EAS 471 $3^{\rm rd}$ Scored Assignment Due: 8 Apr. 2014

Lagrangian Stochastic Simulation of ASL Dispersion

In an earlier computing assignment you used an Eulerian method to calculate the mean concentration field $C(x,z)$ downwind from a continuous crosswind line source located at $x=0, z=h_s=0.46$ m in the horizontally-uniform, neutrally-stratified atmospheric surface layer ("hh_NSL"). In this assignment, complementing assignment 2, we implement a 1st-order Lagrangian stochastic model to calculate the crosswind-integrated concentration profile at fetch $x=100$ m for Project Prairie Grass dispersion experiments (Tables 1, 2) covering a wide range in atmospheric stratification.

Clarification of a symmetry

If Q_ℓ [$\mathrm{kg\,m^{-1}\,s^{-1}}$] is the strength of a line source, then the units of the ratio C/Q_ℓ are

$$\left[\frac{C}{Q_\ell}\right] = \frac{\mathrm{kg\,m^{-3}}}{\mathrm{kg\,m^{-1}\,s^{-1}}} \equiv \mathrm{s\,m^{-2}}\ .$$

Let Q_p [$\mathrm{kg\,s^{-1}}$] be the strength of a steady *point* source, and χ [$\mathrm{kg\,m^{-2}}$] the crosswind-integrated concentration it causes. Then

$$\left[\frac{\chi}{Q_p}\right] = \frac{\mathrm{kg\,m^{-2}}}{\mathrm{kg\,s^{-1}}} \equiv \mathrm{s\,m^{-2}}\ .$$

In neither case is the y-coordinate (Y) of our particles of any interest: both are effectively two-dimensional problems, and furthermore, they are the same problem.

The LS Model

Motion in a horizontally-homogeneous atmospheric surface layer (hh_ASL) can be adequately simulated by assuming the particle's horizontal velocity equal to the mean Eulerian velocity $\overline{u}(Z)$ wherever it happens to be located, while its vertical velocity is random. Specifically, we shall assume particle position (X, Z) evolves in finite steps

$$X(t + dt) = X(t) + U(t)\ dt\ , \tag{1}$$

$$Z(t + dt) = Z(t) + W(t)\ dt \tag{2}$$

where dt is the timestep. The particle's horizontal velocity is

$$U(t) = \overline{u}(Z(t)) \tag{3}$$

while the particle's vertical velocity W evolves in time according to the (unique) well-mixed, one-dimensional 1st-order LS model for Gaussian inhomogeneous turbulence, i.e.

$$dW = \left[- \frac{C_0\ \epsilon(z)}{2{\sigma_w}^2(z)}\ W + \frac{1}{2}\frac{\partial {\sigma_w}^2}{\partial z}\ \left(\frac{W^2}{{\sigma_w}^2} + 1\right)\right]\ dt + \sqrt{C_0\ \epsilon(z)}\ d\xi \tag{4}$$

where $d\xi$ is a Gaussian random variate with $\overline{d\xi} = 0$, $\overline{(d\xi)^2} = dt$. For each of the independent trajectories, the particle's initial vertical velocity should be a random Gaussian number with zero mean and standard deviation $\sigma_w(h_s)$.

Time step

The variable $2\ \sigma_w^2/(C_0\, \epsilon)$ can be interpreted as an effective timescale $T_L(z)$, and the timestep should vary with the particle's position on the height axis, according to

$$dt = \mu\ T_L(Z(t)) \tag{5}$$

where $\mu \ll 1$.

Reflection at ground

Wherever a particle attains a height $Z < z_r$, its trajectory must be reflected according to

$$Z = 2\, z_r - Z\ , \tag{6}$$

$$W = -W\ . \tag{7}$$

The reflection height $z_r \geq z_0$. You may find you can speed up calculations by assigning $z_r \gg z_0$, see instructions below.

