EAS372 Open Book Final Exam 11 April, 2013

Professor: J.D. Wilson Time available: 2 hours Value: 30%

Please check the Terminology, Equations and Data section before beginning your responses. Answer all questions in the Examination Booklet.

A. Multi-choice (11 x 1% $\rightarrow$ 11 %)

1. Which of the following statements is **false** for a barotropic region of the atmosphere?
   (a) $p = p(\rho)$
   (b) ∇p is parallel to $\nabla \rho$, i.e. $\nabla p \cdot \nabla \rho = |\nabla p|\,|\nabla \rho|$
   (c) $\nabla p \times \nabla \rho = 0$
   (d) $\vec{V}_g \cdot \nabla T = 0$
   (e) $\nabla \theta \times \nabla T \neq 0$

2. According to the hypsometric equation (given as data), a 1 K increase in the mean temperature of the 1000-500 hPa layer results in that layer's thickness changing by
   (a) 0.5 K
   (b) 1 K
   (c) 2 K
   (d) -1 K
   (e) -2 K

3. Treating the temperature and pressure as $T = 258$ K and $p = 650$ hPa, calculate the static stability of the 700-600 hPa layer if $\theta_{700} = 288$ K and $\theta_{600} = 296$ K. The correct value (in MKS units) is:
   (a) $+2.4 \times 10^{-4}$
   (b) -2.4×10^{-4}
   (c) $+3.1 \times 10^{-6}$
   (d) -3.1×10^{-6}
   (e) -3.1×10^{-2}

The vertical distribution of winds depicted in the figure implies

4. (a) warm advection
   (b) cold advection
   (c) isotherms are perpendicular to $\mathbf{V_T}$
   (d) thickness contours are perpendicular to $\mathbf{V_T}$
   (e) horizontal divergence

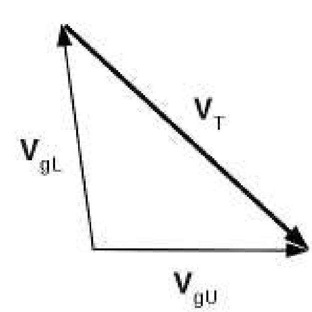

Figure 1: Wind vector at lower (L) and upper (U) levels.

The next few questions pertain to the quasi-geostophic (QG) model. The QG vorticity equation may be written in several equivalent forms, namely

$$
\begin{aligned}
\frac{d_g \eta}{dt} &= f_0 \frac{\partial \omega}{\partial p} , & (1)\\
\frac{\partial \eta}{\partial t} + \vec{V}_g \cdot \nabla_h \eta &= f_0 \frac{\partial \omega}{\partial p} , & (2)\\
\frac{\partial \zeta_g}{\partial t} + \vec{V}_g \cdot \nabla_h \eta &= f_0 \frac{\partial \omega}{\partial p} , & (3)\\
\frac{\partial \zeta_g}{\partial t} + \vec{V}_g \cdot \nabla_h \zeta_g + v_g \beta &= f_0 \frac{\partial \omega}{\partial p} , & (4)\\
\frac{d_g \zeta_g}{dt} + v_g \beta &= f_0 \frac{\partial \omega}{\partial p} , & (5)
\end{aligned}
$$

where d_g/dt is the derivative following the geostrophic wind (recall that f does not vary in time). The QG omega equation is

$$
\left(\nabla_h^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \omega = \frac{f_0}{\sigma} \frac{\partial}{\partial p} \left(\vec{V}_g \cdot \nabla_h \eta \right) + \frac{R}{\sigma\, p} \nabla_h^2 \left(\vec{V}_g \cdot \nabla_h T \right) , \qquad (6)
$$

where $\vec{V}_g = (u_g, v_g)$ is the geostrophic wind, $\eta = \zeta_g + f$ is the absolute vorticity, ζ_g is the geostrophic relative vorticity, σ is the static stability, R the specific gas constant for air and (recall) $\omega = -\rho g w$ where w is the true vertical velocity.

