# General comments regarding NWP Models

initial value problem

Integration times (ie. forecast range) short (~48 hours to 2 weeks or less), so processes with longer time scales excluded (whereas active in climate simulations, e.g. $CO_2$ cycle)

equilibrium problem

Key aspects:

- Domain - global, hemispheric, regional?
- Resolution – horiz. (Δ) and vertical

} trade-off

- Lateral boundary conditions (if needed)
- Dynamics - hydrostatic (considered inapprop. for $\Delta$<10 km) or non-hydrostatic?
- Horiz. discretization - finite difference, finite element, spectral?
- Vertical coordinate - usually related to $p/p_{sfc}$ – and discretization
- Representation of terrain

} related issues

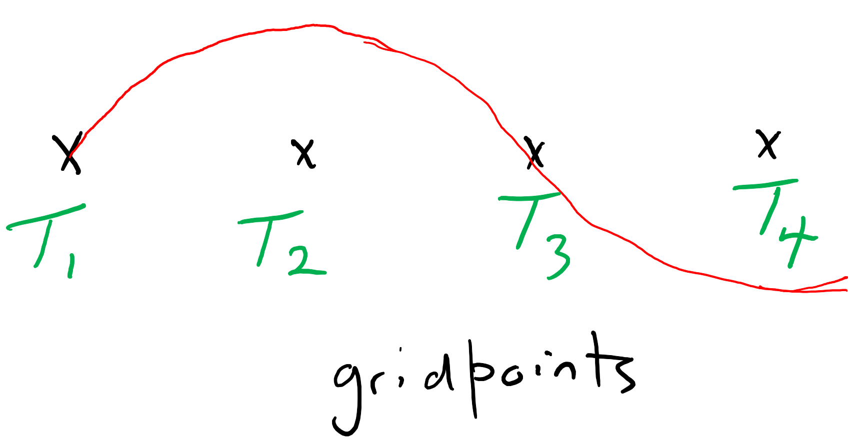
- Coupling to lower boundary - static ocean?, cryosphere?, vegetation?..
- Initialization and data-assimilation (4D-Var now usual)
- Numerics – e.g. order of approx. of operators, control of numeric noise?
- Parameterizations for unresolved processes ("model physics")
  - solar and longwave radiation
  - vertical transport by unresolved motion (esp. in friction layer)
  - unsaturated convection, convective cloud, stratiform cloud
  - coupling to surface (air- ground or ocean exchange fluxes)
  - gravity wave drag

eas372_NWP_CMC_2017.odp
JD Wilson, EAS, U. Alberta
Last modified 27 Mar., 2017

**Aside*** Consider an arbitrary function $f(x)$. Under very broad conditions it can be represented as a superposition of (perhaps a very large number of) sin and cosine waves, i.e. by a "Fourier series" ("Fourier decomposition")

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

We call the sine and cosine waves a (complete) "basis" in that any function in the "function space" can be built up this way.




spectral representation

or $T = A \sin\left[k(x - x_0)\right]$

phase

amplitude

wavenumber

* meaning of "spectral" discretization

# Aside on dynamics

Hydrostatic approximation not realistic if aim is to resolve atmosphere down to scales on which convection occurs. Let total pressure $p = p_0 + \tilde{p}$ where $p_0(z)$ denotes the pressure of a hydrostatic reference atmosphere

- under Boussinesq** approx., vertical acceleration of a parcel depends on deviations $\tilde{T}, \tilde{p}$ of the parcel's state from the reference state $p_0$, $T_0$ at that level...

$$\frac{d\,w}{d\,t} = -\frac{1}{\rho_0}\frac{\partial\,\tilde{p}}{\partial\,z} + g\,\frac{\tilde{T}}{T_0}$$

vert. accel'n PGF buoyancy

- versus hydrostatic approximation

$$0 = -\frac{1}{\rho}\frac{\partial\,p}{\partial\,z} + g$$

**Boussinesq approx. suitable for shallow layer (ABL) only. NWP models (e.g. WRF) fully compressible

Molinari (1993; in *Representation of Cumulus Convection in Numerical Models*, Am. Meteor. Soc.) defines mesoscale models as hydrostatic models with horiz. gridlength $10 \le \Delta \le 50$ km

RDPS GDPS

By this criterion both the Global (25 km) and Regional (10 km) runs of CMC's GEM (Global Environmental Multiscale) NWP model are mesoscale models…

"At a grid spacing of 10 km, the grid scale approaches the preferred scale for instability of convection in nature." (Molinari)

# Reynolds-averaged zonal momentum equation (in Cartesian coords.)

Will use upper case, or where more convenient an overbar, to denote the resolved scale variables, which in principle are volume averages

advection of $U$, *i.e.* $(\vec{U} \cdot \nabla) U$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} + W \frac{\partial U}{\partial z} = \frac{-1}{\rho} \frac{\partial P}{\partial x} + f V + F_u$$

non-linearity

if all other terms vanished, we'd have the geostrophic wind

friction: influence of unresolved scales

or using the Lagrangian derivative

$$\frac{dU}{dt} = \frac{-1}{\rho} \frac{\partial P}{\partial x} + f V + F_u$$

The friction term is (formally) the divergence of the unresolved momentum flux

$$F_u = -\frac{\partial \overline{u'u'}}{\partial x} - \frac{\partial \overline{v'u'}}{\partial y} - \frac{\partial \overline{w'u'}}{\partial z}$$

vertical gradient of the mean vertical flux of $u$-momentum carrried by the unresolved scales of motion

these two terms neglected in the "physics package" ("parameterizations") for GEM (and other NWP models)

## CMC's GEM – Global Environmental Multiscale – model: common elements

- primitive equations model, formulated in "horiz." velocity components (*U, V*), the vertical "velocity" $\dot{\eta} \equiv d\eta/dt$, the virtual temperature** $T_v$ and specific humidity $Q$

- hydrostatic *or* non-hydrostatic (hydrostatic for the GDPS and RDPS, non-hydrostatic for the HRDPS (GEM-LAM 2.5 km) forecasts

- vertical coord* $\eta = \dfrac{P - P_T}{P_S - P_T}$

$P_S$, surface pressure, evolves
$P_T$, pressure at top of domain, fixed
$0 \leq \eta \leq 1$

- top level $P_T = 10$ hPa

*changed July 2014 to "terrain following vertical coordinate of the log-hydrostatic-pressure type vertically discretized on a Charney-Phillips grid" (source: an internal CMC report)

**Temperature of dry air having same $P$ and $\rho$ as sample:

$$T_v = T\,(1 + 0.61\,Q)$$

# CMC's GEM – Global Environmental Multiscale – model: common elements

- in the operational hydrostatic GEM the coordinate $\eta$ is based on total pressure

- in non-hydrostatic version it is based on the dry, hydrostatic component of the pressure (see NAM/WRF model later) as introduced by Laprise (1992, MWR Vol. 120). Note that

$\dot{\eta} = d\eta/dt = 0$ at the surface ( $\eta = 0$ ) and top of model domain ( $\eta = 1$ )

<u>Dynamics/physics terminology</u>

$$\dot{\eta}\frac{\partial u}{\partial \eta}$$

$$\frac{\partial U}{\partial t} = \underbrace{-U\frac{\partial U}{\partial x} - V\frac{\partial U}{\partial y} - W\frac{\partial U}{\partial z} - \frac{1}{\rho}\frac{\partial P}{\partial x} + fV}_{\text{dynamics}} + F_u$$

$$\frac{\partial U}{\partial t} = \left(\frac{\partial U}{\partial t}\right)_{\text{dyn}} + \left(\frac{\partial U}{\partial t}\right)_{\text{phy}}$$

## CMC's GDPS-4.0.0 (Global Deterministic Prediction System) as of 18 Nov. 2014

- run four times a day in analysis mode (centered at 00, 06, 12, 18 UTC)

- run twice a day for forecasts with initial times at 00 and 12 UTC

- in addition to providing analysis and first-guess fields ("background state") to its own forecast component, the GDPS analysis component also provides the initial conditions to the regional deterministic prediction system (RDPS) assimilation cycle

- forecast range to 10 days (Saturday, range to 15 days)

