

1. General comments regarding NWP Models

Integration times (ie. forecast range) short (48 hours to 2 weeks or less) so no need to include processes active in climate simulations, like (eg.) CO₂ cycle.

Key properties:

- Domain - global, hemispheric, regional?
 - Resolution - horiz (Δ) and vertical
 - Lateral boundary conditions (if needed)
 - Dynamics - hydrostatic (considered inapprop. for $\Delta < 10$ km) or non-hydrostatic?
 - Hor. discretization - finite element, finite difference, spectral?
 - Vertical coordinate - usually related to p/p_{sfc} – and discretization
 - Representation of terrain
 - Coupling to lower boundary - static ocean?, cryosphere?, vegetation?..
 - Initialization and data-assimilation (4D-Var now usual)
 - Numerics – e.g. order of approx. of operators, control of numeric noise?
 - Parameterizations for unresolved processes (“model physics”)
 - solar and longwave radiation
 - unresolved scales of motion (turbulence in friction layer, and above)
 - convective cloud, stratiform cloud
 - gravity wave drag
- } trade-off
- } related issues

1.1 Aside on dynamics

Hydrostatic approximation not realistic if aim is to resolve atmosphere down to scales on which convection occurs. Let total pressure $p = p_0 + \tilde{p}$ where $p_0(z)$ denotes the pressure of a hydrostatic reference atmosphere

- under Boussinesq** approx., vertical acceleration of a parcel depends on deviations \tilde{T}, \tilde{p} of the parcel's state from the reference state p_0, T_0 at that level...

$$\frac{d w}{d t} = - \frac{1}{\rho_0} \frac{\partial \tilde{p}}{\partial z} + g \frac{\tilde{T}}{T_0}$$

vert. accel'n PGF buoyancy

- versus hydrostatic approximation

$$0 = - \frac{1}{\rho} \frac{\partial p}{\partial z} + g$$

**Boussinesq approx. suitable for shallow layer (ABL) only. Latest models (e.g. WRF) fully compressible

Molinari (1993; in *Representation of Cumulus Convection in Numerical Models*, Am. Meteor. Soc.) defines mesoscale models as hydrostatic models with horiz. gridlength $10 \leq \Delta \leq 50$ km

By this criterion both the Global (33 km) and Regional (15 km) runs of CMC's GEM (Global Environmental Multiscale) NWP model are mesoscale models...

“At a grid spacing of 10 km, the grid scale approaches the preferred scale for instability of convection in nature.”
(Molinari)

Zonal momentum equation (in Cartesian coords.)

Will use upper case, or where more convenient an overbar, to denote the resolved scale variables, which in principle are volume averages

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} + W \frac{\partial U}{\partial z} = \underbrace{\frac{-1}{\rho} \frac{\partial P}{\partial x}}_{\text{if all other terms vanished, we'd have the geostrophic wind}} + f V + F_u$$

non-linearity

if all other terms vanished, we'd have the geostrophic wind

friction: influence of unresolved scales

or using the Lagrangian derivative

$$\frac{dU}{dt} = \frac{-1}{\rho} \frac{\partial P}{\partial x} + f V + F_u$$

The friction term is (formally) the divergence of the unresolved momentum flux

$$F_u = \frac{\partial \overline{u'u'}}{\partial x} + \frac{\partial \overline{v'u'}}{\partial y} + \frac{\partial \overline{w'u'}}{\partial z}$$

these terms neglected

vertical gradient of the mean vertical flux of u -momentum carried by the unresolved scales of motion

Canadian Meteorological Centre's

Global Environmental Multiscale

NWP model

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CMC's GEM in Regional Configuration

- run at 00, 06, 12, 18Z with forecast range to 48 hours (or 54 hours)
- global domain*
- primitive equations model, formulated in “horiz.” velocity components (U, V), the vertical “velocity” $\dot{\eta} \equiv d\eta/dt$, the virtual temperature** T_v and specific humidity Q
- hydrostatic (a non-hydrostatic version is applied on urban scale)
- horizontal resolution uniform over focal area $\Delta = 15 \text{ km}$ (0.135° lat.), non-uniform outside N. America (has Δ been refined to 10 km?)

- vertical coord
$$\eta = \frac{P - P_T}{P_S - P_T} \quad (P_S \text{ surface evolves, } P_T \text{ top fixed; } 1 \geq \eta \geq 0)$$

- 58 levels, 10 below 850 hPa, top level $P_T = 10 \text{ hPa}$

- timestep **7.5 min**

*even the “regional run” has global domain, but resolution is low outside focal area

**Temperature of dry air having same P and ρ as sample:
$$T_v = T (1 + 0.61 Q)$$

- in the (present) operational hydrostatic GEM the coordinate η is based on total pressure
- but in the (already existing) non-hydrostatic version it is based on the dry, hydrostatic component of the pressure (see NAM/Wrf model later) as introduced by Laprise* (1992, MWR Vol. 120). Note that $\dot{\eta} = d\eta/dt = 0$ at the surface ($\eta = 1$) and top of model domain ($\eta = 0$)

