EAS471, "Atmospheric Modelling" <u>Exam</u> 15 April, 2014

<u>Professor</u>: J.D. Wilson <u>Time available</u>: 150 mins <u>Value</u>: 30%

Please answer in the exam booklet. Symbols have their usual meteorological interpretation. Data, given at the back, should be read before starting the exam.

Multi-choice (18 x $\frac{1}{2}$% = 9%)

1. With T_I^n representing the temperature at position $x = I\,\Delta x$ and time $t = n\,\Delta t$, the Dufort-Frankel scheme for the one-dimensional heat equation is

$$\frac{T_I^{n+1} - T_I^{n-1}}{2\Delta t} = \kappa\,\frac{T_{I+1}^n + T_{I-1}^n - \left(T_I^{n+1} + T_I^{n-1}\right)}{\Delta x^2}\,,$$

where κ is the thermal diffusivity. Which statement is **false**?

(a) κ has unit $[\mathrm{m^2\,s^{-1}}]$
(b) this is an implicit method xx
(c) this is a "leapfrog scheme"
(d) this method cannot be used to obtain T_I^1 from the initial state T_I^0
(e) this discretization is "consistent" with the heat equation

2. Non-linear computational instability (NLCI) of a finite difference scheme for the Navier-Stokes equations is a consequence of which factor(s)?

(a) inconsistency of the difference scheme with the underlying differential equation
(b) discretization of the advection- rather than the flux- form of the equation
(c) wave-wave interaction originating in non-linear terms (e.g. $u\,\partial u/\partial x$) and aliasing ✓✓
(d) truncation error and neglect of small terms in the differential equations
(e) staggering of the grid for different variables

3. Consider the vertical dispersion of a passive tracer gas from a point source in the horizontally-homogeneous atmospheric surface layer, and the suitability of its description by the Random Displacement Model (or Drunkard's Walk)

$$dZ = \frac{\partial K}{\partial z}\,dt + \sqrt{2\,K}\,d\zeta$$

(where dt is the time step, K is the eddy diffusivity, and $d\zeta$ is a Gaussian random variable with variance dt and vanishing mean). Which statement is **false**?

(a) this RDM is exactly consistent with an eddy diffusion treatment using the same eddy diffusivity
(b) the RDM is "grid-free" whereas Eulerian methods (such as solving the eddy-diffusion equation) entail a grid
(c) the RDM automatically conserves tracer mass
(d) the RDM is valid in both the near field and the far field of the source xx
(e) the first term on the right hand side ("drift term") tends to "push" computational particles towards higher heights

4. Suppose $L[\phi(x)] = 0$ (where $L[\ ]$ is a differential operator) is to be solved on $a \leq x \leq b$ using the Galerkin method with basis functions $\theta_j(x)$, viz. $\phi(x) = \sum_j a_j \theta_j(x)$. The (constant) expansion coefficients a_j are optimized by requiring that ____
   (a) $a_i a_j = \delta_{ij}$
   (b) the residual $e(x) = L\left[\sum_j a_j \theta_j(x)\right]$ must vanish
   (c) the residual $e(x)$ must be minimal
   (d) the covariance (over $a \leq x \leq b$) of the residual $e(x)$ with each basis function $\theta_j(x)$ must vanish ✓✓

5. Suppose that at $x = 0$ there were a step change between a uniform, dry upwind surface with surface temperature T_{01} and a cooler uniform, dry downwind surface with surface temperature $T_{02} < T_{01}$. According to the internal boundary layer (IBL) paradigm, the rate of growth dh/dx in the depth $h(x)$ of the cool internal boundary layer is controlled by which factor?
   (a) Reynolds stress $\overline{u'w'}$
   (b) mean vertical velocity $\overline{w}$
   (c) ratio $\sigma_w/\overline{u}(\alpha h)$, ($\alpha$ a constant) ✓✓
   (d) ratio $\overline{u'w'}/u_{*1}^2$ (u_{*1} being the upwind surface friction velocity)
   (e) mean pressure $\overline{p}$

6. Rider, Philip and Bradley performed a local advection experiment at Canberra airport, to test Philip's analytical theory. Having travelled a distance of 350 m over dry tarmac, the wind passed onto short grass. Measurements of daytime wind profiles in the lowest 2 m showed that mean wind speed only 16 m downwind from the boundary had decreased by as much as about $0.5\,\mathrm{m\,s^{-1}}$. Which statement is **irrelevant** or **wrong**, in terms of the probable cause for this deceleration?
   (a) the buoyant production term in the TKE budget would be positive over the grass **xx**
   (b) the grass surface likely had a larger roughness length than the tarmac (more drag)
   (c) the surface energy budget over the grass would differ from that over the tarmac
   (d) the grass surface would have been cooler than the tarmac, causing an inversion