5. Which of the following restrictions or approximations regarding the quasi-geostrophic (QG) model (neglecting diabatic heating) is **false**?
   (a) frictionless, adiabatic, hydrostatic, extra-tropical motion
   (b) linear variation of Coriolis parameter with north-south coordinate y, viz. $f = f_0 + \beta y$
   (c) neglect vertical advection
   (d) neglect horizontal advection of and by the ageostrophic wind
   (e) divergence ($D = \nabla_h \cdot \vec{V} = -\partial\omega/\partial p$) evaluated using the geostrophic wind

6. Suppose $\partial\omega/\partial p = 0$ (no stretching): which of the statements below is **false**?
   (a) following $\vec{V}_g$, absolute vorticity is conserved
   (b) following $\vec{V}_g$, relative vorticity is conserved *only* on the condition that the meridional component $v_g \neq 0$
   (c) neither PVA (positive vorticity advection) nor NVA can occur
   (d) if the alongstream derivative in the natural coordinate system $\partial\eta/\partial s < 0$, then $\partial\eta/\partial t > 0$

7. If PVA (advection of positive absolute vorticity) is increasing with increasing height z, the first term on the right hand side of the ω-eqn is
   (a) negative
   (b) zero
   (c) positive

8. On a region of an isobaric surface where a local maximum of warm advection is occurring (i.e. $\vec{V}_g \cdot \nabla_h T$ negative with large magnitude relative to the surroundings), the second term on the right hand side of the ω-eqn is
   (a) negative
   (b) zero
   (c) positive

9. Qualitatively, the left hand side of the ω-eqn can be interpreted as the 3D Laplacian of ω, and accordingly it can be interpreted by replacing it with the expression
   (a) $\alpha\,\omega$, with proportionality constant $\alpha > 0$
   (b) $\alpha\,\omega$, with proportionality constant $\alpha < 0$

10. Suppose (q, ω) are respectively the specific humidity [kg/kg] and the vertical velocity [Pa/s] at the 700 hPa level (where pressure $p = 7 \times 10^4$ Pa). Which formula for the resolved vertical flux E of water vapour [$\mathrm{kg\,m^{-2}\,s^{-1}}$] is correct?
    (a) $E = -\omega\, q$
    (b) $E = -\omega\, q/g$
    (c) $E = \omega\, q$
    (d) $E = \omega\, q/g$

11. Referring to Figure (3), what label [hPa] would correctly identify the isobar at the north-east corner of the map?
    (a) 1020
    (b) 1022
    (c) 1024
    (d) 1028
    (e) 1032

B. "Live" web weather data (5 x 1 $\rightarrow$ 5%)

1. Give the sign and magnitude [$\mathrm{s^{-1}}$] of the strongest feature in the absolute vorticity field of the GEM-regional 0h prog that was initialized at 00Z today.

2. What was the temperature at Coronation, Alberta at 06 MDT today?

3. To the nearest metre, what was the 850-700 hPa thickness at Churchill, Manitoba (YYQ) at 00Z today?

4. Retrieve and write down the METAR for Toronto Pearson (CYYZ) for 06Z today.

5. What was the potential temperature of the air at the 500 hPa level over Edmonton as of 00Z today.

C. Short answer (2 x 3 % → 6 %)

Please answer **two** of the following questions.

1. Neglecting some small terms, the instantaneous horizontal momentum equation may be written in Cartesian coordinates as

$$\left(\frac{\partial}{\partial t} + \vec{U} \cdot \nabla\right) \vec{V} = \frac{-1}{\rho} \nabla_h P - \; f \, \hat{k} \times \vec{V} \tag{7}$$

   where $\hat{k}$ is the unit vector along the vertical. Evaluate the cross product and give the implied equations for the two horizontal components (u, v).

   We know that upon Reynolds averaging, extra terms arise, and that they represent momentum transport by unresolved scales of motion, i.e. "friction." Which term in Eq. 7 gives rise to the friction terms?

