EAS 471 First Lab (unscored) 2014

"Drunkard's walk" simulation of particle paths in homogeneous turbulence

Write a program to compute an ensemble of N_P (of order 10^3, say) independent particle trajectories $Z(t), X(t)$ originating at the origin (coordinates $x = z = 0$) at time $t = 0$ in unbounded, homogeous, stationary, two-dimensional turbulence. Assume the Eulerian velocity vector is of form $(\overline{u}, w)$ where the mean velocity on the z axis, i.e. $\overline{w}$, is zero. The needed velocity statistics are to be prescribed as:

$$\overline{u} = 1\,\mathrm{m\,s^{-1}}\,, \tag{1}$$

$$\sigma_w = 1\,\mathrm{m\,s^{-1}}\,, \tag{2}$$

$$\tau = 1\,s\,, \tag{3}$$

where σ_w is the standard deviation of w and τ is the turbulence time scale.

Trajectories are to be computed with discrete timestep $dt \ll \tau$, the algorithm being

$$dX = \overline{u}\,dt\,, \tag{4}$$

$$dZ = \sqrt{2\,K\,dt}\;r\,, \tag{5}$$

where r is a standardized Gaussian random number (i.e. a normally distributed random variate with mean value 0 and variance 1) and $K \equiv \sigma_w^2 \tau$ is the eddy diffusivity.

In terms of output from your simulation, please compute the ensemble mean[1] square displacement of particles on the z axis, $\langle Z^2(t) \rangle$, at times $t/\tau = (0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50)$. To speed up the calculation you may wish to vary the time step: e.g. for travel times $t/\tau \leq 1$ you might use $dt = 0.02$, and for longer travel times use $dt = 0.1$.

Plot the root mean square displacement $\sigma_z = \sqrt{\langle Z^2 \rangle}$ versus t. Compare your results (graphically) with the formula

$$\sigma_z = \sqrt{2\,\sigma_w^2\,\tau\,t}\ . \tag{6}$$

Briefly document your lab exercise and submit to Ginny for feedback. You should submit at least two figures, i.e. your curve of $\sigma_z = \sqrt{\langle Z^2 \rangle}$ versus t, and a sample of your trajectories as Z versus t (or versus $x \equiv \overline{u}t$). It will suffice to display only a handful of the actual trajectories (so as to give a "flavour" of what the simulation is doing), but your graph of $\sigma_z(t)$ should be based on a value of N_P that is sufficiently large to yield a reasonably smooth curve.

[1] An ensemble mean is often represented using the angle bracket $\langle\,\rangle$.