Dynamics/physics terminology

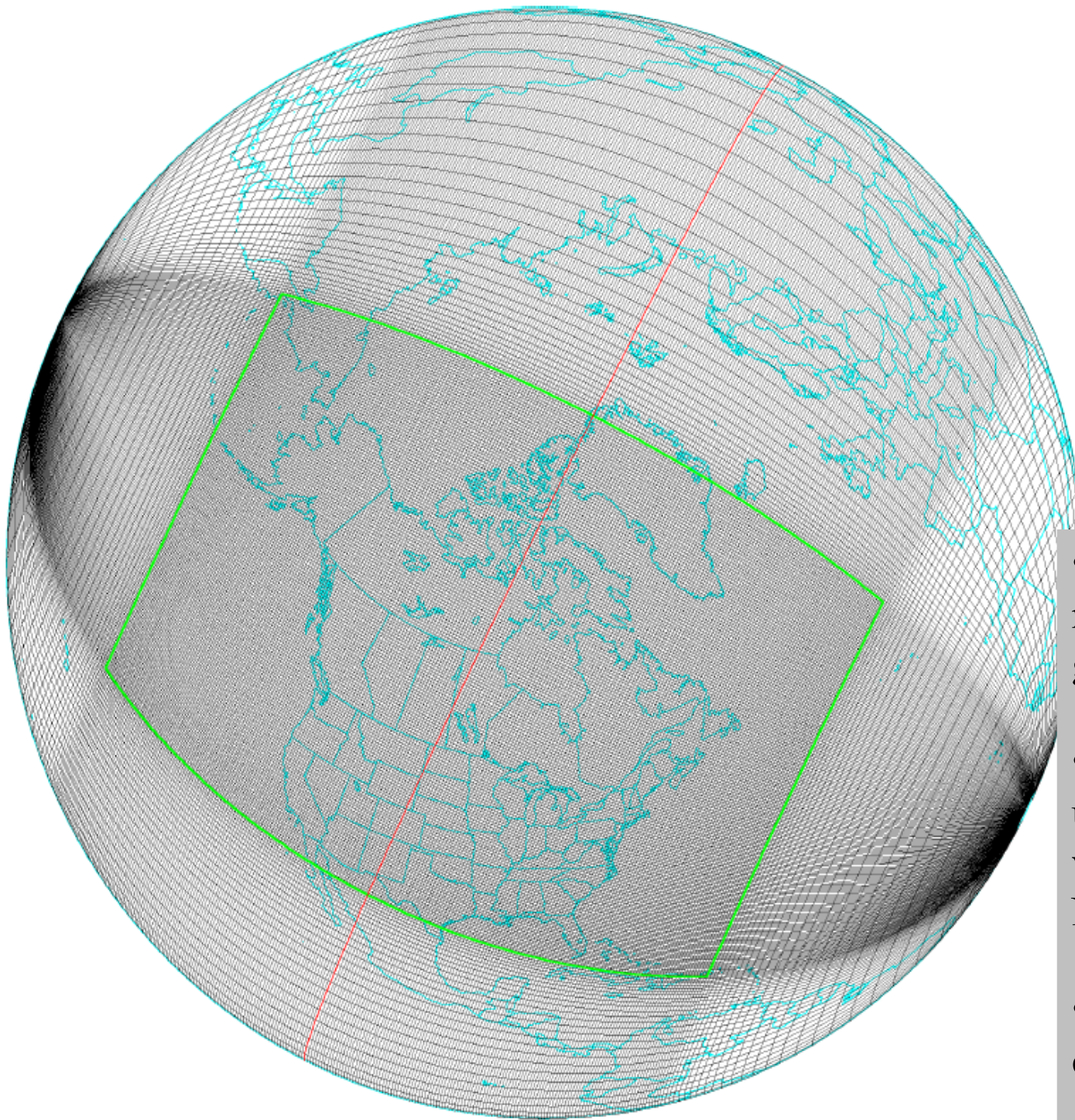
$$\frac{\partial U}{\partial t} = \underbrace{-U \frac{\partial U}{\partial x} - V \frac{\partial U}{\partial y} - W \frac{\partial U}{\partial z} - \frac{1}{\rho} \frac{\partial P}{\partial x} + f V + F_u}_{\text{dynamics}}$$

dynamics



$$\frac{\partial U}{\partial t} = \left(\frac{\partial U}{\partial t} \right)_{\text{dyn}} + \left(\frac{\partial U}{\partial t} \right)_{\text{phy}}$$

Configuration for
the regional run:
uniform high res
over N. America



- 575 x 641 variable-resolution global lat-long grid
- 0.1375° (~ 15 km) uniform-resolution window covering most of N. America
- 432 x 565 grid points in central window
- periodic horiz. b/conds

Schematic for horizontal placement of variables, displayed around a pole.