- global domain

- horizontal resolution $\Delta$ =25 km at mid latitudes

- 79 levels

- timestep 12 min

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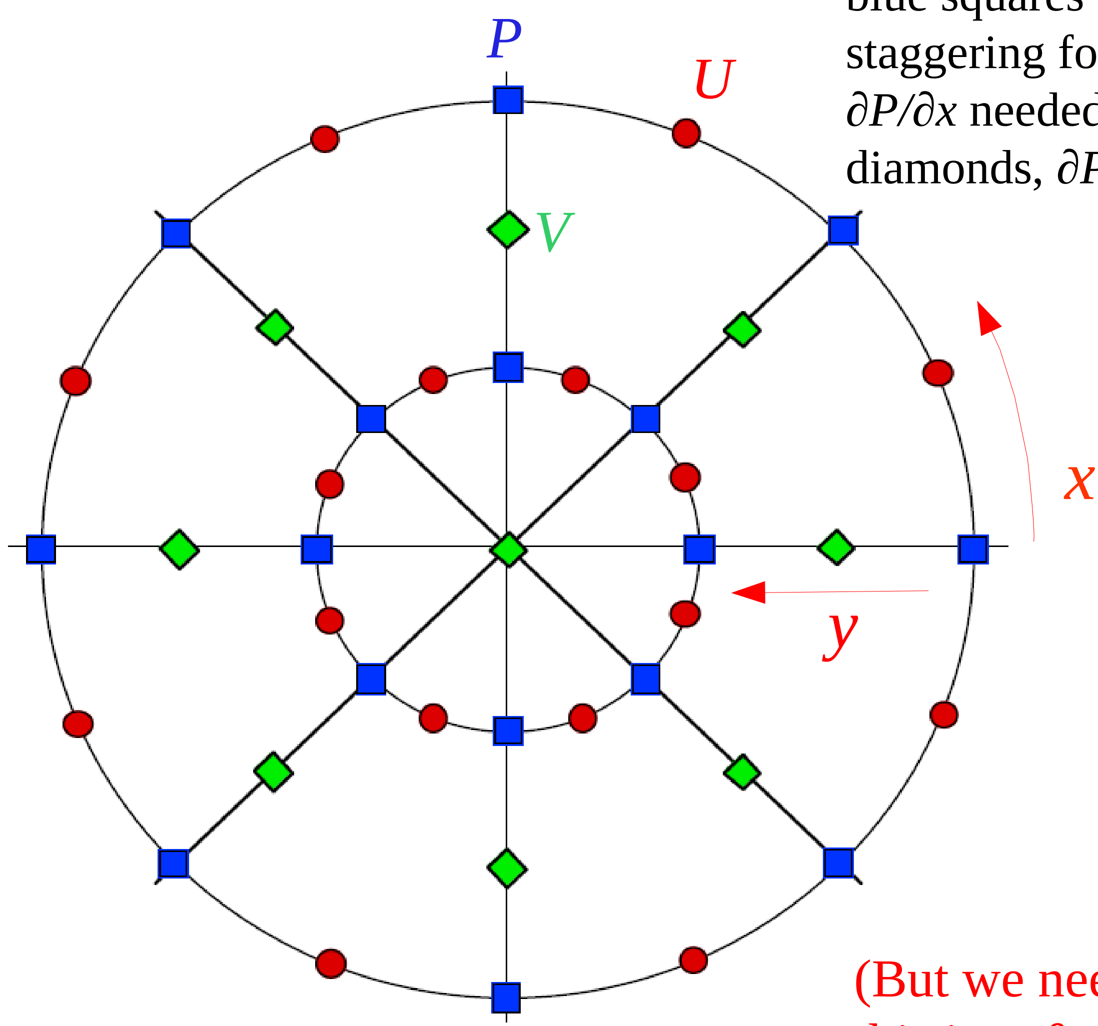
## Spatial discretization (grid) for GDPS

Squares: scalar fields (*P* etc.)
Circles: *U* nodes
Diamonds: *V* nodes

Each cell is a "finite element." With pressure ***P*** placed at the blue squares we have nice staggering for ***U*** (red circles, $\partial P/\partial x$ needed) and ***V*** (green diamonds, $\partial P/\partial y$ needed)

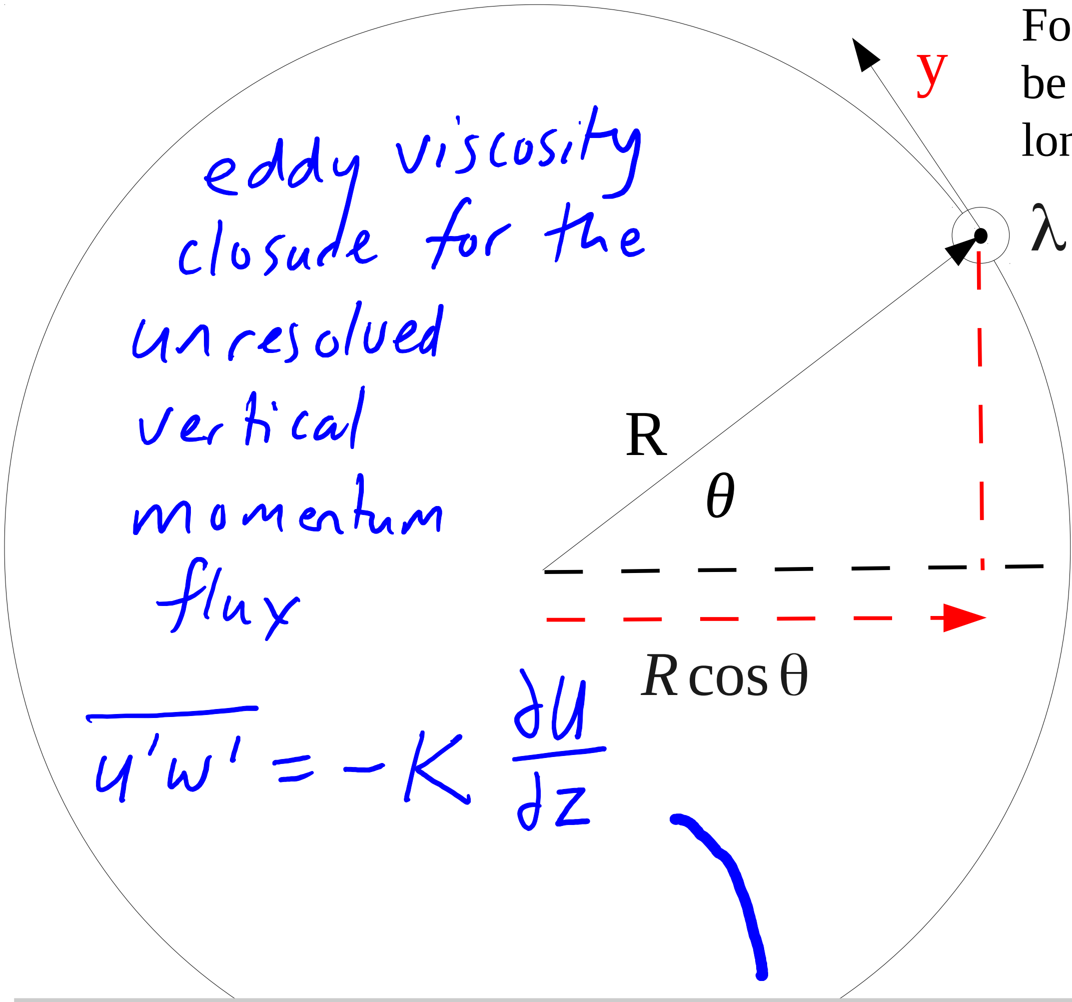



- domain is separated by imaginary lines into finite elements
- values of *U, V, W, P*… at the nodes are the basic unknowns (resolved variables)
- an interpolating function is used to provide the values of *U*, *V* (etc.) wherever needed within each finite element in terms of nodal values (e.g. at *U* gridpoints we need *V* to compute $V\,\partial U/\partial y$

(But we need to transform this into $\theta$, $\lambda$, $P$ coords)

$$\left[\frac{\partial U}{\partial t}\right]_{\mathrm{dyn}} = -U\frac{\partial U}{\partial x} - V\frac{\partial U}{\partial y} - W\frac{\partial U}{\partial z} - \frac{1}{\rho}\frac{\partial P}{\partial x} + f\,V$$

## Transforming the eqn for resolved *U*-mtm into $\theta$, $\lambda$, $P$ coordinates:




eddy viscosity closure for the unresolved vertical momentum flux

$$\overline{u'w'} = -K \frac{\partial U}{\partial z}$$

Following terminology of the previous page, let $\lambda$ be longitude and let $d\lambda$ be an increment in longitude. The corresponding increment in $x$ is:

$$dx = (R\cos\theta)\ d\lambda$$

so $$\frac{\partial}{\partial x} = \frac{1}{R\cos\theta}\frac{\partial}{\partial \lambda}$$

Similarly $$dy = R\ d\theta$$

so $$\frac{\partial}{\partial y} = \frac{1}{R}\frac{\partial}{\partial \theta}$$

$$\frac{\partial}{\partial z} = -\rho g \frac{\partial}{\partial P}$$

$$\left[\frac{\partial U}{\partial t}\right]_{\text{dyn}} = -\frac{U}{R\cos\theta}\frac{\partial U}{\partial \lambda} - \frac{V}{R}\frac{\partial U}{\partial \theta} + \rho g W \frac{\partial U}{\partial P} - \frac{1}{\rho R\cos\theta}\frac{\partial P}{\partial \lambda} + f\ V$$

And the effect of unresolved motion? Treated as:

$$\left[\frac{\partial U}{\partial t}\right]_{\text{phy}} \approx -\frac{\partial\ \overline{w'u'}}{\partial z} = -\frac{\partial}{\partial z}\left(-K\frac{\partial\ U}{\partial z}\right) = \rho g^2 \frac{\partial}{\partial P}\left(\rho K \frac{\partial U}{\partial P}\right)$$

# CMC's RDPS 4.0.0 (Regional Deterministic Prediction System) as of 18 Nov. 2014

• launched at [00,06,12,18] Z+2:00, with forecast range to 48 hours (sometimes 54 hours)

• 25 km analysis from GDPS initializes a 6 h limited area (LAM) forecast starting T-6h (LB). This forecast serves as the background state for the analysis at time T (LF). Same procedure is applied to a **global GEM driving model** (33 km). The synchronous global driving analysis (DF) and forecast allow obs. outside the LAM domain to influence the LAM forecasts through the lateral boundary conditions (LBCs).