Mean horizontal velocity

The mean (Eulerian) velocity should be computed using the Monin-Obukhov profile

$$\frac{k_v\,\overline{u}(z)}{u_*} = \ln\,\frac{z}{z_0} - \psi_m\left(\frac{z}{L}\right) + \psi_m\left(\frac{z_0}{L}\right) \tag{8}$$

where

$$\psi_m = \begin{cases} -5z/L & L \geq 0\,, \\ \\ 2\,\ln\left(\frac{1+\phi_m{}^{-1}}{2}\right) + \ln\left(\frac{1+\phi_m{}^{-2}}{2}\right) - 2\,\text{atan}\left(\phi_m{}^{-1}\right) + \frac{\pi}{2} & L < 0\,, \end{cases} \tag{9}$$

with

$$\phi_m(z/L) = \left(1 - 28\,\frac{z}{L}\right)^{-\frac{1}{4}}\,. \tag{10}$$

Standard deviation of the vertical velocity

Use the MO profiles

$$\frac{\sigma_w}{u_*} = \begin{cases} 1.25 & L \geq 0\,, \\ 1.25\ \left(1 - 3\,z/L\right)^{1/3} & L < 0\,. \end{cases} \tag{11}$$

TKE dissipation rate

The product $C_0\epsilon$ should be specified according to:

$$\frac{2\,\sigma_w{}^2}{C_0\,\epsilon} = \frac{0.5\,z}{\sigma_w}\left(1 +\ 5\frac{z}{L}\right)^{-1},\ L \geq 0\,, \tag{12}$$

$$\frac{2\,\sigma_w{}^2}{C_0\,\epsilon} = \frac{0.5\,z}{\sigma_w}\left(1 -\ 6\frac{z}{L}\right)^{\frac{1}{4}},\ L < 0\,. \tag{13}$$

These equations, in conjunction with the above formulae for σ_w, can be rearranged to obtain compute $\sqrt{C_0\,\epsilon(z)}$.

Deriving mean concentration from trajectories

Your program will compute an ensemble of $N_P \sim 10^5$ (or more) random paths from the source to a distance sufficiently far downstream from the point of observation that the trajectory can be abandoned, e.g. to $X = 105$ m. The profile of mean concentration is to be estimated from the mean particle residence times in a stack of sampling "volumes" centred on $x = 100$ m and having resolution D_x, D_z. The detectors can be indexed j and effectively, when each particle passes $x = 100$ m within detector j we increment the residence time counter t_j for that detector by the amount $D_x/\overline{u}(z_j)$. When all trajectories have been computed,

$$\left(\frac{u_* \chi}{Q} \right)_j = \frac{u_* \, t_j}{N_P \, D_x \, D_z} \, . \tag{14}$$

Each computed path contributes to the mean residence time only when it passes $x = 100$ m within the j^{th} layer. (Note the ultimate irrelevance of D_x, which cancels.).

Write-up

For this assignment, **no page limit applies**.

- Provide a flowchart describing your algorithm
- Provide a diagram that compares the normalized mean profiles $\overline{u}(z)/\overline{u}(h_s)$ for the four PPG runs and (likewise) a diagram comparing the four profiles of $\sigma_w(z)/\overline{u}(h_s)$
- In the case of PPG57, compare your Lagrangian simulations with the outcome of assignment 2 (advection-diffusion equation)
- In the case of PPG57, examine the impact on your solutions of increasing the reflection height z_r from z_0 to $50z_0$

- For each PPG experiment, plot your solution $u_* \chi(z)/Q$ versus z alongside the experimental points

PPG Data

Table 1: Normalized cross-wind integrated concentration $u_* \chi/Q$ [m^{-1}] observed at distance $x = 100$ m from the source (height $h_s = 0.46$ m) in several Project Prairie Grass runs.

z [m]	Run 57	Run 33	Run 50	Run 59
17.5	1.1E-4	1.3E-4	2.3E-4	0
13.5	4.5E-4	4.8E-4	7.1E-4	0
10.5	1.08E-3	1.17E-3	1.72E-3	0
7.5	2.42E-3	2.80E-3	3.41E-3	0.07E-3
4.5	0.55E-2	0.58E-2	0.61E-2	0.21E-2
2.5	0.86E-2	0.92E-2	0.85E-2	1.05E-2
1.5	1.06E-2	1.08E-2	0.96E-2	1.75E-2
1.0	1.12E-2	1.16E-2	1.00E-2	2.14E-2
0.5	1.17E-2	1.22E-2	1.07E-2	2.40E-2

Table 2: Micro-meteorological parameters for the above Project Prairie Grass runs.

	Run 57	Run 33	Run 50	Run 59
u_* [$\mathrm{m\,s^{-1}}$]	0.5	0.55	0.44	0.14
L [m]	-239	-93	-26	7
z_0 [m]	0.0058	0.0075	0.0033	0.005