2. Provide an overview of the dynamics and physics of Canada's NWP model "GEM"

3. In an unsaturated and horizontally-homogeneous atmospheric boundary layer the Reynolds-averaged thermodynamic equation simplifies to

$$\frac{\partial \overline{T}}{\partial t} = - \frac{\partial \, \overline{w'T'}}{\partial z} \tag{8}$$

   where $\overline{w'T'}$ is the (kinematic) sensible heat flux density, and any contribution due to radiative flux divergence has been neglected. Compute the rate of warming [K hr^{-1}] assuming the ABL is 1000 m deep and the surface heat flux density $\left(\overline{w'T'}\right)_0 = 0.2\,\mathrm{K\,m\,s^{-1}}$.

4. In the isobaric coordinate system the continuity equation is

$$\nabla_h \cdot \vec{V} + \frac{\partial \omega}{\partial p} = 0 \; . \tag{9}$$

   With reference to this equation, explain the statement that "the vertical velocity takes on a local maximum or minimum value at a level of non-divergence."

D. Interpretation of weather situation. (1 x 8→ 8%)

On 7 March 2007 Edmonton experienced rapid warming (Figure 2). Give your own interpretation of the weather situation over Western Canada around that time, as conveyed by Figures (3-7).

Terminology, Equations and Data.

- Nomenclature: $\vec{U}$ is a 3D velocity, $\vec{V}$ is the "horizontal" velocity (strictly, the components lying in a constant pressure surface), and $V_g = (u_g, v_g)$ is the geostrophic velocity. Actual and potential temperature, T and θ. Air density and pressure ρ, p.

- Unless otherwise stated, ∇ is to be interpreted as ∇_h (sometimes also designated ∇_p), the gradient on a constant pressure surface.

- $A_f = -\, v\, \frac{\partial f}{\partial s}$

  Rate of horizontal advection of the property f, expressed in natural coordinates. The unit vector $\hat{s}$ for the s-axis points downstream and parallel to the flow contours (eg. height contours), and v is the wind *speed*.

- $\Delta Z = Z_2 - Z_1 = \frac{R}{g}\, \overline{T}\, \ln \frac{p_1}{p_2}$

  The hypsometric equation (where $Z_2 > Z_1$). The left hand side is the (Z_2 to Z_1 hPa) thickness expressed in [dam], and $\overline{T}$ is the weighted mean temperature of the layer (weighting factor is p^{-1}). $R = 287\,\mathrm{J\,kg^{-1}\,K^{-1}}$ is the specific gas constant for dry air, and $g = 9.81\,\mathrm{m\,s^{-2}}$ is the gravitational acceleration.

- $\sigma = \frac{-RT}{p}\, \frac{\partial \ln \theta}{\partial p}$

  The static stability parameter [$\mathrm{Pa^{-2}\,s^{-2}}$] where (p, T) are the pressure and temperature; R is the specific gas constant; and θ is the potential temperature

Hourly Data Report for March 7, 2007

Time	Temp °C	Dew Point Temp °C	Rel Hum %	Wind Dir 10s deg	Wind Spd km/h	Visibility km	Stn Press kPa	Hmdx	Wind Chill	Weather
00:00	-8.5	-10.2	87	14	19	24.1	92.77		-16	Mostly Cloudy
01:00	-8.1	-9.9	87	15	17	24.1	92.65		-15	Mostly Cloudy
02:00	-7.7	-9.4	88	16	15	24.1	92.59		-14	Mostly Cloudy
03:00	-8.4	-10.0	88	14	11	24.1	92.53		-14	Mostly Cloudy
04:00	-9.1	-10.7	88	17	9	24.1	92.47		-14	Mostly Cloudy
05:00	-9.6	-10.9	90	20	13	19.3	92.38		-16	Cloudy
06:00	-9.8	-11.1	90	17	13	19.3	92.32		-16	Cloudy
07:00	-8.6	-10.4	87	17	11	16.1	92.19		-14	Mostly Cloudy
08:00	-7.2	-9.0	87	17	9	19.3	92.10		-12	Mostly Cloudy
09:00	-4.9	-8.0	79	16	7	24.1	92.05		-8	Mostly Cloudy
10:00	-3.7	-6.3	82	15	9	24.1	92.01		-7	Mostly Cloudy
11:00	0.0	-4.3	73	14	11	24.1	91.93		-4	Mainly Clear
12:00	2.2	-2.0	74	15	13	24.1	91.87			Mainly Clear
13:00	3.5	-1.5	70	17	19	24.1	91.78			Mainly Clear
14:00	4.3	-0.3	72	16	11	24.1	91.69			Mainly Clear
15:00	4.7	-0.8	67	15	7	24.1	91.64			Mainly Clear
16:00	4.5	-0.7	69	16	7	24.1	91.60			Mainly Clear
17:00	3.0	0.3	82	16	6	24.1	91.56			Mainly Clear
18:00	3.7	-0.7	73	12	6	24.1	91.55			Mostly Cloudy
19:00	1.9	-0.3	85	15	11	24.1	91.51			Mostly Cloudy
20:00	2.7	-0.5	79	18	11	24.1	91.44			Mostly Cloudy
21:00	2.2	-0.5	82	16	6	24.1	91.38			Mostly Cloudy
22:00	1.4	-1.1	83		0	24.1	91.38			Mostly Cloudy
23:00	1.0	-1.7	82	18	11	24.1	91.34			Mostly Cloudy