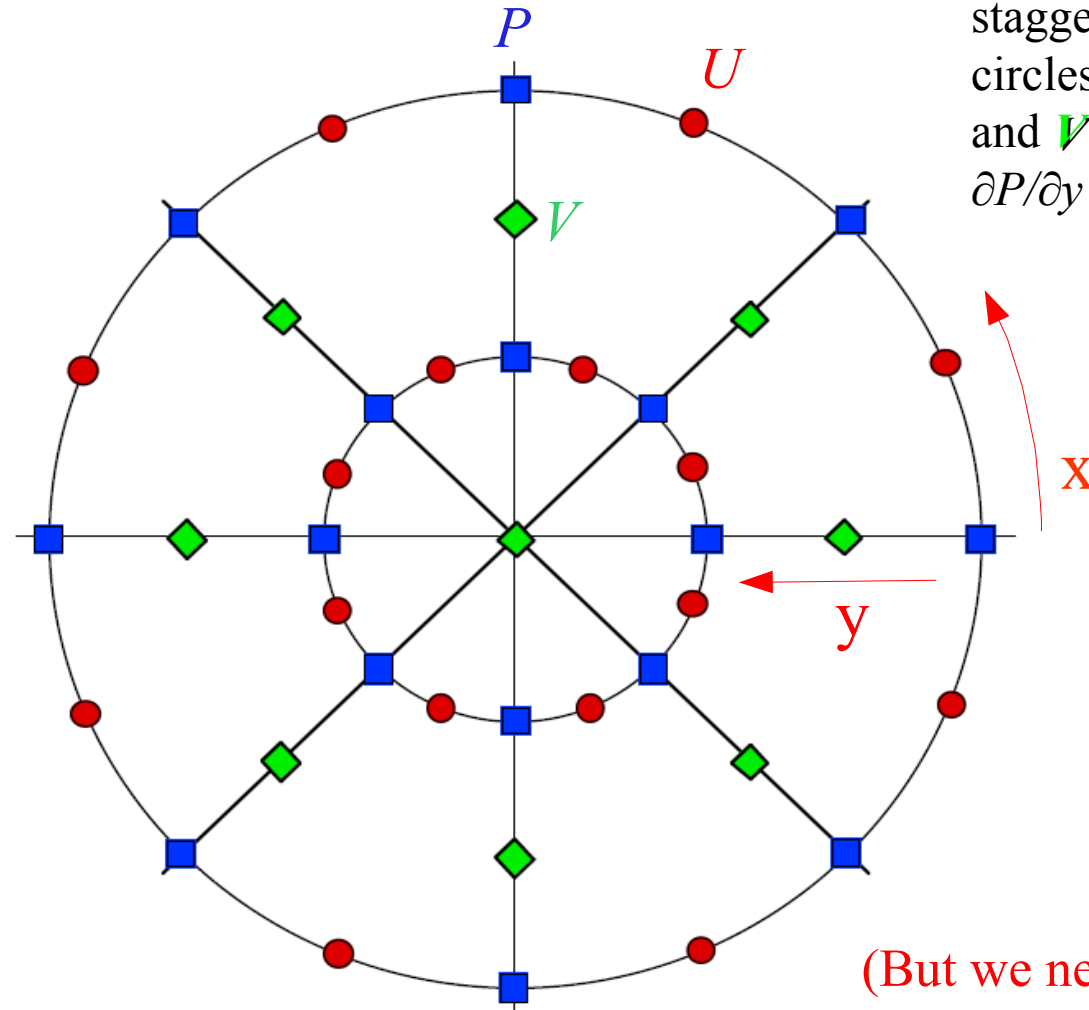
Squares: scalar fields. Circles: $U = \lambda$ – component of wind images.

Diamonds: $V = \theta$ – component of wind images.

- the domain is separated by imaginary lines into a number of finite elements

- elements are assumed interconnected at a discrete number of nodal points on their boundaries. Values of $U, V, W, P...$ at the nodes are the basic unknowns (resolved variables)

- an interpolating function is used to provide the values of U, V (etc.) wherever needed within each finite element in terms of nodal values (e.g. at U gridpoints we need V to compute $V \partial U / \partial y$)

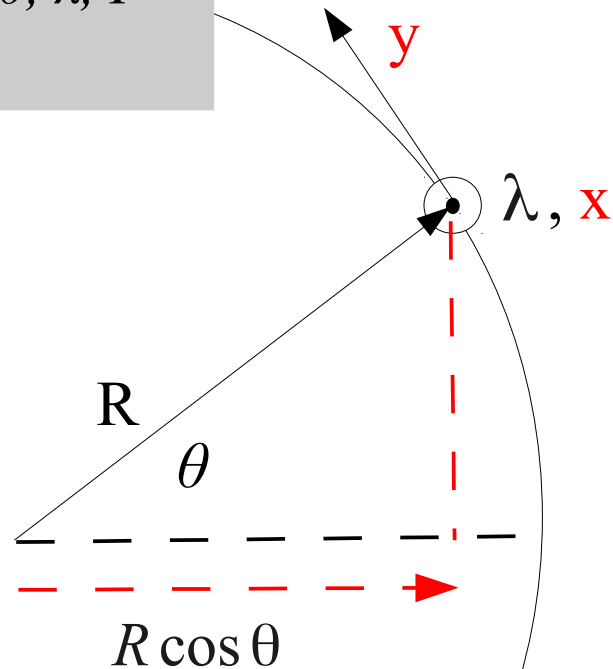


Each cell is a “finite element.” With pressure P placed at the blue squares we have nice staggering for U (red circles, $\partial P / \partial x$ needed) and V (green diamonds, $\partial P / \partial y$ needed)

(But we need to transform this into θ, λ, P coords)

$$\left(\frac{\partial U}{\partial t} \right)_{\text{dyn}} = -U \frac{\partial U}{\partial x} - V \frac{\partial U}{\partial y} - W \frac{\partial U}{\partial z} - \frac{1}{\rho} \frac{\partial P}{\partial x} + f V$$

Transforming the eqn for resolved U -mtm into θ, λ, P coordinates:



Following terminology of the previous page, let λ be longitude and let $d\lambda$ be an increment in longitude. The corresponding increment in x is:

$$dx = (R \cos \theta) d\lambda$$

$$\text{so } \frac{\partial}{\partial x} = \frac{1}{R \cos \theta} \frac{\partial}{\partial \lambda}$$

$$\text{Similarly } dy = R d\theta$$

$$\text{so } \frac{\partial}{\partial y} = \frac{1}{R} \frac{\partial}{\partial \theta}$$