7. In micro-meteorological models of so-called "disturbed flows", each "property" is subject to advection (e.g. $\overline{u}\,\partial\overline{T}/\partial x$ etc.). For an abstract mean variable $\phi = \phi(x, z)$, and letting $\phi_0(z)$ designate its profile at the "inflow" (i.e. upwind) boundary, which of the listed conditions is most suitable as a boundary condition on ϕ at the (downwind) "outflow" boundary (i.e. at large x)
   (a) $\phi = 0$
   (b) $\nabla \cdot \phi = 0$
   (c) $\phi = \phi_0(z)$
   (d) $\partial\phi/\partial z = 0$
   (e) $\partial\phi/\partial x = 0$ ✓✓

8. In the context of (modelling) the vertical velocity W ($= dZ/dt$) of a particle in turbulence, the generalized Langevin equation

$$dW = a(Z, W)\, dt \;+\; b(Z)\, d\zeta$$

(with coefficients a, b and Gaussian forcing $d\zeta$) implies a Fokker-Planck equation

$$\frac{\partial p(z, w, t)}{\partial t} = -\frac{\partial}{\partial z}\,(w\,p) - \frac{\partial}{\partial w}\,(a\,p) + \frac{b^2}{2}\,\frac{\partial^2 p}{\partial w^2}\,.$$

The former permits to compute an ensemble of (random) paths, while the latter gives the (deterministic) evolution of the joint density function $p(z, w, t)$ for particle position and velocity. Which statement is **false**?

(a) $p(z, w, t)$ has the unit $[\mathrm{m}^{-1}\,(\mathrm{m\,s}^{-1})^{-1}]$
(b) $p(z, w, t)$ is a probability density defined in a $z - w$ phase space
(c) (wp) is a convective flux of probability along the z-axis of phase space
(d) (ap) is a diffusive flux of probability along the w-axis of phase space xx
(e) the final term on the right hand side is the divergence of a diffusive flux of probability

9. The conservation equation for vertical velocity variance $\overline{w'^2}$ in steady state, horizontally-homogeneous atmospheric boundary-layer turbulence is

$$\frac{\partial \overline{w'^2}}{\partial t} = 0 = 2\,\frac{g}{T_0}\,\overline{w'T'} - \frac{\partial\,\overline{w'^3}}{\partial z} + \frac{2}{\rho_0}\,\overline{p'\,\frac{\partial w'}{\partial z}} + \nu\,\frac{\partial^2\,\overline{w'^2}}{\partial z^2} - \epsilon_{ww}$$

where ϵ_{ww} represents viscous conversion of kinetic energy (in w') to heat (and T_0, ρ_0 are mean temperature and density of the layer). Which statement is **false**?

(a) $\overline{w'^2}$ ($\equiv \sigma_w^2$) does not obey Monin-Obukhov scaling xx
(b) the term $2\,(g/T_0)\,\overline{w'T'}$ is "buoyant production"
(c) $-\,\partial\overline{w'^3}/\partial z$ is a (turbulent) transport term
(d) $\nu\,\partial^2\overline{w'^2}/\partial z^2$ is the molecular transport term ($\nu =$ const.)
(e) the term involving pressure fluctuations transfers kinetic energy in w' to or from the u', v' kinetic energy pools ("redistribution")

10. Which of the following statements in regard to the "convective boundary layer," or models of it, is untrue?

(a) mean horizontal wind speed may be treated as being height-independent ($U =$ const.)
(b) the probability density function for vertical velocity fluctuations is Gaussian xx
(c) mean wind direction may be treated as height-independent
(d) a "mixed layer" (constant potential temperature) lies between the (unstable) surface layer and the capping inversion
(e) a fraction $A \sim 0.2 - 0.4$ of the horizontal plane is (on average) rising, the complementary fraction $1 - A$ descending

11. Consider a plume of gas which is emitted continuously from a point source in the boundary layer, and let the mean concentration of the gas be $C(x, y, z)$ [$\mathrm{kg\,m^{-3}}$]. Assuming that the x-axis is parallel to the mean wind, what is signified by the property

$$\langle z \rangle = \frac{\int_S \; z\, C(x, y, z)\, dy\, dz}{\int_S \; C(x, y, z)\, dy\, dz}$$

where $\int_S \, dy\, dz$ signifies an area integral over $\infty \le y \le \infty$, $0 \le z \le \infty$?

