s(t)$. The particular case of the “step response” corresponds to the specification: at $t = 0$, $V_o = V_s = 0$, while for $t > 0$, $V_s = \text{constant}$.

- $y(t) = Y_2 + (Y_1 - Y_2) \exp(-\frac{t}{\tau})$

Response of a 1st order (RC lowpass type) system to step $Y_1 \rightarrow Y_2$ in input.

- $Q_H = \rho c_p \frac{T_1 - T_2}{r_H}$

Ohm's law model for sensible heat exchange.

- $N_u = 2 + 0.54 R_e^{0.5}$ ($R_e \leq 300$), $N_u = 0.34 R_e^{0.6}$ ($50 \leq R_e \leq 1.5 \times 10^5$)

Nusselt number for a sphere in air (forced convection).

- $r_H = \frac{d}{D_H N_u}$ [$s\ m^{-1}$]

Resistance r_H for heat transfer.

- $G(f) = \left(\frac{A_o}{A_s} \right)^2 = \frac{1}{1 + (f/f_0)^2}$

“Power gain” (ie. ratio of square of output amplitude A_o to square of input amplitude A_s) of an RC lowpass filter having half-power frequency $f_0 = \frac{1}{2\pi RC}$.

¹Symbols κ, D_H are both used for this quantity.

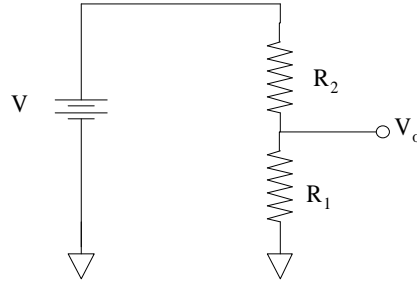


Figure 1: For this circuit it is implicit that no current is drawn from the output terminal. The output voltage $V_o = V \frac{R_1}{R_1+R_2}$

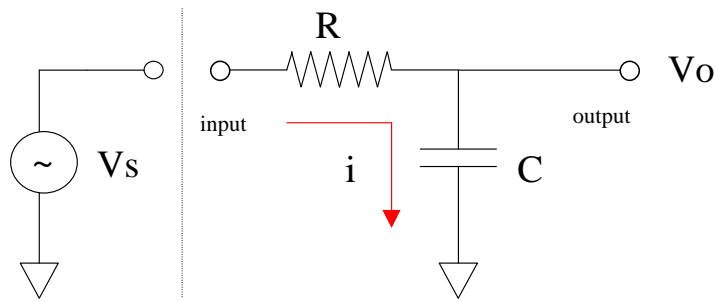


Figure 2: RC circuit driven by a sine wave generator (amplitude V_s), where again, it is implicit that no current is drawn from the output terminal. The amplitude V_o at the output terminal can be computed from $V_o^2 = V_s^2 / [1 + (f/f_o)^2]$ where $f_o = \frac{1}{2\pi RC}$ is the “half-power frequency”.

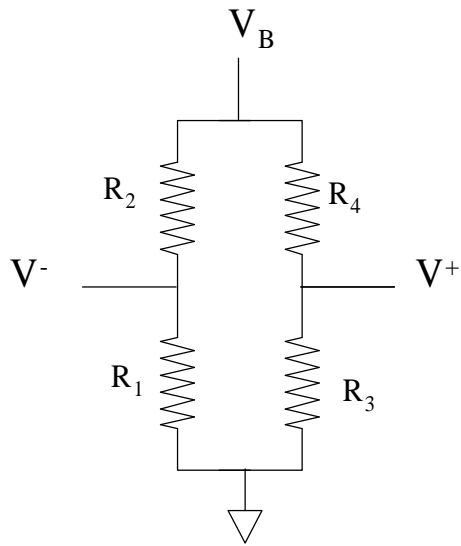


Figure 3: Error detection circuit. The “error voltage” is $V^+ - V^-$.

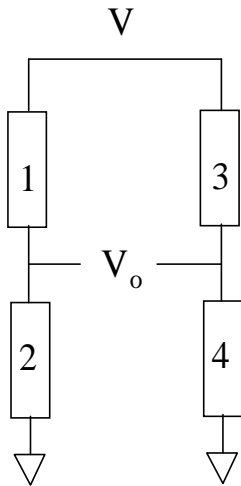


Figure 4: Template for construction of a differential temperature sensor.