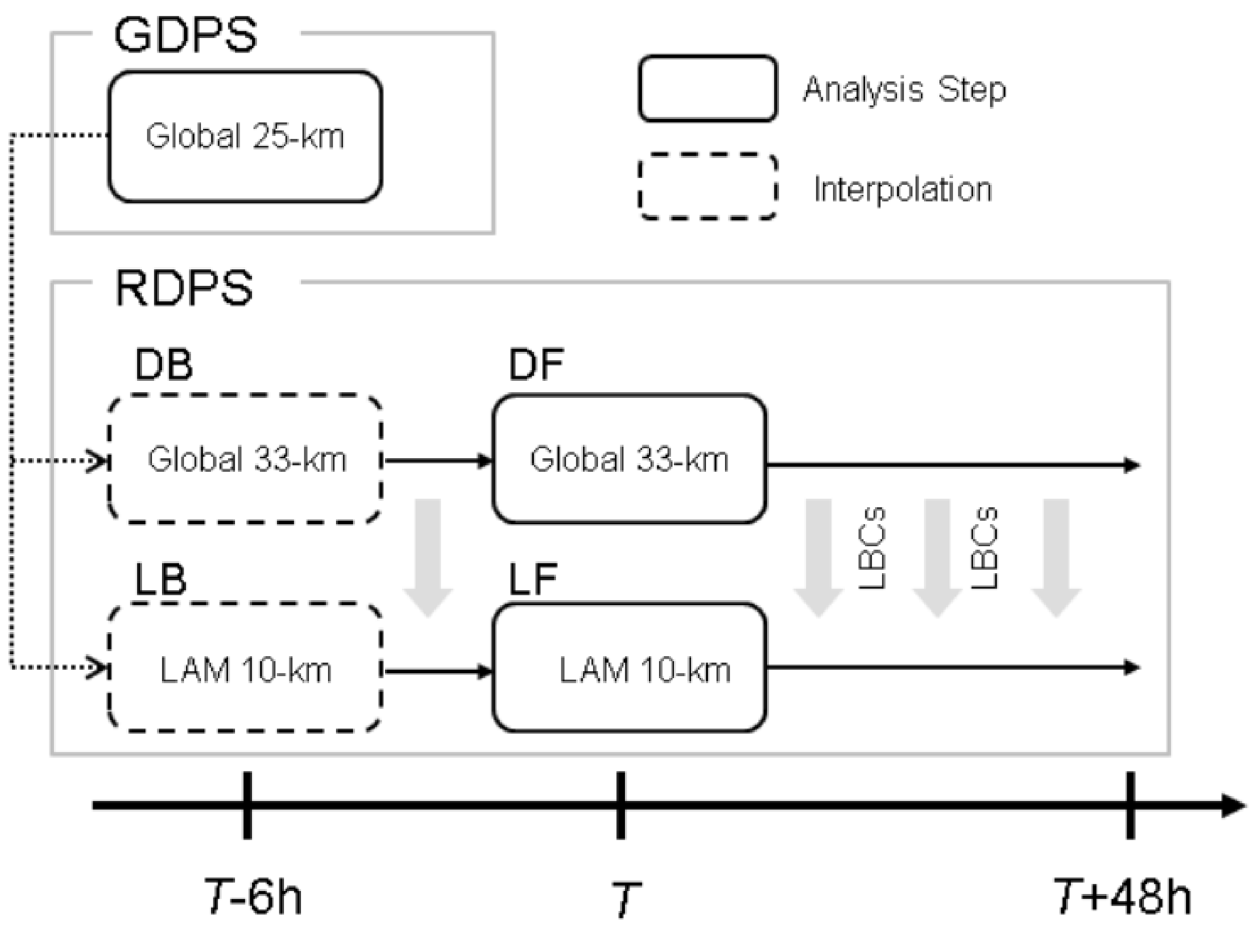
• core is a LAM calculation

• LAM grid uniform Δ=10 km

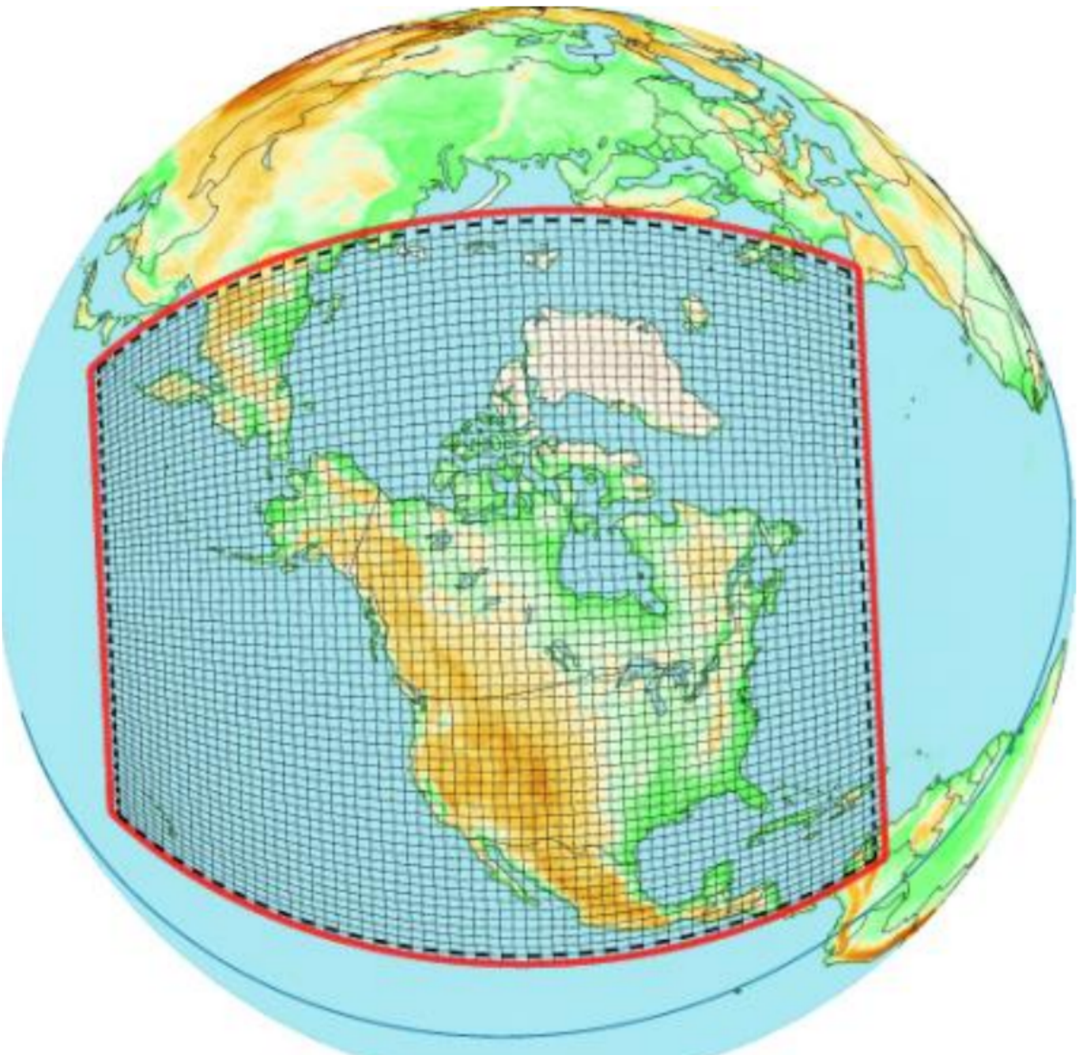
• b/conds for LAM provided by global "driving" model