Figure 2: Hourly observations at Edmonton International Airport, 7 March 2007.

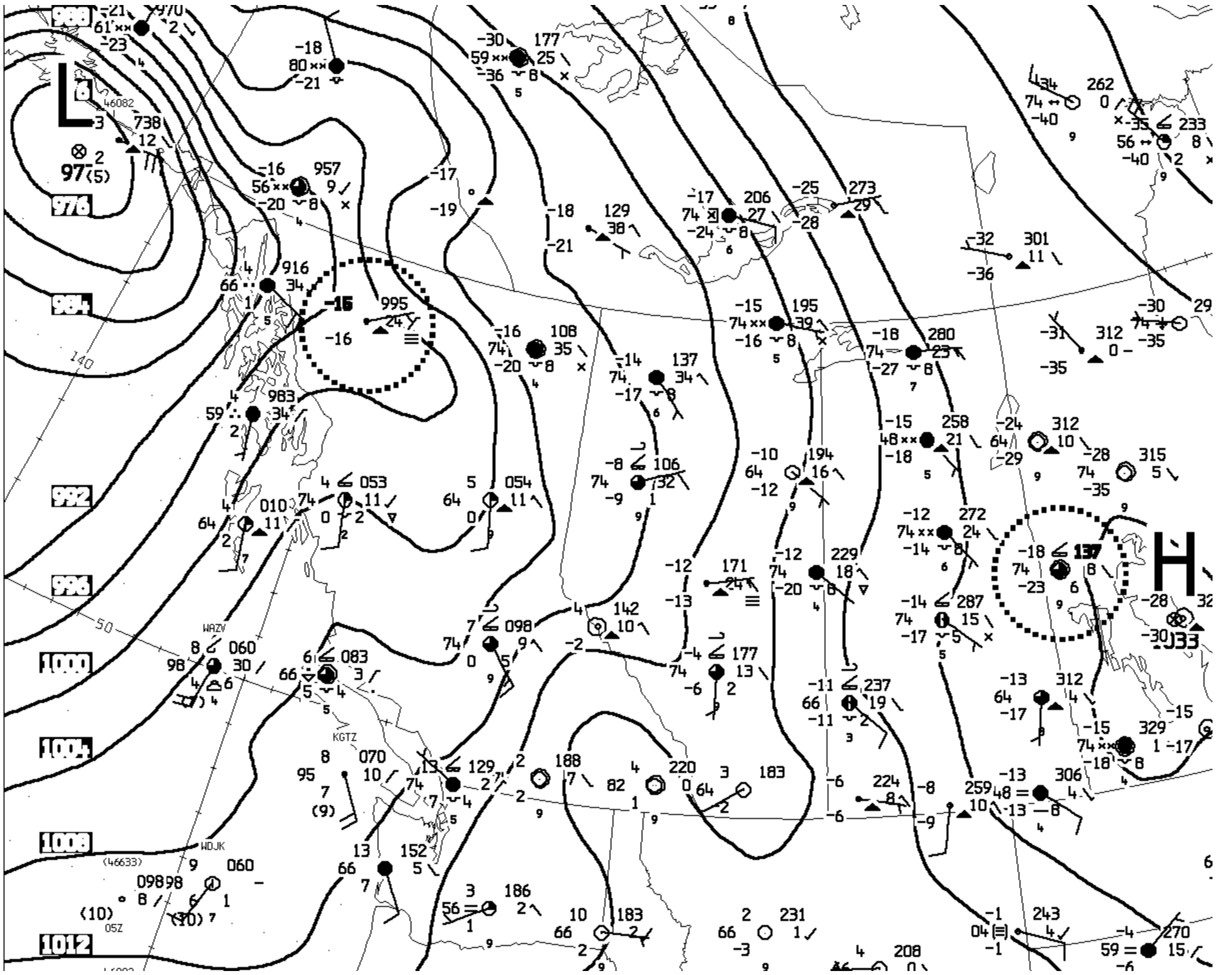

Figure 3: CMC surface analysis, 06Z, 7 March 2007.

