EAS327 "Environmental Instrumentation" 6 April,2004

Professor: J.D. Wilson   Time available: 80 mins   Value: 20%

Instructions: Closed book exam. Please record your answers in the exam booklet. Pertinent data and diagrams are at the back.

Multi-choice (20 x $\frac{1}{2}$% $\rightarrow$ 10%)

1. Consider a leaf, modelled as a thin disc of diameter $d = 3$ cm, having uniform temperature T_ℓ, and exposed in an airstream of speed u whose temperature and vapour pressure are T_a, e_a. The heat transfer resistance for loss of heat to the airstream is r_H. If the leaf was wet, the resistance to the vapour flux would be $r_v \approx r_H$, but water vapour diffusing from the stomatal cavities (where vapour pressure $e = e_*(T_\ell)$) through the stomata (pores) would encounter an additional transfer resistance, the "stomatal resistance" r_s. If the leaf surface is dry, r_H, r_s moderate the transpiration rate E

   (a) in parallel, with effective resistance $r_H r_s/(r_H + r_s)$

   (b) in series, with effective resistance $r_H + r_s$ ✓✓

   (c) with effective resistance $r_H - r_s$

   (d) with effective resistance $r_s - r_H$

   (e) none of the above

2. A bulk model of transpiration from this leaf would regard the driving force for the transpiration flux E as

   (a) T_ℓ

   (b) $T_\ell - T_a$

   (c) u

   (d) E/u

   (e) $e_*(T_\ell) - e_a$ ✓✓

3. If measurements showed that the sensible and latent heat fluxes (defined as positive if directed from the leaf to the airstream) were $Q_H = 80$ W m^{-2}, $Q_E = 100$ W m^{-2} then as a first approximation the net radiation at the leaf surface (positive for energy gained by the leaf) must be

   (a) ~ 0

   (b) ~ 20 W m^{-2}

   (c) ~ 180 W m^{-2} ✓✓

   (d) ~ -20 W m^{-2}

   (e) ~ -180 W m^{-2}

4. An eddy covariance measurement of evapotranspiration E over a uniform surface directly evaluates the covariance $\overline{w'\rho_v'}$ of fluctuations w', ρ_v' in the vertical velocity and absolute humidity. The velocity and humidity sensors should be

   (a) linear for ease of computations

   (b) precisely co-located (separated by less than a micrometer)

   (c) slow, so as to filter out rapid peaks in the signals

   (d) fast enough to measure the fastest fluctuations, and separated by the smallest distance sufficient to ensure flow disturbance by the ρ_v' sensor does not interfere with the w' signal ✓✓

   (e) power-line grounded

5. A chilled-mirror dewpoint hygrometer probably could not be used to provide the humidity signal for eddy covariance because

   (a) it produces a non-linear voltage signal

   (b) it has a narrow full scale range of humidities

   (c) it has a slow response ✓✓

   (d) it is dependent on availability of a controlled-temperature water bath

   (e) the dewpoint fluctuation T_d' signal it provides cannot be related to the (needed) absolute humidity fluctuation ρ_v'

6. A thermistor $R(T)$ has negative temperature coefficient $\Delta R/\Delta T$. If it is placed as R_2 in this circuit

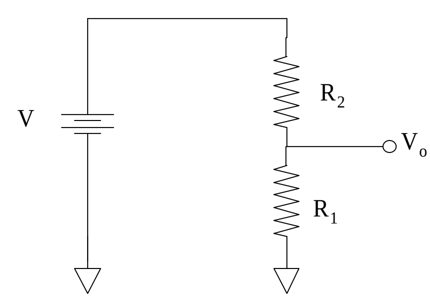
   (a) output voltage V_o increases with increasing T ✓✓

   (b) output voltage V_o decreases with increasing T

   (c) output voltage V_o is independent of T

   (d) output voltage V_o is a maximum when $T = 0$ C

   (e) output voltage V_o is offscale



7. Over the range $-20 \leq T_d \leq 20$ C the calibration curve for a certain dewpoint hygrometer is $V = 2.07\ T_d + 19.84$, where the signal V is in millivolts and dewpoint T_d in Celcius. If the signal $V = 0.71$ mV the dewpoint is