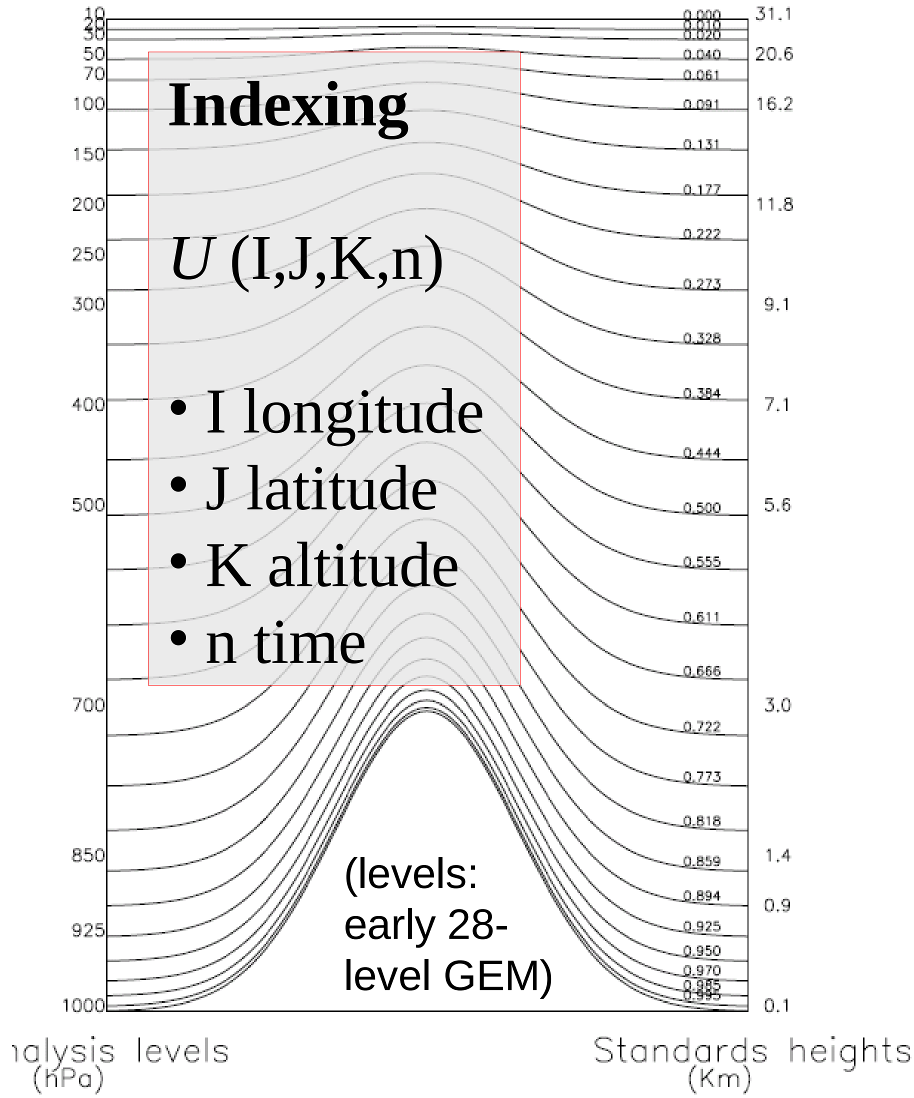
• both have same 80 levels (7 or more below 850 hPa)

• timestep of the LAM: 5 min



## **Domain** of the limited area model in RDPS (Δ=10 km)






**Indexing**

*U* (I,J,K,n)

- I longitude
- J latitude
- K altitude
- n time

Derivatives are approximated by finite differences, e.g. $\frac{\partial U}{\partial x} = \frac{U(\text{I+1,J,K,n}) - U(\text{I-1,J,K,n})}{x(\text{I+1,J}) - x(\text{I-1,J})}$

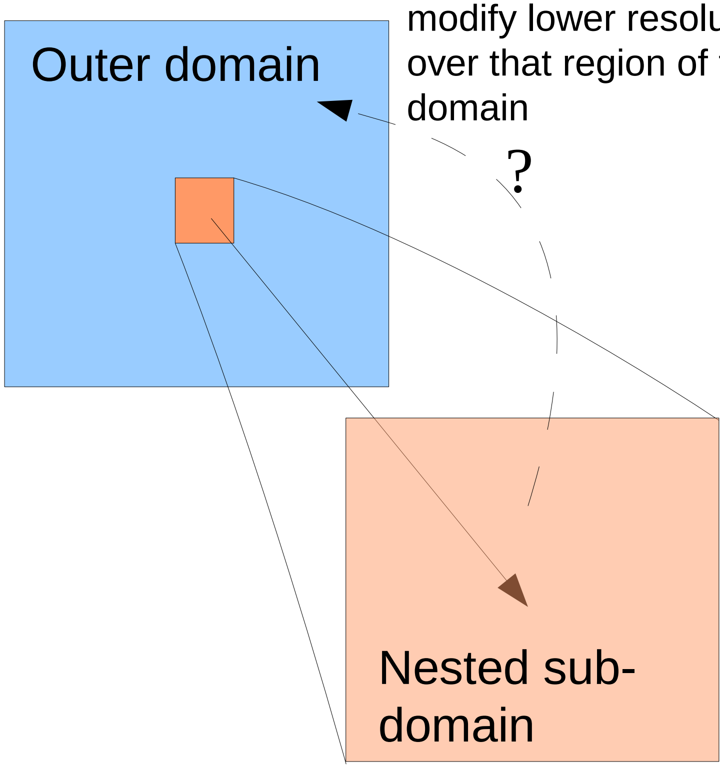
with the result that the governing differential equations are transformed into a coupled set of

non-linear algebraic equations

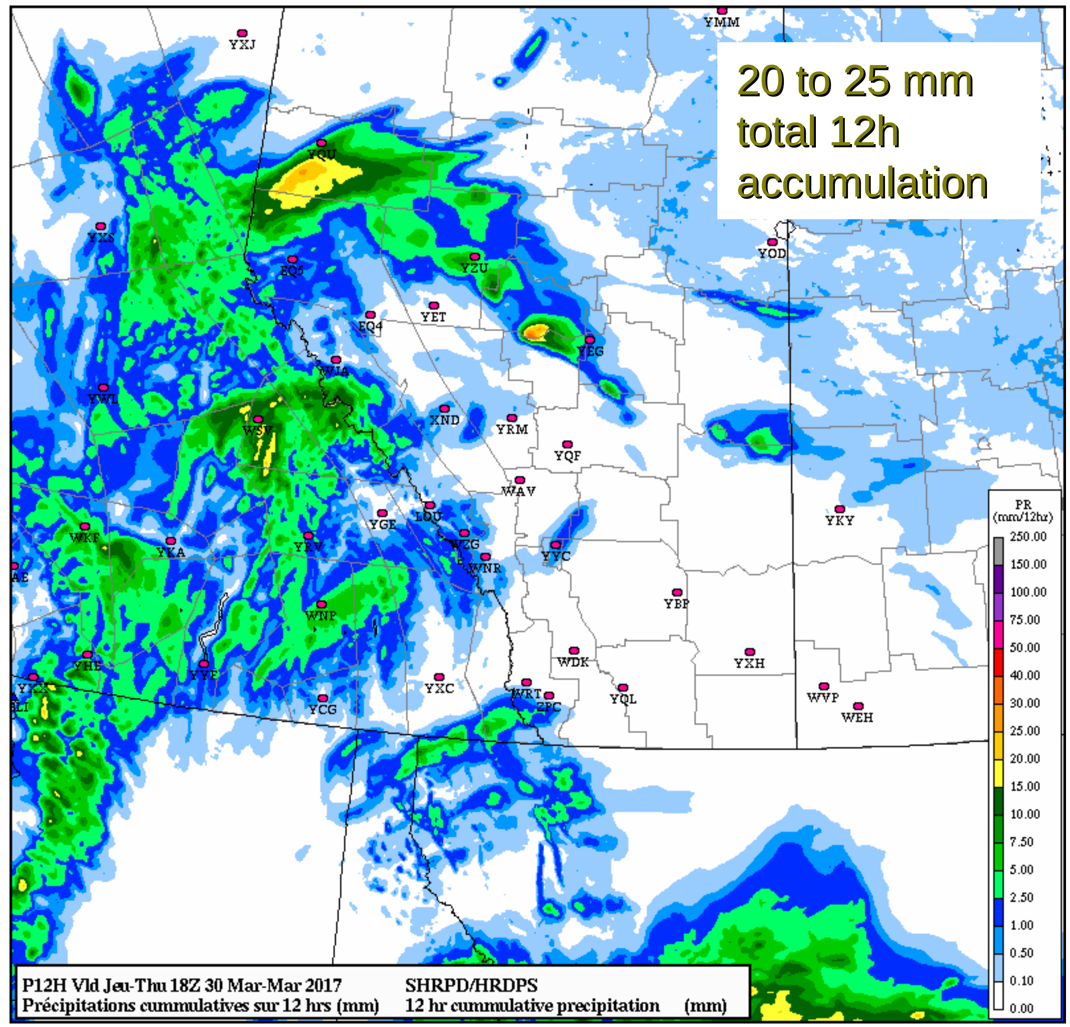
# High Resolution Deterministic Prediction System (HRDPS)

- one way nested LAM (Limited Area Model) non-hydrostatic implementation of GEM's eqns
- horizontal resolution 2.5 km
- 4 x daily on four domains:
  - arctic, east, west & maritimes

two-way nesting, if permitted, allows hi-resolution fields computed on sub-domain to modify lower resolution fields over that region of the outer domain




12h prog. LAM 2.5 km west vld 18 Z Thurs 30 Mar. 2017



# Summary of GEM forecast system (GDPS configuration)

From: Belair et al., 2005, Monthly Weather Review

These specs. pertain to the twice-daily Global runs (to 10 days*) – note the coarser resolution and timestep relative to the Regional run

*A 15-day run is made on Saturdays

## Dynamics/numerics

- Hydrostatic primitive equations;
- Global uniform resolution of ~~0.45°~~ longitude and ~~0.30°~~ latitude ~~(800 × 600);~~ 1024x800
- Variable vertical resolution with 80 levels; model top at 10 hPa;
- Time step 12 min);
- Cell-integrated finite-element discretization on Arakawa C grid;
- Terrain-following hydrostatic pressure vertical coordinate;
- Two-time-level semi-implicit time scheme;
- 3D semi-Lagrangian advection; (see over)
- $\nabla^6$ horizontal diffusion on momentum variables; increased horizontal diffusion (sponge) for the four uppermost levels;
- Periodic horizontal boundary conditions;
- No motion across the lower and upper boundaries.