(a) alongwind flux of the gas [$\mathrm{kg\,s^{-1}}$] at x, equal to the source strength Q
(b) height of the centre of mass of the plume at x ✓✓
(c) mean height of reflection of the gas plume off the surface
(d) height at which the mean horizontal velocity U is a maximum (low level jet)
(e) effective reflection height of fluid elements released at the source

12. Gridpoint computations for the influence of unresolved scales of motion in the ABL on the resolved absolute humidity ρ_v will involve the equation

$$\left(\frac{\partial \rho_v}{\partial t}\right)_{\text{physics}} = - \; [.] \; \overline{w' \rho_v'} \qquad (1)$$

The missing operator "[.]" is ____

(a) $\partial/\partial z$ ✓✓
(b) $\partial/\partial x$
(c) $U\,\partial/\partial x + \; V\,\partial/\partial y$ (U, V the resolved horizontal velocity components)
(d) $W\,\partial/\partial z$ (W the resolved vertical velocity)
(e) $K\,\partial/\partial z$ (K the eddy diffusivity)

13. The sheltering effect of a long thin porous windbreak can be computed by solving the Reynolds-averaged Navier-Stokes ("RANS") equations, in which it is necessary to parameterize the interaction of the flow with the windbreak. Assuming the barrier (height H) is located at $x = 0$ and oriented along the crosswind (y) axis, its drag on the wind may be represented by a sink S_U on the right hand side of the mean streamwise (U-) momentum equation,

$$S_U = - \; k_r \; U\,\sqrt{U^2 + W^2}\;\delta(x - 0)\; s(z - H)\;,$$

where k_r is a dimensionless "resistance coefficient" of the barrier; the localizing step function $s(z - H)$ is unity for $z \le H$ and zero otherwise. The primary effect of the momentum sink is to induce a streamwise pressure gradient $\partial P/\partial x$ that is ____ of the barrier

(a) adverse upwind and favourable downwind
(b) favourable upwind and adverse downwind
(c) infinite "at" the barrier ($x = 0,\; z \le H$) and zero upwind or downwind
(d) favourable (accelerating) both upwind and downwind
(e) adverse (decelerating) both upwind and downwind ✓✓

14. Suppose a surface-layer flow along the x-axis encounters an infinitely-long crosswind barrier (a fence or shelterbelt) standing at $x = 0$. If a subscript "0" designates a flow property far upwind of the barrier, which of the following statements is true? (Notes: k is turbulent kinetic energy; surface layer flows are non-divergent, viz. $\nabla \cdot \mathbf{u} = 0$, and their density ρ can be treated as constant)

    (a) $\int_{z_0}^{\infty} k(x,z)\, dz \;=\; \int_{z_0}^{\infty} k_0(z)\, dz$ for $x > 0$

    (b) $\int_{z_0}^{\infty} \overline{u}(x,z)\, dz \;=\; \int_{z_0}^{\infty} \overline{u}_0(z)\, dz$ for $x > 0$ ✓✓

    (c) surface friction velocity $u_*(x) = u_{*0}$ for $x > 0$ (note: wind drag on the ground $\tau = \rho u_*^2$)

    (d) $\overline{w}(x,z) = 0$ for all x, z

    (e) $\partial\overline{u}/\partial x \gg \partial\overline{w}/\partial z$ for $x \gg 0$

15. Sometimes the rate Q of transfer of a gas to the atmosphere from a small ground-level area source can be most conveniently estimated by "inverse dispersion," i.e. inferred from the mean concentration rise C caused by the source at one or more downwind points. An effective basis for inverse dispersion is the bLS ("backward Lagrangian stochastic") method, whose essence is to deduce the C/Q ratio from the touchdown points (x_0, y_0) and vertical velocities (w_0) of an ensemble of N_P backwards-in-time trajectories that all pass through the concentration detector providing C,

$$\frac{C}{Q} = \frac{1}{N_P} \sum_i \frac{2}{|w_0|}$$

    where the sum runs over all touchdowns within the boundary of the source. Provided that the wind field is horizontally-homogeneous, the trajectories can be computed using a simple Lagrangian model driven by Monin-Obukhov profiles of mean horizontal velocity $\overline{u}(z)$, vertical velocity variance $\sigma_w^2(z)$ and TKE dissipation rate $\epsilon(z)$ — in which case the method can be called "MO-bLS." Which statement regarding MO-bLS is **false**?