   (a) 21.31 C

   (b) 0.71 C

   (c) - 9.24 C ✓✓

   (d) outside the range of calibration

   (e) none of the above

8. Many electromagnetic-absorption gas analysers use a light-chopper to direct a beam alternately down each of two paths to the detector. Extracting the information from (ie. demodulating) the resultant "chopper-modulated" square-wave voltage signal entails creating a signal proportional to

   (a) the mean value of the wave
   (b) the probability density function of the wave
   (c) the maximum of the wave
   (d) the mimimum of the wave
   (e) the peak-to-peak amplitude of the wave ✓✓

9. If used in "absolute mode" to determine the concentration C_G of a gas "G" in the measurement path, the reference path of a dual-path infra-red gas analyser ought to be filled with

   (a) any gas mixture that does not contain the gas "G" ✓✓
   (b) a gas mixture containing a precisely known mixing ratio of gas "G"
   (c) infra-red photons
   (d) gas "G" at 100% absolute humidity
   (e) absolutely nothing

10. Assuming fixed concentration C_{G0} of the measured gas "G" in the reference path and an unknown concentration C_G of "G" in the measurement path, the "differential mode" of use of a dual-path infra-red gas analyser is designed to provide

    (a) high precision measurements of $C_G - C_{G0}$ over a narrow full scale range in $C_G - C_{G0}$ ✓✓
    (b) high precision measurements of C_G over a wide full scale range in C_G
    (c) high precision measurements of C_G over a narrow full scale range in C_G
    (d) low precision measurements of C_G over a narrow full scale range in C_G
    (e) high accuracy measurements of $C_G - C_{G0}$ over a wide full scale range in C_{G0}

11. The "photosynthetically-active" radiation band (PAR) spans $0.4 \leq \lambda\ \mu\text{m} \leq 0.7$. If one wished to measure the hemispheric energy flux density available to a leaf in this waveband, the most suitable type of sensor is ____

    (a) a thermometric sensor
    (b) a photo-voltaic sensor whose sensitivity is constant within, and zero outside, the PAR band
    (c) a thermometric sensor viewing the hemisphere through a filter with passband $0.4 \leq \lambda\ \mu\text{m} \leq 0.7$
    (d) an allwave net radiometer
    (e) both (b) and (c) are reasonable possibilities ✓✓

12. Suppose a certain surface has longwave emissivity $\epsilon = 0.8$ ($\equiv$ longwave absorptivity a), and is at temperature $T_s = 20$ C. Suppose also that the hemispheric longwave radiation flux incident upon the surface is $L \downarrow = 300$ W m^{-2}. Then a radiation sensor measuring the upwelling longwave flux $L \uparrow$ would "see" $L \uparrow =$ ____ W m^{-2}

   (a) 60
   (b) -60
   (c) 334
   (d) 394 ✓✓
   (e) 418

13. A certain random variable x has pdf $f(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$. The mean value of x is ____

   (a) 1/4
   (b) 1/3
   (c) 1/2
   (d) 2/3 ✓✓
   (e) 3/4

14. For a wet bulb thermometer operating ideally

   (a) a steady state prevails
   (b) the energy balance of the thermometer is overwhelmingly dominated by the convective terms Q_H, Q_E
   (c) magnitude of the net radiation to the thermometer $|Q_*| << |Q_H|, |Q_E|$
   (d) $|Q_H| = |Q_E|$
   (e) all of these statements are true of an ideal wet bulb thermometer ✓✓

15. A familiar example of a dimensionless number is the ratio $\pi = C/D$ of the circumference C of a circle to its diameter D. We say the circumference "scales on diameter". In flow problems, we may scale the product $U\ L$ of a characteristic velocity U and a characteristic length L on the kinematic viscosity ν of the fluid, to form a dimensionless number UL/ν which is called the

   (a) Grashof number
   (b) Prandtl number
   (c) Raleigh number
   (d) Reynolds number ✓✓
   (e) Fourier number

16. If we estimate the values of variables A, B to be $A = a \pm \epsilon_a$, $B = b \pm \epsilon_b$, then the absolute uncertainty in our estimate $a - b$ of the difference $A - B$ is correctly expressed as ____

   (a) $\epsilon_a - \epsilon_b$
   (b) $\epsilon_a + \epsilon_b$ ✓✓
   (c) $(a - b) * [1 \pm (\epsilon_a/a + \epsilon_b/b)]$
   (d) $(a - b) * [1 \pm (\epsilon_a/a - \epsilon_b/b)]$
   (e) $(a - b) * [1 \pm (\epsilon_a - \epsilon_b)]$