## Physics

- Planetary boundary layer based on TKE with statistical representation of subgrid-scale cloudiness (MoisTKE);
- Fully implicit vertical diffusion;
- Stratified surface layer, distinct roughness lengths for momentum and heat/humidity;
- Four types of surface represented: land, water, sea ice, and glaciers;
- Solar/infrared radiation schemes with cloud-radiation interactions based on predicted cloud radiative properties;
- Kuo Transient scheme for shallow convection;
- Kain–Fritsch scheme for deep convection;
- Sundqvist scheme for nonconvective condensation.

# Semi-Lagrangian treatment of advection

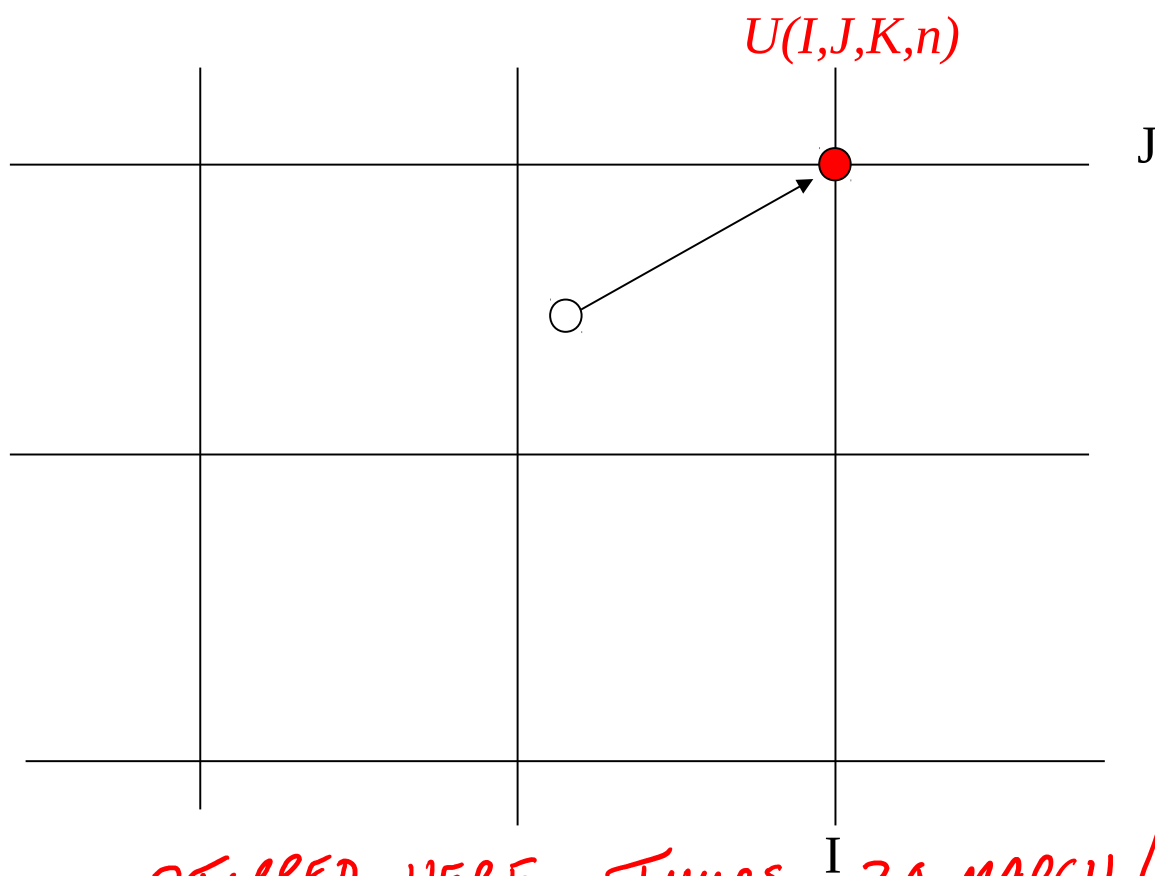
... a strategy to overcome the limitation imposed by the Courant condition, which demands

$$\frac{|U|\ \Delta t}{\Delta x} \leq 1$$

$$\frac{|V|\ \Delta t}{\Delta y} \leq 1$$

Evaluate this with
$\Delta x$ = 25 km
$\Delta t$ = 720 s
$U$=50 m s$^{-1}$

*Computed path of a fluid element backwards in time from t to t-Δt such that U(I,J,K,n) is evaluated by taking the value at the upwind point (open circle) for time level n-1. The latter is evaluated by cubic interpolation from the gridded values*




• *of course other factors, notably pressure gradient and Coriolis force, demand an adjustment to this advected value*