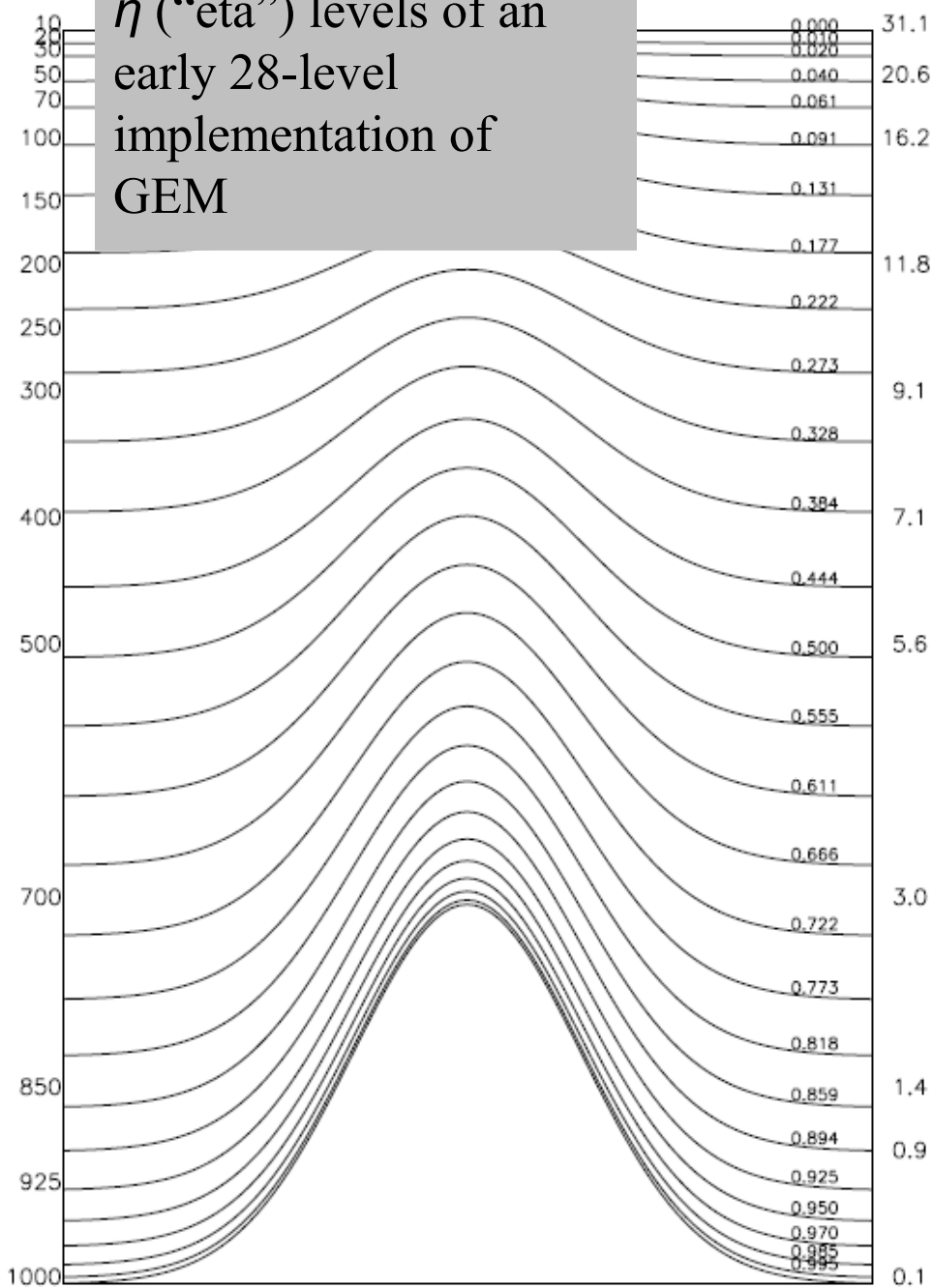
$$\frac{\partial}{\partial z} = -\rho g \frac{\partial}{\partial P}$$

$$\left(\frac{\partial U}{\partial t} \right)_{\text{dyn}} = -\frac{U}{R \cos \theta} \frac{\partial U}{\partial \lambda} - \frac{V}{R} \frac{\partial U}{\partial \theta} + \rho g W \frac{\partial U}{\partial P} - \frac{1}{\rho R \cos \theta} \frac{\partial P}{\partial \lambda} + f V$$

And the effect of unresolved motion?
Treated as:

$$\left(\frac{\partial U}{\partial t} \right)_{\text{phy}} \approx -\frac{\partial \overline{w'u'}}{\partial z} = \frac{\partial}{\partial z} \left(K \frac{\partial U}{\partial z} \right) = \rho g^2 \frac{\partial}{\partial P} \left(\rho K \frac{\partial U}{\partial P} \right)$$

η (“eta”) levels of an early 28-level implementation of GEM



Analysis levels
(hPa)

Standards heights
(Km)

Indexing

$U(I,J,K,n)$

- I longitude
- J latitude
- K altitude
- n time

Derivatives can be approximated by finite differences, with the result that these governing differential equations are transformed into a coupled set of non-linear algebraic equations

$$\frac{\partial U}{\partial x} = \frac{U(I+1,J,K,n) - U(I-1,J,K,n)}{x(I+1,J) - x(I-1,J)}$$

From: Belair et al., 2005,
Monthly Weather Review

These specs. pertain to the twice-daily Global runs (00Z run to 10 days; 12Z run to 6 days*) – note the coarser resolution and timestep relative to the Regional run

*A 15-day run is made on Saturdays

TABLE 1. Summary of the GEM forecast system.