17. Suppose the radiation sensor in an infra-red CO_2 analyzer has time constant τ. A light chopper, rotating at frequency f [cycles s^{-1}], alternately focuses on the detector a beam of radiation that has passed through a reference chamber in which the concentration of CO_2 is zero, and a beam that has passed through the sample chamber. The amplitude of the detector output, for constant (but non-zero) CO_2 concentration, will be a maximum if ____

   (a) $f = \tau$
   (b) $f >> 1/\tau$
   (c) $f = 1/\tau$
   (d) $\tau >> 1/f$
   (e) $\tau << 1/f$ ✓✓

18. If the mean value m_x of a random variable "x" is calculated from a sample of size N ($N > 20$), ie. as $m_x = (x_1 + x_2 + x_3 + ... + x_N)/N$, then if "$x$" has standard deviation σ_x, the standard deviation of the mean m_x is

   (a) $N\ \sigma_x$
   (b) σ_x/N
   (c) $\sqrt{N}\ \sigma_x$
   (d) $\sigma_x/\sqrt{N}$ ✓✓
   (e) none of the above

19. As far as its spectral properties are concerned, the photon beam used by an electromagnetic absorption gas analyser to determine concentration of a gas "G" in the photon path is most likely

   (a) a wide band $\lambda_1 \leq \lambda \leq \lambda_2$ in the ultraviolet
   (b) a narrow band $\lambda_1 \leq \lambda \leq \lambda_2$ in the ultraviolet
   (c) a wide band $\lambda_1 \leq \lambda \leq \lambda_2$ in the infra red
   (d) a narrow band $\lambda_1 \leq \lambda \leq \lambda_2$ in the infra red
   (e) a narrow band $\lambda_1 \leq \lambda \leq \lambda_2$ within which "G" has an absorption peak ✓✓

20. Suppose the axis of a propellor anemometer lies along the x-axis parallel to a unidirectional stream with velocity u_x. According to our analysis of the function of a propellor anemometer, the pitch angle $\theta(r)$ of the blade, defined such that $\theta = 0^o$ if the wind is perpendicular to the surface of the blade, must vary with radius r along the blade according to $\theta(r) \; = 90 - \arctan(\omega r/u_x)$, where ω is the frequency of rotation of the blade (proportional to u_x, presumably). Therefore at the root of the blade ($r = 0$) the surface of the blade should lie

   (a) perpendicular to the fluid motion

   (b) parallel to the fluid motion ✓✓

   (c) at an angle of attack of 30^o relative to the fluid stream

   (d) at an angle of attack of 45^o relative to the fluid stream

   (e) at an angle of attack of 60^o relative to the fluid stream

Short Answer (10 %)

Answer any **two** questions from this section. Give diagrams where appropriate to clarify your working, which should be shown. Justify any assumptions or simplifications you make. (**Answers at end of document**.)

1. Prove by substitution that the function

$$C(x,z) = \frac{\alpha}{\beta\, x}\, \exp\left(-\,\frac{z\,U}{\beta\, x}\right) \tag{1}$$

(where α, β, U are positive constants) is a solution of the partial differential equation

$$0 = -\frac{\partial}{\partial x}\,(UC) - \frac{\partial}{\partial z}\,\left(-\,\beta\, z\,\frac{\partial C}{\partial z}\right) \tag{2}$$

Comment as completely as you are able, but without specific interpretation on the physical situation to which it may apply, on the mathematical nature of this differential equation: eg. what is its order?, which are the "independent" variables?, anything notable about the character of the coefficients?, etc.

2. The torque balance on a cup anemometer may be written

$$I\,\frac{d^2\theta}{dt^2} = \Gamma \tag{3}$$

where I is the moment of inertia, $\ddot{\theta} = d^2\theta/dt^2$ is the rate of angular acceleration of the cups ($\dot{\theta} = d\theta/dt$ is the angular velocity of the cups), and Γ is the aerodynamic torque.

a) Let γ denote the ratio u_c/S of the tangential velocity of the cups to the windspeed S. If the anemometer is in steady state motion in a steady stream,

$$\gamma^2 - 2G\gamma + 1 = 0 \tag{4}$$

where

$$G = \frac{c_{df} + c_{db}}{c_{df} - c_{db}} \tag{5}$$

Determine the steady state calibration factor γ if the drag coefficients for front and back sides of the cups are $c_{df} = 0.75,\ c_{db} = 0.5$.