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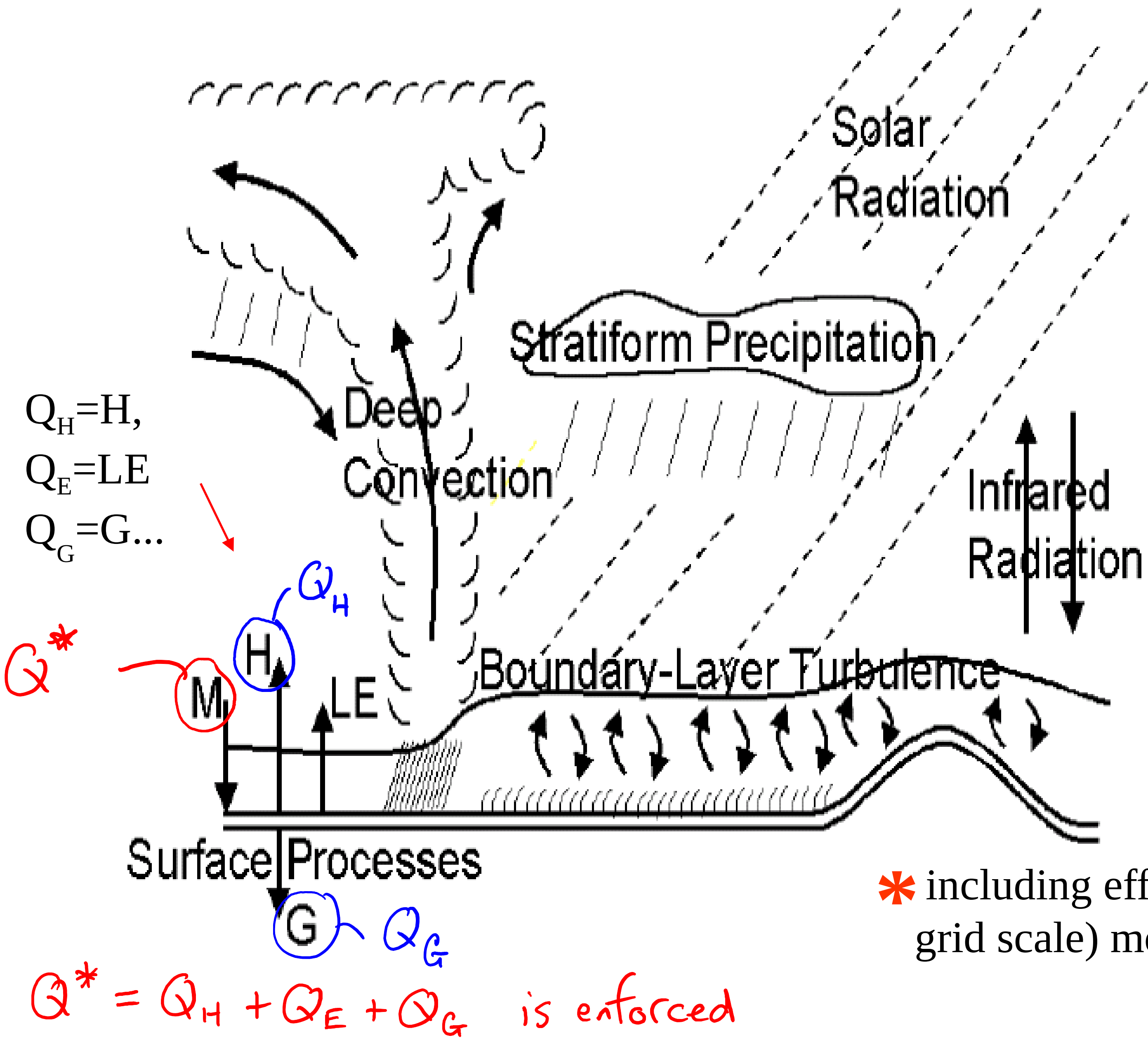
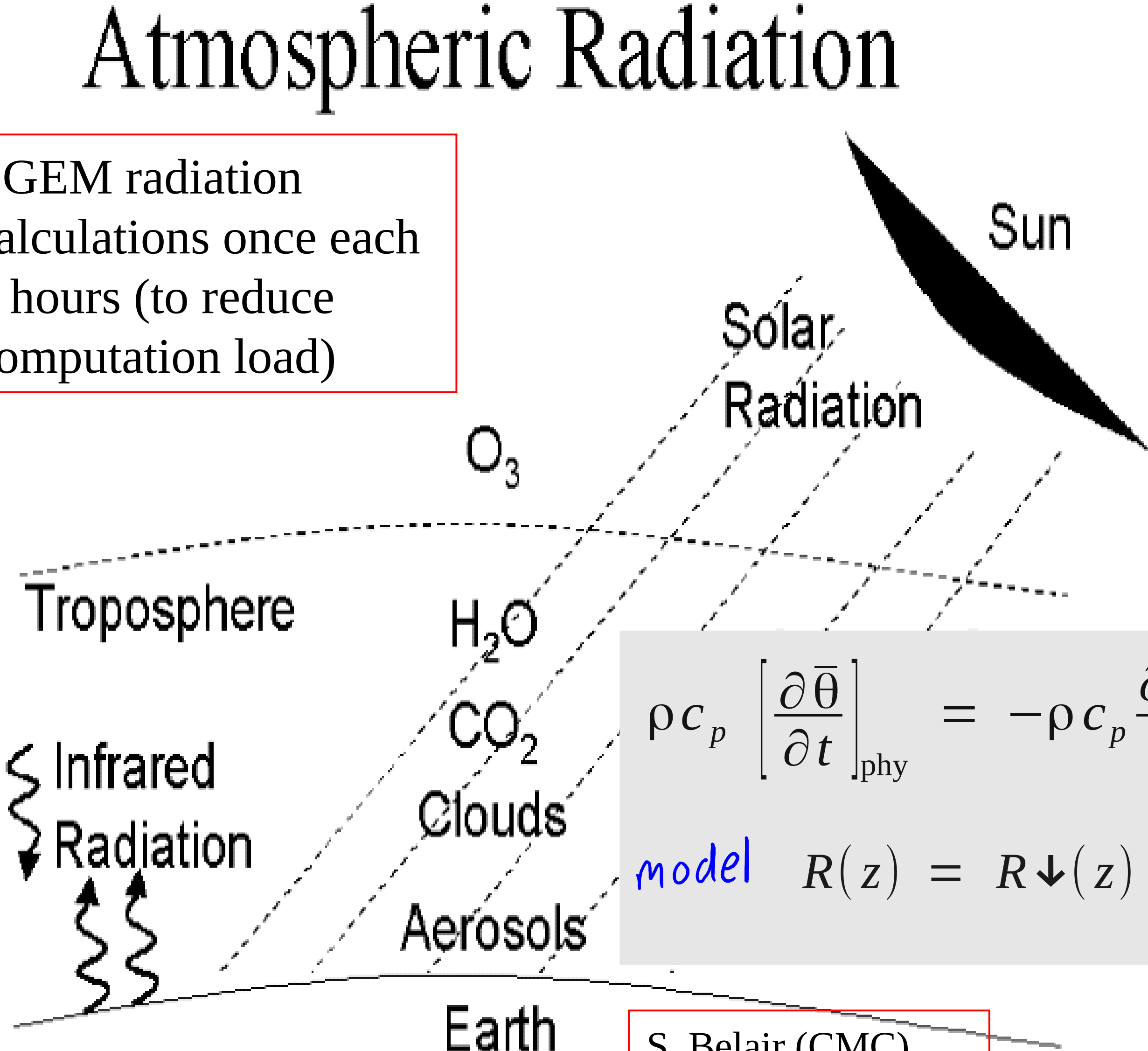
# Overview of Physical Processes* parameterized in GEM

Thanks to Stephane Belair (CMC) for permission to use this and other sketches

$Q_H$=H,
$Q_E$=LE
$Q_G$=G...




* including effects of unresolved (sub-grid scale) motion




• GEM radiation calculations once each 2 hours (to reduce computation load)

S. Belair (CMC)

SOLAR

- single waveband
- sun-earth geometry
- multiple scattering
- absorption by "<u>model</u> clouds" in rel'n to diagnosed fractional sky coverage & effective cloud liquid water content
- climatol. profiles of ozone, CO2; no scheme for aerosols

$$\rho c_p \left[\frac{\partial \bar{\theta}}{\partial t}\right]_{\text{phy}} = -\rho c_p \frac{\partial \overline{w'\theta'}}{\partial z} - \frac{\partial R}{\partial z}$$

model $$R(z) = R\downarrow(z) - R\uparrow(z)$$

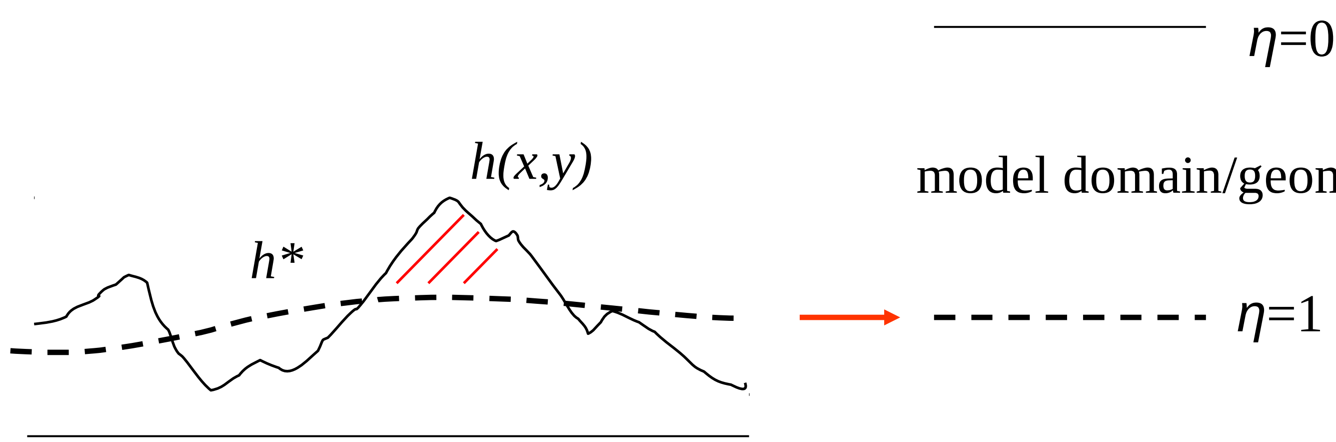
Notice the lateral fluxes $\overline{u'\theta'}, \overline{v'\theta'}$ are not considered

"Two stream model"
($R$ is the net radiation)

LONGWAVE

- four wavebands; interaction with water vapour, $O_3$ , $CO_2$ , clouds
- climatological $O_3$; [$CO_2$] treated as uniform

# Parameterizing effects of terrain




Resolved terrain *h*(x,y)* “disappears” in the (terrain following) eta ($\eta$) coordinate system. How are mountains “felt”? New terms appear in the momentum equations when they are transformed into the $\eta$ coord. system (“metric terms”)*

**Additional - parameterized - effects of terrain**

- unresolved gravity waves transport momentum vertically, and slow the winds aloft (particularly in winter). In introducing their parameterization improved the models' climatological winter winds