Dynamics/numerics
<ul style="list-style-type: none">● Hydrostatic primitive equations;● Global uniform resolution of 0.45° longitude and 0.30° latitude (800×600);● Variable vertical resolution with 58 levels; model top at 10 hPa;● Time step of 900 s (i.e., 15 min);● Cell-integrated finite-element discretization on Arakawa C grid;● Terrain-following hydrostatic pressure vertical coordinate;● Two-time-level semi-implicit time scheme;● 3D semi-Lagrangian advection; (see over)● ∇^6 horizontal diffusion on momentum variables; increased horizontal diffusion (sponge) for the four uppermost levels;● Periodic horizontal boundary conditions;● No motion across the lower and upper boundaries.
Physics
<ul style="list-style-type: none">● Planetary boundary layer based on TKE with statistical representation of subgrid-scale cloudiness (MoisTKE);● Fully implicit vertical diffusion;● Stratified surface layer, distinct roughness lengths for momentum and heat/humidity;● Four types of surface represented: land, water, sea ice, and glaciers;● Solar/infrared radiation schemes with cloud-radiation interactions based on predicted cloud radiative properties;● Kuo Transient scheme for shallow convection;● Kain–Fritsch scheme for deep convection;● Sundqvist scheme for nonconvective condensation.

Semi-Lagrangian treatment of advection?

Strategy to overcome limitation imposed by the Courant condition, which demands

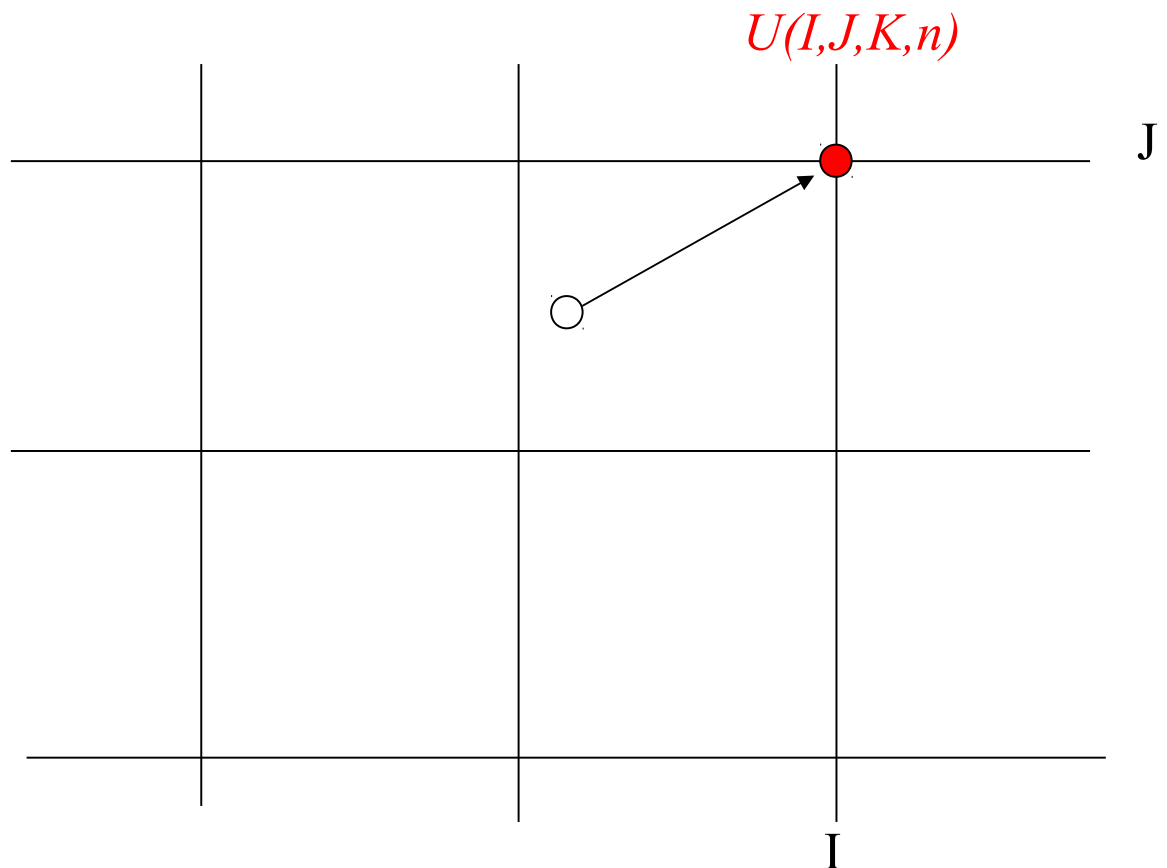
$$\frac{|U| \Delta t}{\Delta x} \leq 1$$