b) Suppose the moment of inertia of the cup assembly is $I = 10^{-4}$ kg m^2, that the cups have diameter $d = 0.05$ m, and that the aerodynamic drag of the wind upon them acts at radius $r = 0.1$ m from the axis of rotation. Using a 2-cup model, and assuming the wind blows directly at the front and back sides (respectively) of the two cups, compute the angular acceleration $\ddot{\theta}$ [radians s^{-2}] at the instant $t = 0$ when the cups are *released from rest* in an airstream of density $\rho = 1$ kg m^{-3} and speed $S = 2$ m s^{-1}.

c) Taking the lower of your two estimates for the steady state calibration factor $\gamma = u_c/S$, and noting that $u_c = r\,\dot{\theta}$, crudely estimate the time τ required to reach steady rotation rate in this (steady) wind, by assuming the initial acceleration *persists*.

3. The "Bowen-ratio energy-balance" method for determining evapotranspiration (E) can be expressed by the equations

$$\begin{aligned} E &= \frac{Q^* - Q_G}{L_v\,(1 + \ \beta)} \\ \beta &= \frac{Q_H}{Q_E} \end{aligned} \tag{6}$$

where β is the Bowen ratio, L_v is the latent heat of evaporation, and the notation for the terms in the energy balance is conventional.

Given the wet- and dry-bulb temperatures of Table (1), determine the evapotranspiration rate if $Q^* = 500$ W m^{-2}, and $Q_G = 50$ W m^{-2}. Assume the pressure $p = 93$ kPa.

Table 1: Data to be used for calculation of the Bowen ratio.

z [m]	T[C]	T_w[C]
2.5	20.5	14.5
2.0	21.0	15.0

Data:

- Kinematic viscosity of air: $\nu \approx 1.5 \text{ x } 10^{-5}$ [m^2 s^{-1}]
- Thermal diffusivity[1] of air: $\kappa \equiv D_H \approx 2.1 \text{ x } 10^{-5}$ [m^2 s^{-1}]
- Specific heat capacity of air at constant pressure: $c_p \approx 1000$ [J kg^{-1}K^{-1}]
- Latent heat of vaporization: $L_v = 2.5 \text{ x } 10^6$ [J kg^{-1}]
- Saturation vapour pressure $e_S(T)$ [Pa] versus temperature T [C]:

$$e_S(T) = 611.2 \;\exp\left(\frac{17.67\;T}{243.5 + T}\right) \tag{7}$$

- The equation to determine the vapour pressure e [Pa] from wet- and dry-bulb temperatures T_w, T is

$$e = \;e_S(T_w) - \;\gamma\;(T - T_w) \tag{8}$$

- Psychrometric constant

$$\gamma = \frac{p\;c_p}{0.622\;L_v} \tag{9}$$

- Ideal gas law: $p = \rho\;R\;T$

  p (or P) [Pascals], pressure; ρ, [kg m^{-3}] the density; T [Kelvin], the temperature; and $R = 287$ [J kg^{-1} K^{-1}], the specific gas constant for air.

- Ideal gas law for water vapour: $e = \rho_v\;R_v\;T$

  e [Pascals], partial pressure; ρ_v, [kg m^{-3}] the absolute humidity; T [Kelvin], the temperature; and $R_v = 462$ [J kg^{-1} K^{-1}], the specific gas constant for water vapour.

- $L \uparrow = \;\epsilon\,\sigma\,T^4$

  Stefan-Boltzmann law. $L \uparrow$ [$W\;m^{-2}$], the emitted longwave energy flux density; ϵ, the emissivity of the surface (dimensionless); $\sigma = 5.67 \text{ x } 10^{-8}$ [$W\;m^{-2}\;K^{-4}$], the Stefan-Boltzmann constant; T [K], the surface temperature.

- $Q_H = \rho\;c_p\;\frac{T_1 - T_2}{r_H}$

  Ohm's law model for sensible heat exchange.

- $E = \frac{\rho_{v1} - \rho_{v2}}{r_v}$

  Ohm's law model for water vapour exchange (ρ_v the absolute humidity).

[1] Symbols κ, D_H are both used for this quantity.

- $F = c_d \, A \, \rho \, U^2$

  Aerodynamic force (F, Newtons) on a body whose "projected frontal area" is A, when placed in a fluid stream of density ρ and speed U. The dimensionless coefficient c_d is called the "drag coefficient." The projected frontal area is the area projected on the plane normal to the fluid stream.