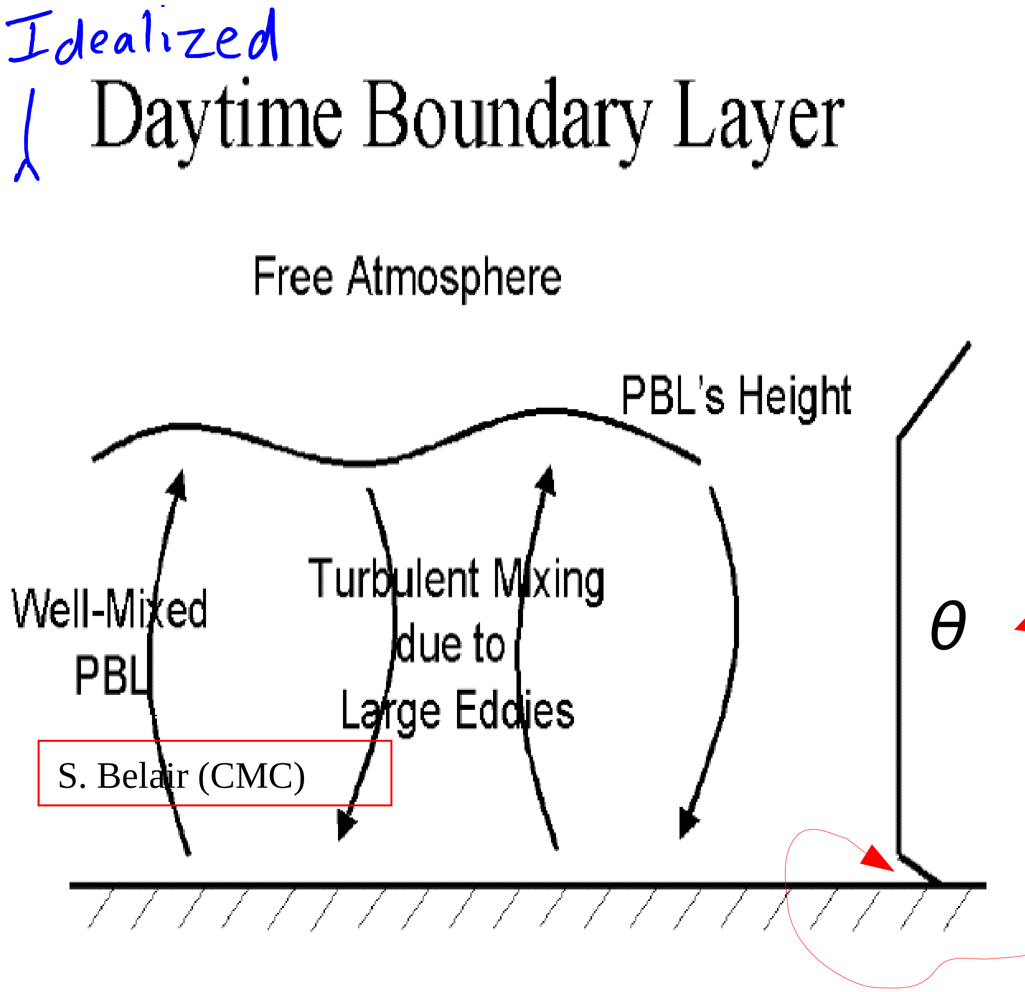
- “blocking” parameterization recognizes influence of unresolved terrain - reduces the low level winds in mountainous regions*

*e.g. Wilson (2002; J. Atmos. Sci., Vol. 59)

# Parameterizing subgrid (unresolved) transport in the boundary-layer

• sub-grid scale motion transports heat, vapour, momentum… (eg. redistributes heat and vapour added at ground). Consider vertical exchange only, i.e. the "grid-point computations" involve local column only, no lateral coupling.

• in analogy with molecular mixing, subgrid transport is represented as "diffusion." Eddy diffusivity *K* is function of kinetic energy of turbulence, and stratification




Depth $\delta$ of the friction layer ("Planetary Boundary Layer") is diagnosed from GEM variables incl. sfc heat flux $Q_{H\,0}$ & wind-speed. $\delta$ changes throughout daily cycle

Mean potential temperature $\theta$ may be almost const. with height in interior of daytime friction layer, due to good mixing

strongly stratified **surface layer** at base of PBL (described by Monin-Obukhov similarity theory)

# Parameterizing unresolved vertical fluxes by eddy viscosity/diffusion paradigm

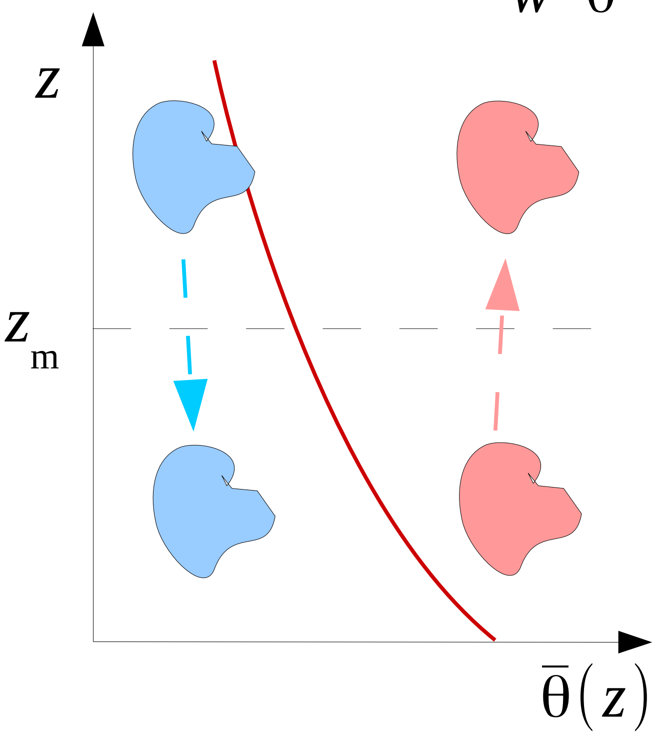
• mean vertical convective heat flux due to the unresolved vertical motion is average of the *w'T '* product (or one can equally write *w' θ'*)… thus, need $\overline{w'\theta'}$

• unresolved fluctuations *w'* carry heat, vapour, $CO_2$, etc. to and from the surface

• eddy-diffusion model postulates that the direction of the mean flow of heat will be from warm to cold, and introduces as proportionality constant an "eddy diffusivity" (for heat) with the same units as, but vastly greater magnitude than, the molecular diffusivity. That is, one adopts the model

$$\overline{w'\theta'} = -K_h \frac{\partial \overline{\theta}}{\partial z}$$

(Dimensionally, $K_h$ is [velocity x length]; numerically, it vastly exceeds the molecular thermal diffusivity; furthermore, it is a property of the flow, not of the fluid itself)



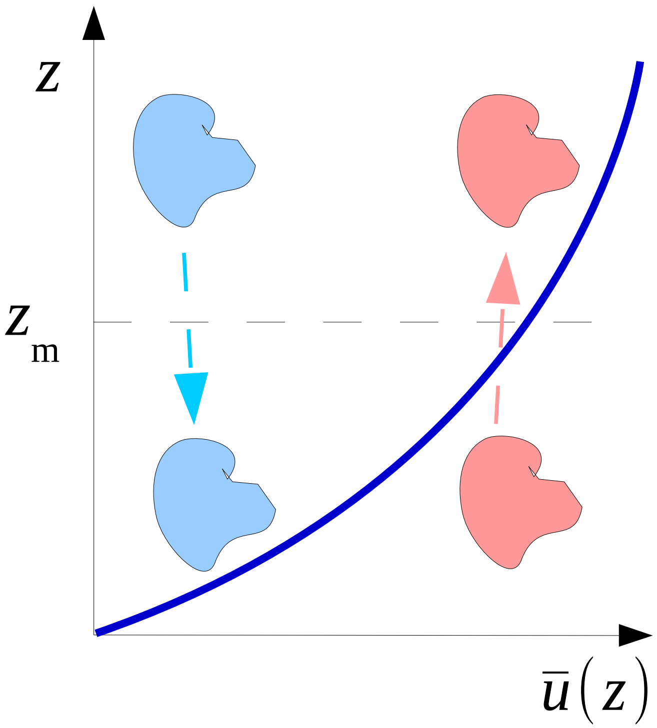

In the case shown (unstable stratification), the covariance (eddy heat flux) is +ve (upward). Why*?

• No net volume flux across measurement level $z_m$

• For each cold parcel ($\theta' <0$) crossing $z_m$ downward ($w'<0$) a warm parcel ($\theta' > 0$) of equal volume crosses $z_m$ moving upward ($w'>0$)

*This is is not a proof, merely a *plausibility* argument

## Parameterizing unresolved vertical fluxes by eddy viscosity/diffusion paradigm




• On average, for each fast-moving parcel ($u'>0$) crossing $z_m$ downward ($w'<0$) a slow-moving parcel ($u'<0$) of equal volume crosses $z_m$ moving upward ($w'>0$)

• Thus the eddy momentum flux (covariance) is negative. The simplest model, "first order closure," is

$$\overline{w'u'} = -K_m \left[\frac{\partial \bar{u}}{\partial z} + \frac{\partial \bar{w}}{\partial x}\right]$$

Now we have the problem of how to rationally model the eddy viscosity and eddy diffusivity! Their magnitude must depend on some measure of the "amount" of vertical motion (loosely, of "mixing"), and this is often expressed by the "turbulent kinetic energy"

$$k = \frac{\sigma_u^2 + \sigma_v^2 + \sigma_w^2}{2}$$

(one half the sum of the variances of the unresolved velocity components)

The eddy diffusivity and eddy viscosity are usually assumed proportional or even equal, and typically written $K_{m,h} \propto \lambda(z)\sqrt{k}$ , where λ is the "length scale"