$$\frac{|V| \Delta t}{\Delta y} \leq 1$$

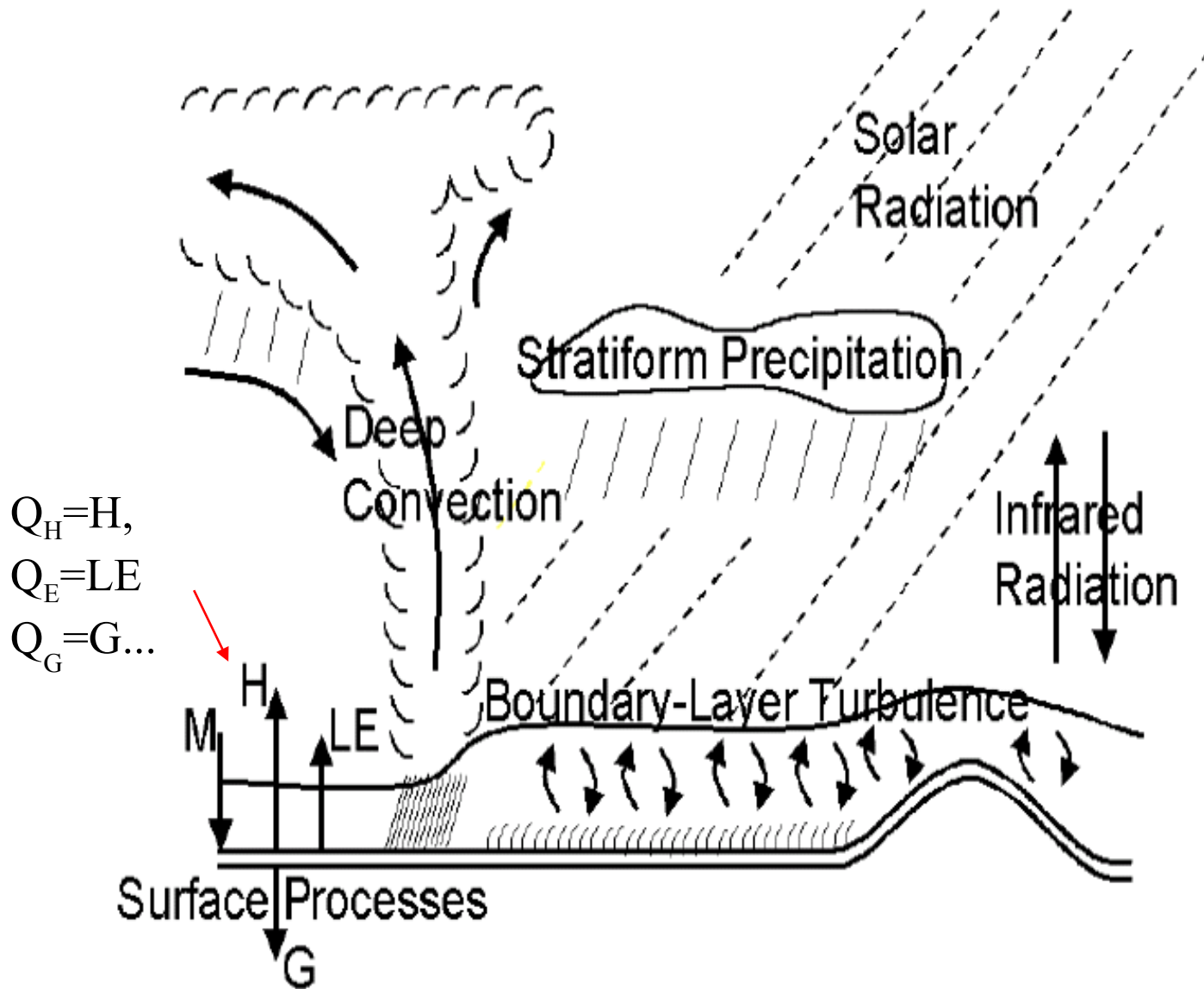
Evaluate this with
 $\Delta x = 33 \text{ km}$
 $\Delta t = 900 \text{ s}$
 $U = 50 \text{ m s}^{-1}$

Computed path of a fluid element backwards in time from t to $t - \Delta t$ such that $U(I, J, K, n)$ is evaluated by taking the value (evaluated by cubic interpolation from the gridded values) at the upwind point (open circle) for time level $n-1$

- of course other factors, notably pressure gradient and Coriolis force, demand an adjustment to this advected value*



3. Overview of Physical Processes ^{*} parameterized in GEM



Thanks to Stephane Belair (CMC) for permission to use this and other sketches

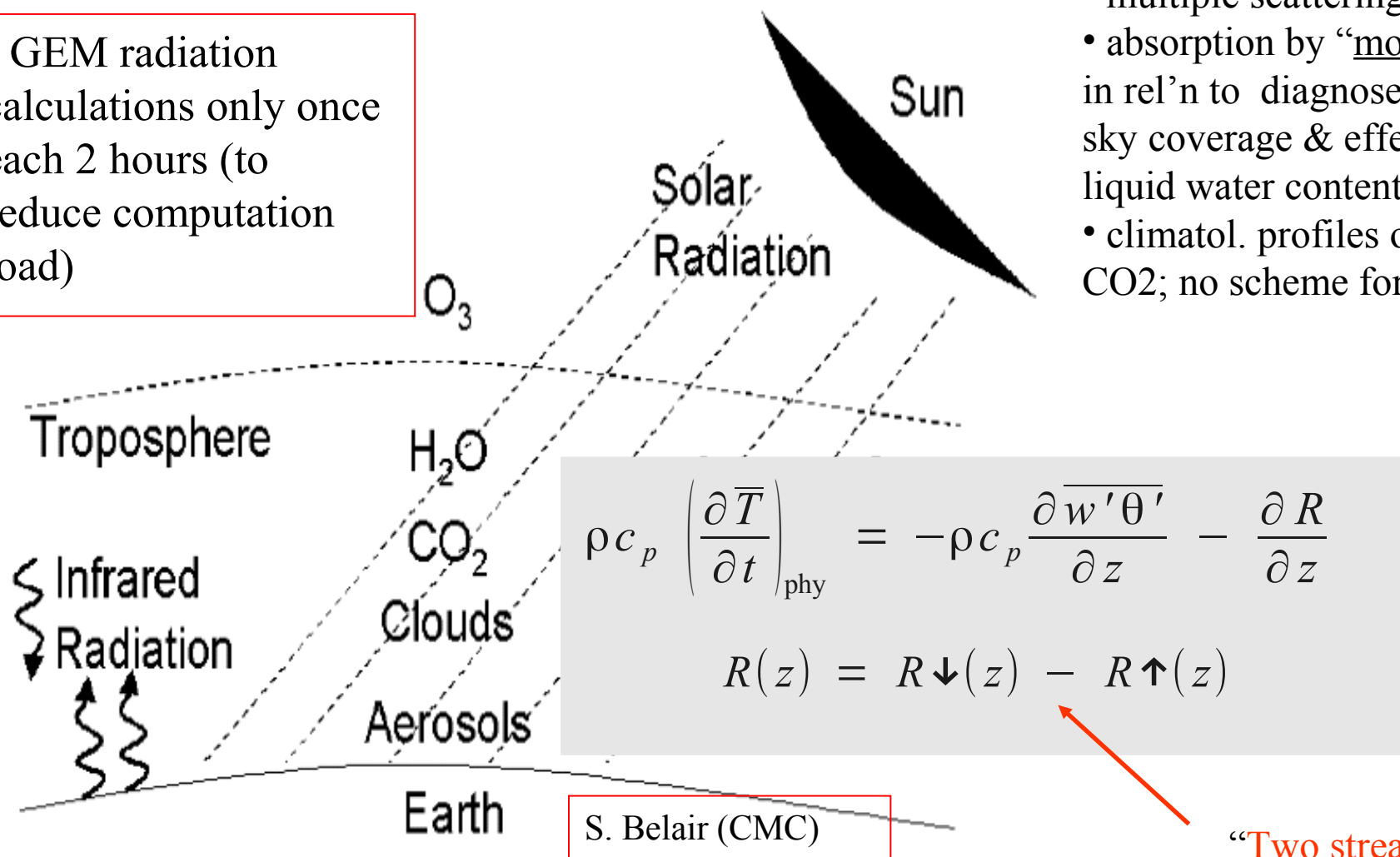
^{*} including effects of unresolved (sub-grid scale) motion