- $Q^* = Q_H + Q_E + Q_G + Q_S$

  The surface energy balance. All fluxes are in $[W \; m^{-2}]$. Q^* the net radiation, positive if directed towards the ground surface; Q_H, Q_E the sensible heat flux and the latent heat flux, positive if directed away from the ground surface; Q_G the soil heat flux, positive if directed away from the ground surface; Q_S, the storage term.

- If x is governed by the quadratic equation $a \, x^2 + b \, x + c = 0$ then the following formula gives the two possible solutions for its numerical value

$$x = \frac{-b}{2a} \pm \frac{1}{2a} \sqrt{b^2 - 4 \, a \, c} \tag{10}$$

Answers to Short Answer Section

1. First question was fairly mechanical substitution (along the same lines of working as the practise example given on the web site). Those who tried this tended to get good scores on it... crucial to be tidy so instructor can check your working.

2. By substitution, $\boxed{G = 5}$ (1 mark)

   Solving the quadratic by the formula given, $\boxed{\gamma = 5 \pm 4.90}$ (1 mark)

   You were given the torque balance, and the equation for aerodynamic drag... making appropriate simplifications, since the cups are at rest, one has (1 mark for using the following equation, even if you did not explicitly write it out in this form):

   $$\boxed{I\,\ddot{\theta} = \; r\;A\;\rho\;S^2\;\;(c_{df} - c_{db})} \tag{11}$$

   where $A = \pi(d/2)^2 = 1.963$ x 10^{-3} m^2. Many forgot to put in the factor r (radius, ie. distance to point of action of the aerodynamic force) on the r.h.s. and many did not correctly calculate the area of the circle with diameter d ("*that* is a very serious crime - but there is no need for a warrant from Bombay"...)

   Putting in the numbers, $\boxed{\ddot{\theta} = 1.96 \text{ radians s}^{-2}}$ (1 mark)

   Cup speed for the lower solution is $u_c = (5 - 4.90)S = 0.2$ m s^{-1} and therefore since $u_c = r\,\dot{\theta}$, we have $\boxed{\dot{\theta} = 2 \text{ radians s}^{-1}}$. Given the solution above for angular acceleration, the time taken to "spin up" to this rotation rate $\dot{\theta} = 2$ radians s^{-1} is $\boxed{\tau = \dot{\theta}/\ddot{\theta} = 1 \text{ s}}$ (1 mark)

   If correct answers were not obtained, part marks were assigned for useful (and correct) reasoning, or diagrams... don't forget (too) to give units...!

3. We see right away that if we can determine the Bowen ratio β then the rest is easy... and you are given the formula for β!...

   Q_H should be easy, as we know $Q_H = \rho\; c_p\; \Delta T/r_H$... and since we have pressure p given and the temperature is about 293 K, $\boxed{\rho = 1.11 \text{ kg m}^{-3}}$. But is not knowing r_H going to be a problem? Wait and see...

   We know $Q_E = L_v E = L_v \Delta\rho_v/r_V$. So if we *assume* $\boxed{r_H = r_v}$ then

   $$\boxed{\beta = \;\frac{\rho c_p}{L_v}\,\frac{\Delta T}{\Delta \rho_v}} \tag{12}$$

   The ΔT is given immediately from the data table. But we'll need to compute ρ_v for each height... how?... first get the vapour pressure e for each level. For example at $z = 2$ m we have $\boxed{e(2) = e_*(15) - \gamma\;(21 - 15)} \;=\; \boxed{1704} - \gamma\;6$ where one will have needed to use the formula for saturation vapour pressure versus temperature (given).

   Clearly though, we need gamma: the formula was given. You know pressure p so,

$\boxed{\gamma = pc_p/(0.622L_v) = 59.8 \text{ Pa C}^{-1}}$.

Thus $e(2) = 1.345$ kPa from which $\rho_v(2) = 1345/(462 * 293) = 9.94$ x 10^{-3} kg m^{-3}.

By the same steps, $\rho_v(2.5) = 9.54$ x 10^{-3} kg m^{-3}. Substituting, $\boxed{\beta = 0.56}$... so $Q_E = 288$ W m^{-2} and dividing by L_v one has $E = 1.2$ x 10^{-4} kg m^{-2} s^{-1}