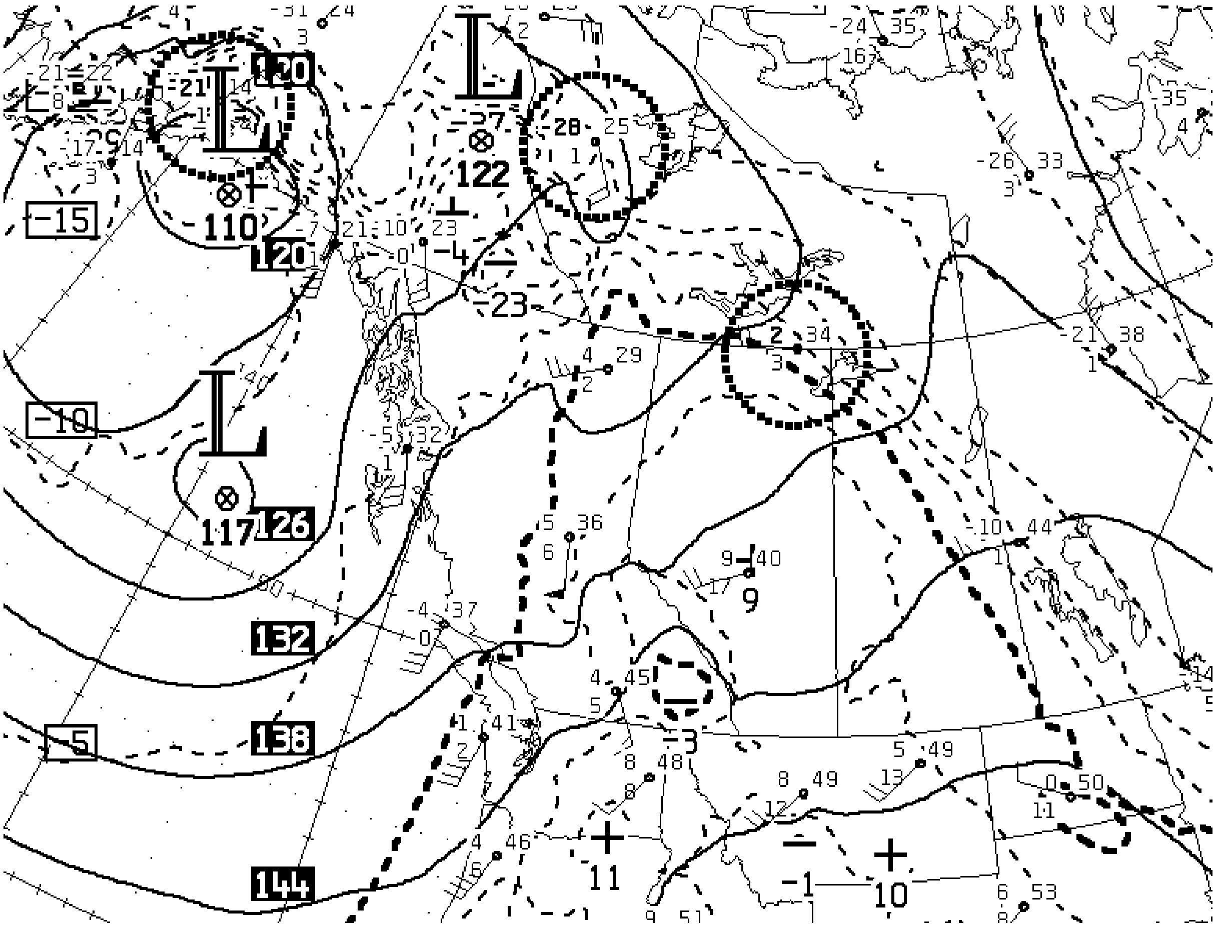

Figure 4: CMC 850 mb analysis, 12Z, 7 March 2007.

