A. First law of thermodynamics

$$dq = c_p\, dT - \alpha\, dp \qquad [\mathrm{J\ kg^{-1}}]$$

where $\alpha = 1/\rho$ "specific volume"

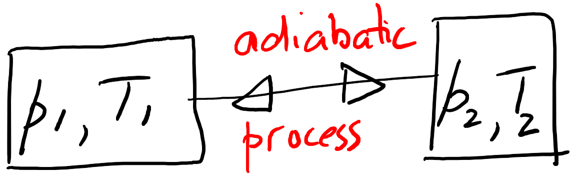
$$dq = c_v\, dT + p\, d\alpha$$

dq is heat added to the system

$$c_p - c_v = R_d \quad \text{(for ideal gas)}$$

$$c_p \simeq 1000\ \mathrm{J\ kg^{-1}\ K^{-1}}$$

Adiabatic process $dq = 0$



$$\left.\begin{aligned} 0 &= c_p\, dT - \frac{1}{\rho}\, dp \\ p &= \rho R_d T \end{aligned}\right\}$$

$$c_p\, dT = R_d T \frac{dp}{p}$$

$$\frac{dT}{T} = \frac{R_d}{c_p} \frac{dp}{p}$$

$$d \ln T = \frac{R_d}{c_p}\, d \ln p$$

$$\int_{T_1}^{T_2} d \ln T = \Big[\ln T\Big]_{T_1}^{T_2} = \ln T_2/T_1$$

$$= \frac{R_d}{c_p} \int_{p_1}^{p_2} d \ln p = \frac{R_d}{c_p} \ln p_2/p_1$$

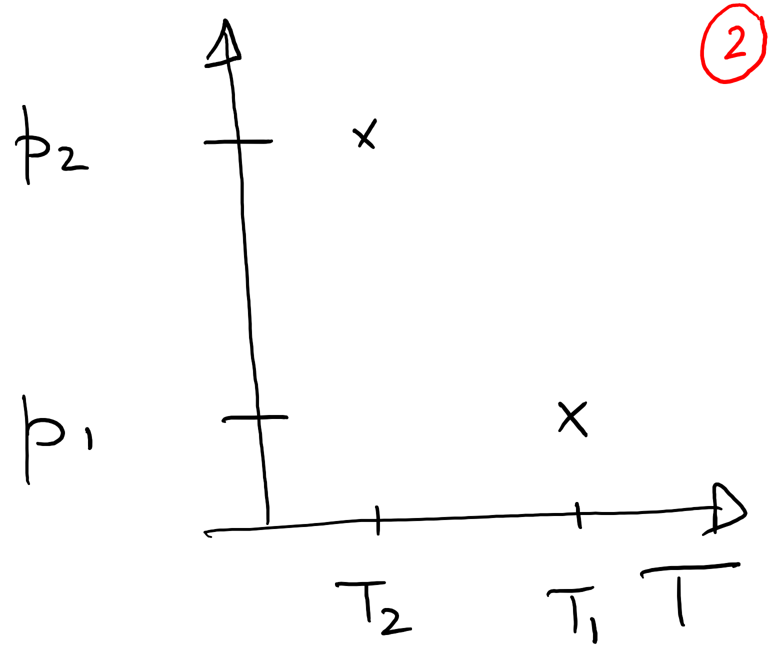
$$= \ln \left(p_2/p_1\right)^{R_d/c_p}$$

$$\frac{T_2}{T_1} = \left(p_2/p_1\right)^{R_d/c_p}$$

Call p_1 the "reference level"

(typically 1000 hPa)

$$T_1 = T_2 \left(p_1/p_2\right)^{R_d/c_p}$$



$$\theta = T\left(p_{ref}/p\right)^{R_d/c_p}, \quad R_d/c_p = 0.287 = \text{"}\gamma\text{"}$$

Dry adiabat on a thermodyamic chart is an isoline of potential temperature