3.1 Atmospheric Radiation

- GEM radiation calculations only once each 2 hours (to reduce computation load)

SOLAR

- single waveband
- sun-earth geometry
- multiple scattering
- absorption by “model clouds” in rel’n to diagnosed fractional sky coverage & effective cloud liquid water content
- climatol. profiles of ozone, CO₂; no scheme for aerosols

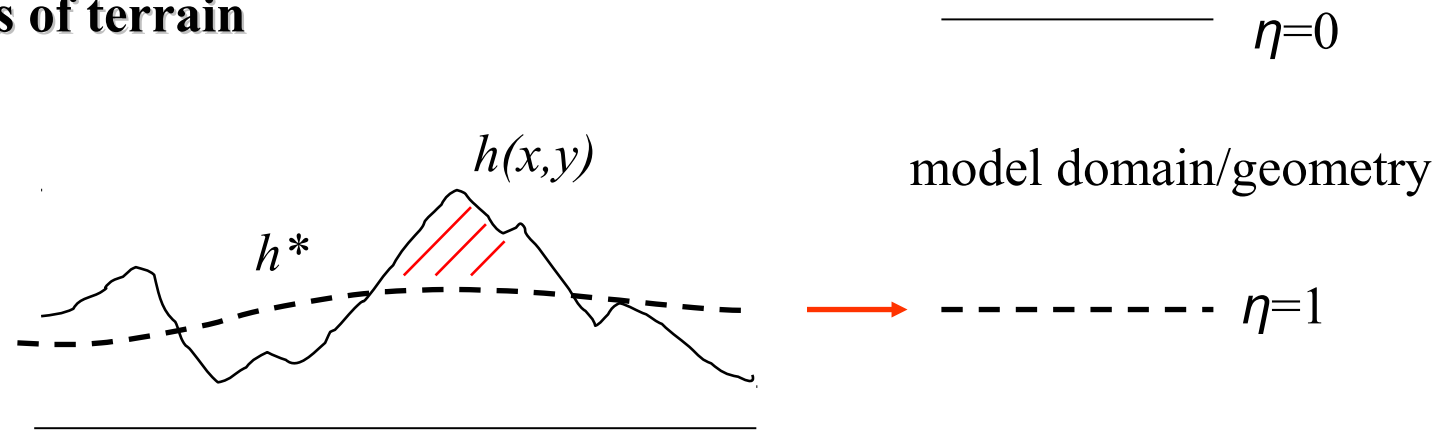


“Two stream model”
(R is the net radiation)

LONGWAVE

- four wavebands; interaction with water vapour, O₃, CO₂, clouds
- climatological O₃; [CO₂] treated as uniform

3.2 Explicit effects of terrain



Resolved terrain $h^*(x,y)$ “disappears” in the (terrain following) eta (η) coordinate system. How are mountains “felt”? New terms appear in the momentum equations when they are transformed into the η coord. system (“metric terms”)*