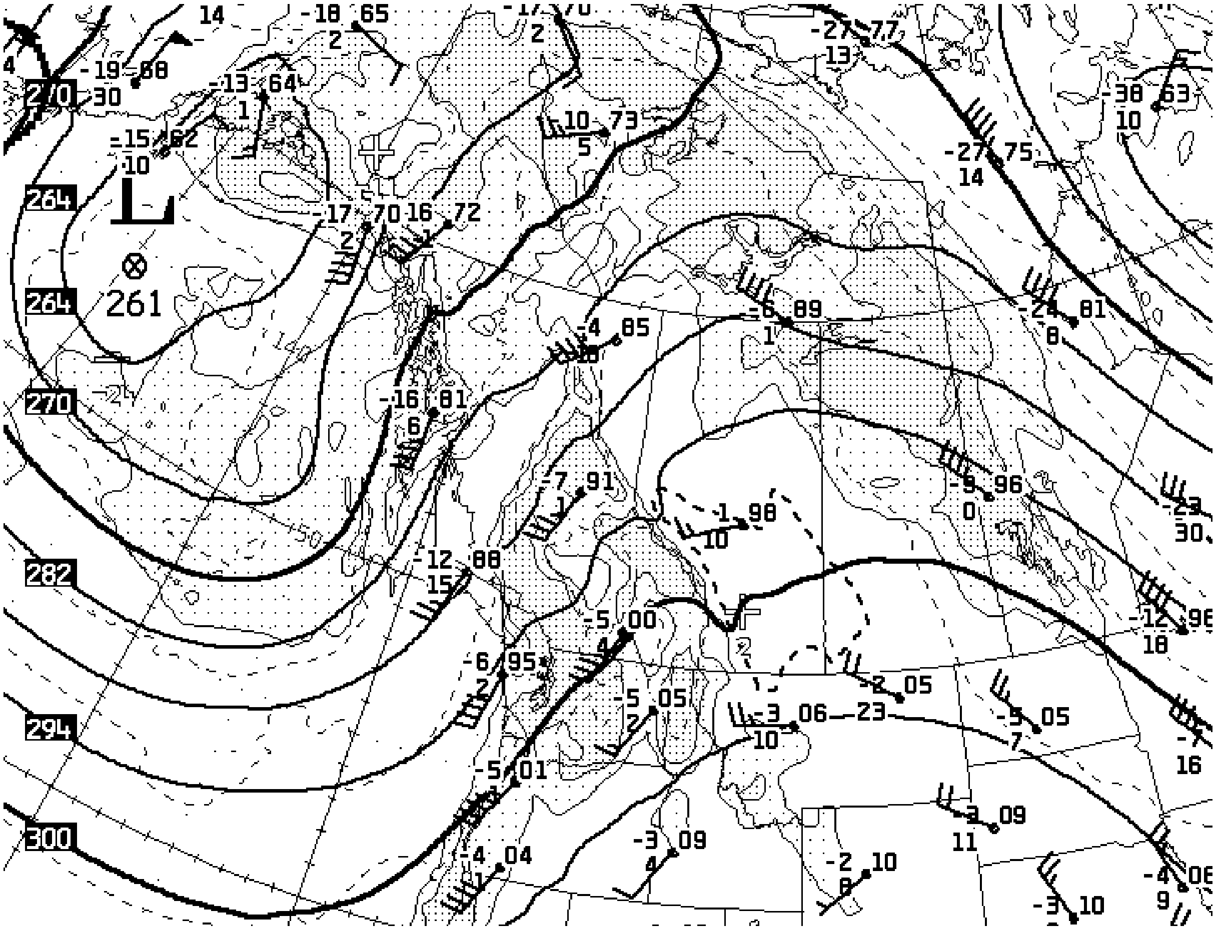

Figure 5: CMC 700 mb analysis, 12Z, 7 March 2007. Heavy (light) stippling, $T - T_d \leq 2^{o}$C (5^{o}C).

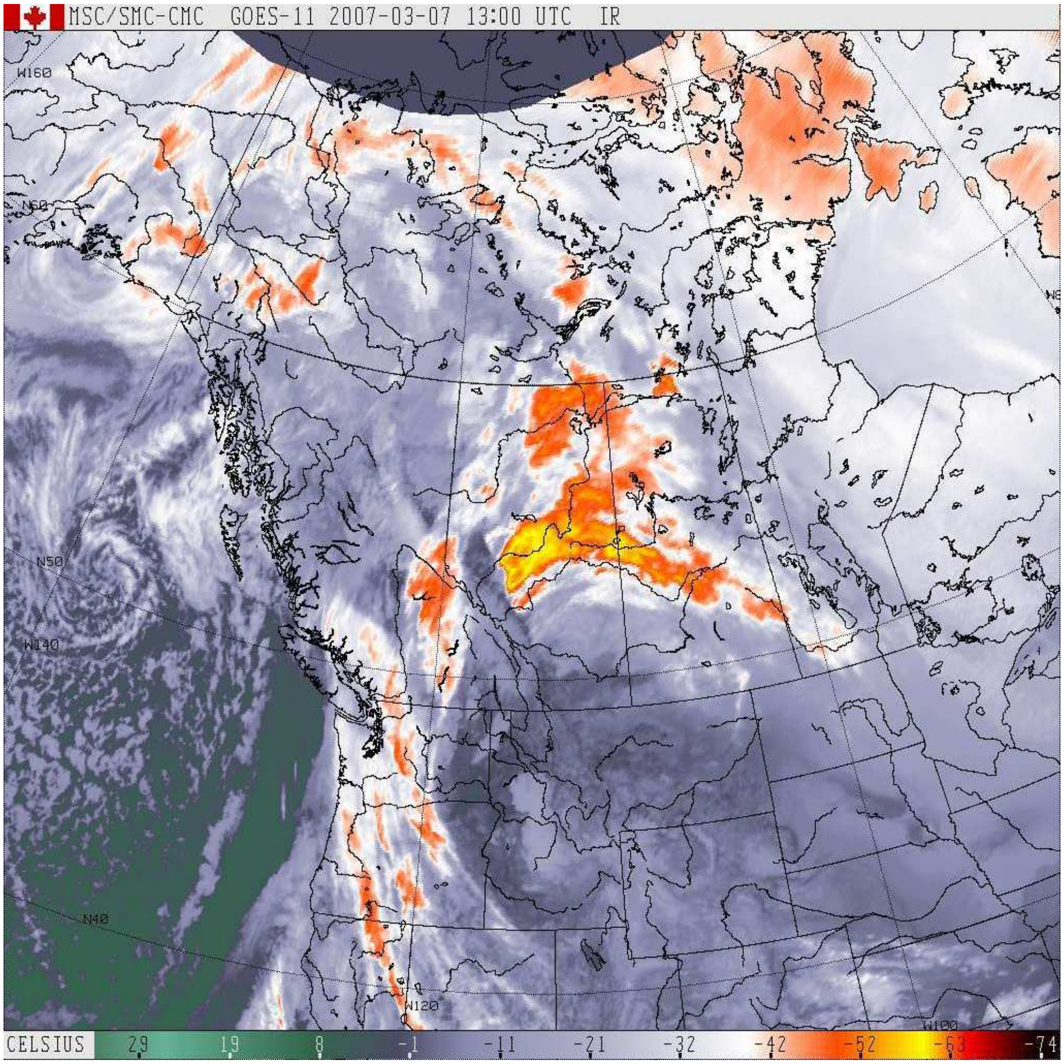

Figure 6: GOES west IR image, 12Z, 7 March 2007.