    (a) C and Q are in linear proportion (doubling Q doubles C, if nothing else changes)
    (b) the averaging interval for C (and for meteorological inputs such as u_*, L) could be as short as 30 seconds, without loss of accuracy **xx**
    (c) periods of extreme stability or instability are unsuitable
    (d) the significance of $2/|w_0|$ is that it is related to the time $2\delta z/|w_0|$ that the particle spends within a (shallow) ground-based layer of depth δz during a touchdown
    (e) when Q^{bLS}/Q is averaged over many (say ~ 10) suitable intervals, the level of accuracy for $\langle Q^{\text{bLS}}/Q \rangle$ is found to be about 10-20%

16. Assume a particle is subject to convection on the z-axis, and that there is no diffusive contribution to the motion. Which option for the transition density function

$$p(z_2, t + \Delta t | z_1, w_1, t)$$

for changes in state over a finite interval Δt is correct? (w is the velocity coordinate of the $z - w$ phase space, and $\delta(\, , \,)$ is the Dirac delta function)

(a) $p(z_2, t + \Delta t | z_1, w_1, t) = 0$
(b) $p(z_2, t + \Delta t | z_1, w_1, t) = w_1 \delta t$
(c) $p(z_2, t + \Delta t | z_1, w_1, t) = \delta(z_2, z_1)$
(d) $p(z_2, t + \Delta t | z_1, w_1, t) = \delta(z_2, z_1 + w_1 \Delta t)$ ✓✓

17. In GCMs and weather models the rate of sensible heat exchange Q_H between the atmosphere and the underlying surface is modelled by a "bulk transfer law"

$$Q_H \; = \; \alpha \, \rho_a \, c_p \, U \, (T_s - T_a)$$

where T_s is the surface temperature, T_a is the temperature at the lowest model gridpoint, U is wind speed at the lowest gridpoint, ρ_a is air density, $c_p \, [\mathrm{J\,kg^{-1}\,K^{-1}}]$ is the specific heat of air at constant pressure, and α is a dimensionless transfer coefficient (whose value reflects characteristics of the surface, such as its roughness length, and the height of the lowest gridpoint). Which theory or methodology might one exploit to determine the value of the coefficient α?

(a) a Galerkin method
(b) a finite difference method
(c) a Monte Carlo method
(d) Shannon's sampling theorem
(e) Monin-Obukhov similarity theory ✓✓

18. Which of the following NWP models uses a spectral representation of the horizontal structure of the atmosphere, and is formulated in (vertical) vorticity and (horizontal) divergence?

(a) NCEP's GFS (Global Forecast System) ✓✓
(b) NCEP's NAM-WRF (North American Mesoscale)
(c) CMC's GEM (Global Environmental Multiscale) "regional"
(d) CMC's GEM (Global Environmental Multiscale) "global"

Paragraph answer: $\qquad$ 3 x 2% = 6%

1. Why do NWP models often feature a "nested" higher-resolution domain within an outer domain, and what is the function of the solution on the outer domain?

   To save computing time/improve accuracy over a subregion; outer solution provides b/conds

2. List processes that are normally classified as belonging to the set of model "parameterizations" (or "grid point computations" or "model physics") in NWP models

   Solar radiation, longwave radiation, clouds and precipitation, drag on unresolved terrain, gravity waves, unresolved (eddy) transport in the ABL, surface-atmosphere exchange fluxes,...

3. The "convective velocity scale"

   $$w_* = \left[\frac{g}{\theta_0} \, \delta \, \left(\overline{w'\theta'}\right)_{\rm sfc} \right]^{1/3}$$

   where θ_0 is mean potential temperature, δ is boundary layer depth and $\overline{w'\theta'})_{\rm sfc}$ is the kinematic surface heat flux density. Assuming plausible summer daytime values for all quantities, compute implied values for w_* and for the convective time scale δ/w_*.

   $\delta \sim 1000 - 2000$ m, $\overline{w'\theta'} \equiv Q_H/(\rho c_p) \sim [100 - 200 \ \mathrm{W\,m^{-2}}]/1000 \rightarrow w_* \sim 2 \ \mathrm{m\,s^{-1}} \rightarrow \delta/w_* \sim 500 - 1000$ s. Reasonable variations of the numbers were accepted (but not, for instance, $\delta = 10$ km).

Long answer: $3 \times 5\% = 15\%$

*Answer any **three** questions from this section. Use point or essay format, as you wish. Do document your steps and your reasoning, as marks will be assigned for partial answers.*

1. Describe the logic and the qualitative results given by Delage's eddy viscosity/diffusivity model of the cooling of a horizontally-homogeneous nocturnal boundary layer (NBL) from an initially neutral state. (Some relevant equations are given as data).