Additional - parameterized - effects of terrain

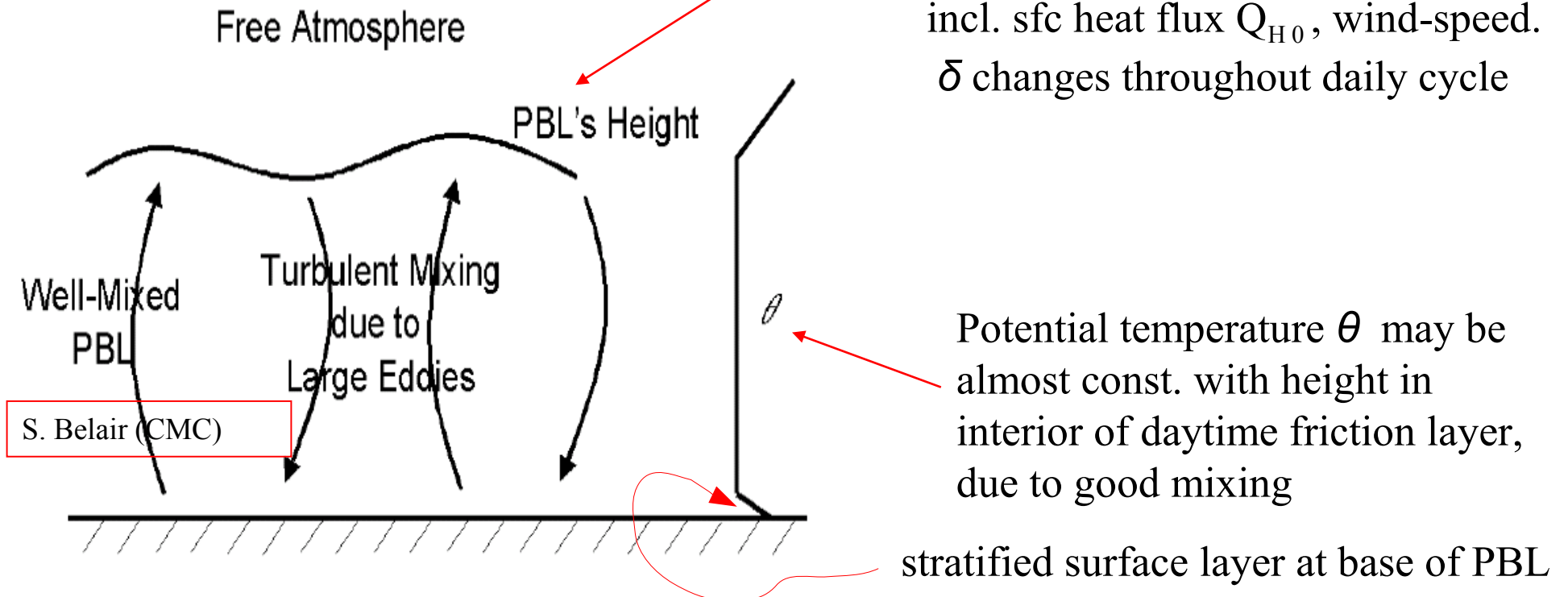
- gravity wave drag slows stratospheric winds (GEM dynamical eqns suppress gravity waves, thus need for parameterization; Dr. Sutherland EAS/Physics supervising studies of this)
- “blocking” parameterization recognizes influence of unresolved terrain - reduces the low level winds in mountainous regions*

*e.g. Wilson (2002; J. Atmos. Sci., Vol. 59)

3.3 Boundary-layer (ie. friction layer) turbulence

- sub-grid scale motion transports heat, vapour, momentum... (eg. transports and redistributes heat and vapour added at ground). Consider vertical exchange only, i.e. the “grid-point computations” involve local column only, no lateral coupling.
- in analogy with molecular mixing, subgrid transport is represented as “diffusion.” Eddy diffusivity K is function of kinetic energy of turbulence, and stratification

Daytime Boundary Layer



The mean vertical convective heat flux due to the unresolved vertical motion is simply the average of the $w'T'$ product (or one can equally write $w'\theta'$)... thus, the quantity $\overline{w'\theta'}$ is of interest. It is these unresolved fluctuations w' that carry heat, vapour, CO₂, etc. to and from the surface. They are important, but, unresolved. The eddy-diffusion model postulates that the direction of the mean flow of heat will be from warm to cold, and introduces as proportionality constant an “eddy diffusivity” (for heat) with the same units as, but vastly greater magnitude than, the molecular diffusivity. That is, one adopts the model

$$\overline{w'\theta'} = -K \frac{\partial \bar{\theta}}{\partial z}$$

(Dimensionally, K is [velocity x length]; numerically, it vastly exceeds the molecular diffusivity; furthermore, it is a property of the flow, not of the fluid itself)

Now we have the problem of how to rationally model the eddy diffusivity! Its magnitude must depend on some measure of the “amount” of vertical motion (loosely, of “mixing”), and this is often expressed by the “turbulent kinetic energy”

$$k = \frac{\sigma_u^2 + \sigma_v^2 + \sigma_w^2}{2}$$

(one half the sum of the variances of the unresolved velocity components)

The eddy diffusivity is typically written as $K \propto \lambda \sqrt{k}$, where λ is the “length scale”

3.4 GEM's coupling to the surface

- enforce surface energy balance $Q^* = K^* + L^* = Q_{H0} + Q_{E0} + Q_{S0}$
- detailed map of (time-evolving) surface type/condition
- prognostic variables for surface and soil temperatures, and soil moisture: working towards replacing “ISBA” scheme (Interaction Soil Biosphere Atmosphere) with “CLASS” (Canadian Land Surface Scheme**): three soil layers, vegetation canopy, interaction of radiation and vegetation canopy (surface albedo), vertical diffusion of heat and moisture between the soil layers, treatment of snow on canopy, inclusion of precip infiltration, runoff, and drainage
- static analyzed ocean/lake ice field and ocean/lake temperature (SST)
- lake/ocean surface roughness length (“ z_0 ”) responds to surface windspeed

3.5 GEM's treatment of clouds and precip – see table on a previous page

**in development since 1980's by a large team... incl. Dr. R. Grant (U.Alberta, Renewable Resources) a contributor. See Versegny (2000, *The Canadian Land Surface Scheme (CLASS): its history and future*, Atmos.-Ocean, Vol. 38, pp. 1-13)