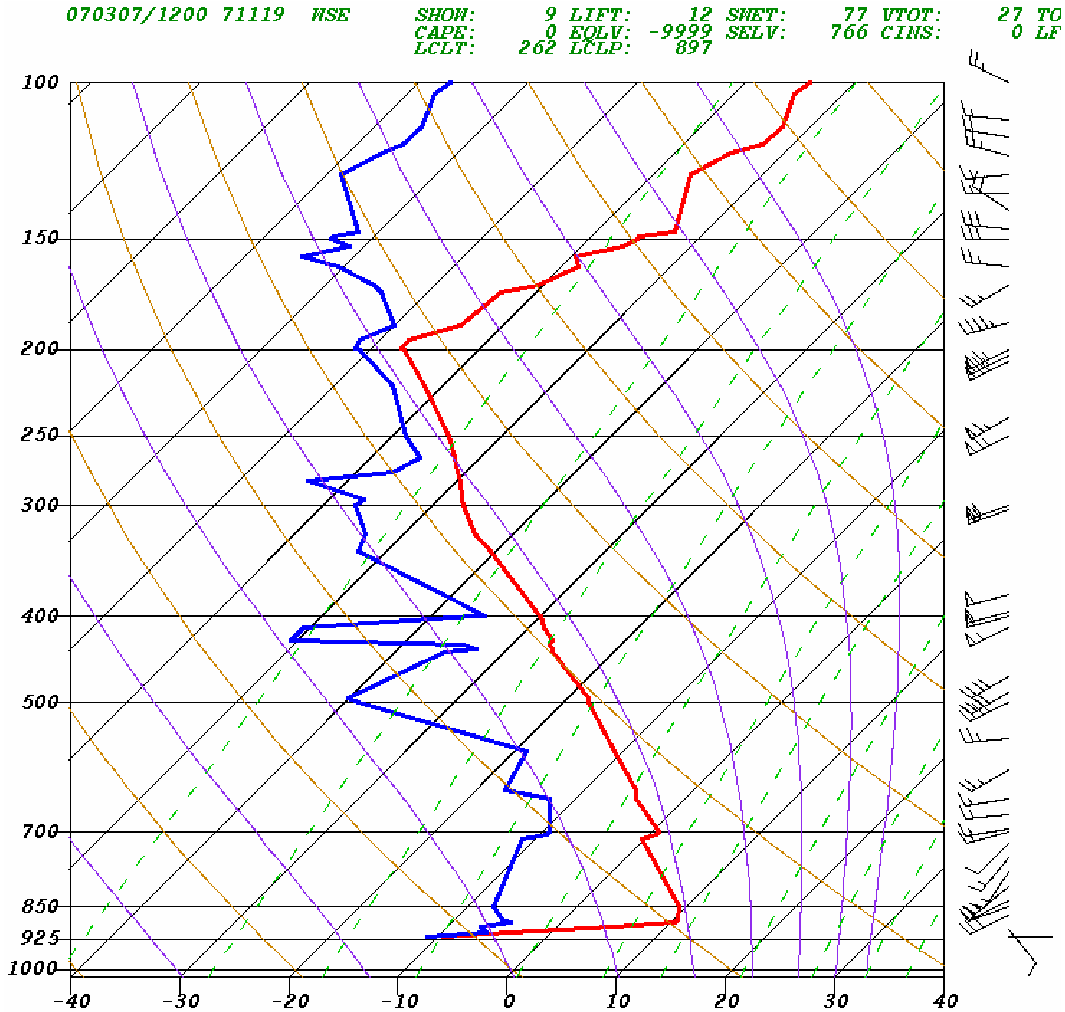

Figure 7: Edmonton (Stony Plain) sounding, 12Z, 7 March 2007.