## TKE budget equation – assuming horizontal homogeneity

so no advection

Buoyant production, $P_B$

Turbulent & pressure transport, $T_T$

$$\frac{\partial k}{\partial t} = -\overline{u'w'}\frac{\partial U}{\partial z} - \overline{v'w'}\frac{\partial V}{\partial z} + \frac{g}{\theta_0}\overline{w'\theta'} - \epsilon - \frac{\partial}{\partial z}\overline{w'\left(\frac{p'}{\rho} + \frac{u'u'+v'v'+w'w'}{2}\right)}$$

Shear production, $P_S$

Dissipation (conversion of TKE to heat)

In the general case there are many more production terms (involving horiz. gradients)

Assume $\overline{u'w'} = -K\frac{\partial U}{\partial z}$, $\overline{v'w'} = -K\frac{\partial V}{\partial z}$, $\overline{w'\theta'} = -K\frac{\partial \bar{\theta}}{\partial z}$, $\epsilon = \frac{(\alpha k)^{3/2}}{\lambda}$

(note the assumption that eddy viscosity and eddy diffusivity are equal)

Resulting TKE equation:

$$\frac{\partial k}{\partial t} = K\left[\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2\right] - \frac{g}{\theta_0}K\frac{\partial \bar{\theta}}{\partial z} - \frac{(\alpha k)^{3/2}}{\lambda} + \frac{\partial}{\partial z}K\frac{\partial k}{\partial z}$$

$T_T$

Shear production, $P_S$

Buoyant production, $P_B$

Flux Richardson number: $R_i^f = -\frac{P_B}{P_S} = \frac{g}{\theta_0}\frac{\partial\bar{\theta}/\partial z}{(\partial U/\partial z)^2 + (\partial V/\partial z)^2}$

Unstable, $R_i^f < 0$

Stable, $R_i^f > 0$

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OPTIONAL

## Exercise: calibrating the TKE budget equation (a manipulation using calculus & algebra)

$$\frac{\partial k}{\partial t} = K \left[ \left( \frac{\partial U}{\partial z} \right)^2 + \left( \frac{\partial V}{\partial z} \right)^2 \right] - \frac{g}{\theta_0} K \frac{\partial \bar{\theta}}{\partial z} - \frac{(\alpha k)^{3/2}}{\lambda} + \frac{\partial}{\partial z} K \frac{\partial k}{\partial z}$$

In the ideal neutral surface layer

- $k = \text{const.}$ $= \frac{1}{2} \left[ c_u^2 + c_v^2 + c_w^2 \right] u_*^2$, $c_u$ etc. dimensionless, $c_u = \frac{\sigma_u}{u_*}$ (e.g. $c_w \simeq 1.3$)
- $K = k_v u_* z$
- $U = \frac{u_*}{k_v} \ln \frac{z}{z_0}$ (coords. chosen such that $V$=0)
- $\bar{\theta} = \text{const.}$
- $\lambda = k_v z$

Assuming stationarity, express the coefficient $\alpha$ in the TKE eqn. in terms of $u_*^2 / k$

## GEM's formulation** of the eddy viscosity/diffusivity $K$ and length scale $\lambda$

$$K = c\,\lambda\,\sqrt{k} = \frac{\lambda^{\mathrm{neut}}(z)}{1-R_i^f}\,\sqrt{c^2 k}$$

$1-R_i^f$ , "stability (correction) function"

$c$ , dimensionless constant

Require that if $R_i^f \rightarrow 0$ with $z \rightarrow 0$

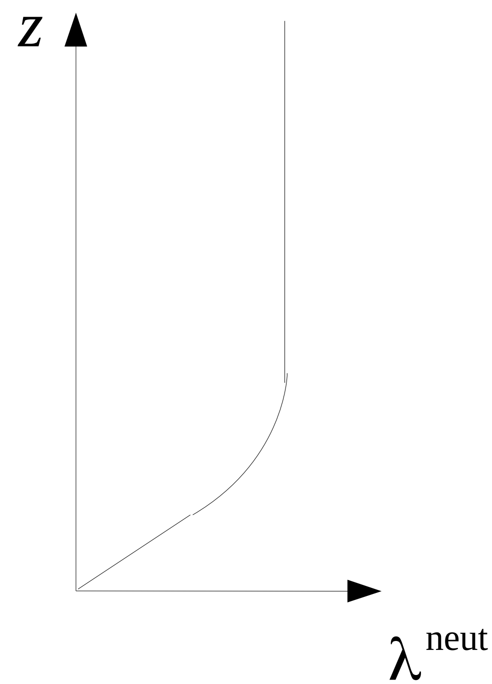
then $K \rightarrow K^{\mathrm{neut}} = k_v\, u_*\, z$

Thus require $c^2 = [u_*^2/k]^{\mathrm{neut}}$

$$\lambda^{\mathrm{neut}} = \min[k_v\, z,\ 200\,\mathrm{m}]$$

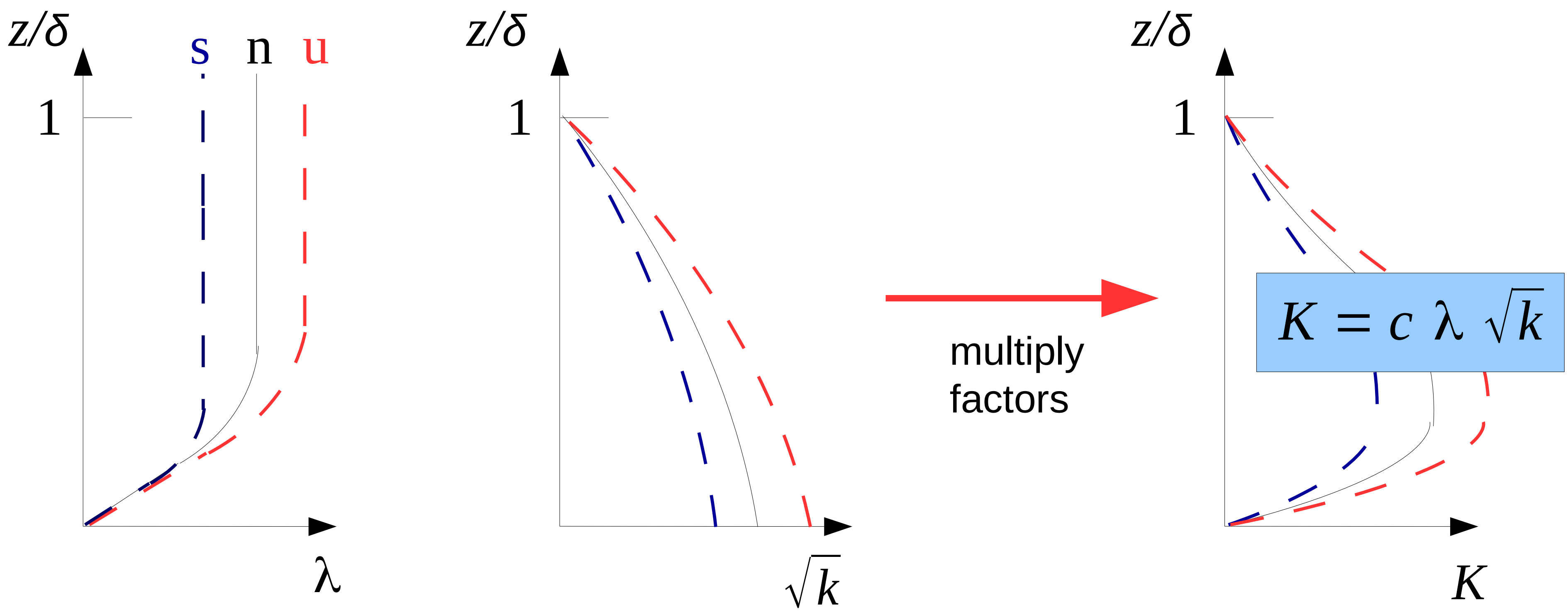
(*arbitrarily imposed maximum value for the length scale in a neutral ABL*)

The $c^2$ on this page is the $\alpha$ of the previous page




**http://collaboration.cmc.ec.gc.ca/science/rpn/physics/physic98.pdf

GEM's formulation** of the eddy viscosity/diffusivity $K$ and length scale $\lambda$



# GEM's coupling to the surface

- enforce surface energy balance $$Q^* = K^* + L^* = Q_{H0} + Q_{E0} + Q_{S0}$$
- detailed map of (time-evolving) surface type/condition
- prognostic variables for surface and soil temperatures, and soil moisture
- "ISBA" scheme (Interaction Soil Biosphere Atmosphere): three soil layers, vegetation canopy, interaction of radiation and vegetation canopy (surface albedo), vertical diffusion of heat and moisture between the soil layers, treatment of snow on canopy, inclusion of precip infiltration, runoff, and drainage
- static analyzed ocean/lake ice field and ocean/lake temperature (SST)
- lake/ocean surface roughness length ("$z_0$") responds to surface windspeed

GEM's treatment of clouds and precip – see table on a previous page