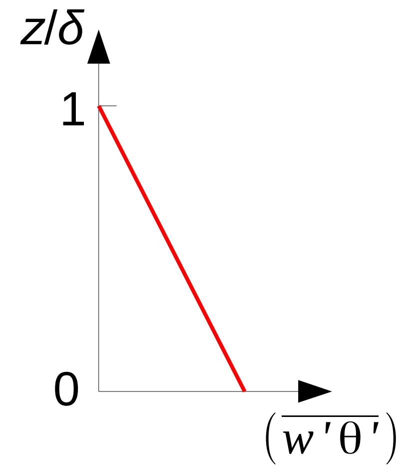
Exercise – heat budget of the ideal ABL

By averaging the equation $\frac{D\theta}{Dt} = 0$



we found

$$\frac{\partial\overline{\theta}}{\partial t} + \overline{u}\,\frac{\partial\overline{\theta}}{\partial x} + \overline{v}\,\frac{\partial\overline{\theta}}{\partial y} + \overline{w}\,\frac{\partial\overline{\theta}}{\partial z} = -\frac{\partial\overline{u'\theta'}}{\partial x} - \frac{\partial\overline{v'\theta'}}{\partial y} - \frac{\partial\overline{w'\theta'}}{\partial z}$$

Assuming horizontal uniformity (i.e. horizontal gradients vanish) and that $\overline{w} = 0$,

this simplifies to $\frac{\partial\overline{\theta}}{\partial t} = -\frac{\partial\overline{w'\theta'}}{\partial z}$

Suppose the eddy heat flux decays linearly with height across the boundary layer

$$\overline{w'\theta'} = \left(\overline{w'\theta'}\right)_0 \left[1 - z/\delta\right]$$

Then (1) if the rate of warming is 2 K hr^{-1} while the ABL depth is δ = 500 m, what is the value of the kinematic eddy heat flux density at the surface, $\left(\overline{w'\theta'}\right)_0$? And (2) if $Q_{H0} \equiv \rho\, c_p \left(\overline{w'\theta'}\right)_0 = 300\ \mathrm{W\,m^{-2}}$ and δ = 1750 m, what is the rate of warming?