   Solves the coupled 1D, time-dependent U, V, θ equations (given as data), with the eddy viscosity/diffusivity specified as $K = \lambda(z)\sqrt{c_e\, k}$. A simplified TKE ($k$) equation is solved, while the turbulence length scale is specified by a simple formula

$$\frac{1}{\lambda} = \frac{1}{k_v z} + \frac{c}{L} + \frac{1}{\lambda_\infty} \, .$$

   (c being a constant) which gives the correct surface layer variation, imposes a maximum value $\lambda \leq \lambda_\infty$ for the neutral (initial) state, and limits the length scale more strongly in stable stratification.

   Both wind components, and θ, are held constant far aloft. The initial state is neutral, i.e. $\theta(z, 0) = \theta_{00} =$ const., and the surface temperature is "driven" by an imposed cooling rate $\theta(0, t) = \theta_{00} - \gamma t)$. A ground-based inversion develops. The resulting effects of stable stratification are impressed through the TKE equation (in which the buoyany production term will be a sink for TKE) and through the second term in the formula for λ: as L (which is positive) becomes numerically small, that term potentially dominates the expression for λ, limiting its growth.

   In addition to the growth of an inversion, a low level windspeed "jet" develops. Near ground the TKE remains high due to the large wind shear, but aloft it decays. As there is no mechanism other than eddy transport to "communicate" the influence of surface cooling aloft, the inversion can be said to limit its own depth, because it suppresses vertical mixing. (No student chose to answer this question.)

2. Calder gave the solution

$$\frac{u_* \, \chi(x,z)}{Q} = \frac{1}{k_v \, x} \exp\left(- \frac{U_0 \, z}{k_v \, u_* \, x} \right)$$

for the normalized mean crosswind-integrated concentration at (x, z) due to a continuous point source of strength Q on ground at $x = 0$, valid under the restrictions that the mean horizontal wind velocity is treated as height independent (value, U_0) and that the eddy diffusivity is $K(z) = k_v u_* z$ (it is assumed there is no flux to ground, i.e. the ground is non-absorbing).

Deduce the apparent (effective) mean wind speed U_0 for Project Prairie Grass run 57 ($u_* = 0.5\,\mathrm{m\,s^{-1}}$, $z_0 = 0.0058$ m), given the fact that at distance $x = 100$ m downwind from the source the (observed) normalized XWIC at $z = 1$ m was

$$\left(\frac{u_* \chi(100, 1)}{Q} \right)^{\mathrm{obs}} = 0.0112 \; .$$

Compare your inferred wind speed U_0 with the "true" windspeed $U(h)$ at the PPG source height ($h = 0.46$ m) that is implied by the log wind profile (given as data).

Substituting the known information we have

$$0.0112 \times 0.4 \times 100 = \exp\left[- U_0 \, \frac{1}{0.4 \times 0.5 \times 100} \right]$$

and taking the logarithm of each side

$$\ln(0.448) = -0.8029 = -\, U_0 \, \frac{1}{20}$$

so that $U_0 = 16.06\mathrm{m\,s^{-1}}$. This is a surprisingly large value, because according to the log wind profile — which is very reliable — the given values of u_*, z_0 imply

$$\overline{u}(h_s) = \frac{u_*}{k_v} \ln \frac{h_s}{z_0} = \frac{0.5}{0.4} \ln \frac{0.46}{0.0058} = 5.47 \mathrm{\ m\,s^{-1}}$$

(it is legitimate to ignore the effect of stratification because for Run 57, $|L|$ is very large).

3. Calibrate the coefficient c_e in the eddy viscosity

$$K = \lambda \sqrt{c_e \, k} \ ,$$

by requiring that the steady-state, horizontally-homogeneous TKE equation

$$0 = K \left(\frac{\partial U}{\partial z} \right)^2 + \frac{\partial}{\partial z} \left(K \, \frac{\partial k}{\partial z} \right) - \frac{(c_e \, k)^{3/2}}{\lambda}$$

is exactly satisfied in an idealized neutrally stratified atmospheric surface layer, for which

$$\begin{aligned} \lambda(z) &= k_v \, z \, , \\ U(z) &= \frac{u_*}{k_v} \, \ln \left(\frac{z}{z_0} \right) \, , \\ k &= \frac{c_u^2 + c_v^2 + c_w^2}{2} \, u_*^2 \, . \end{aligned}$$

One could begin by recalling that for the neutrally stratified atmospheric surface layer

$$K = k_v \, u_* \, z \ ,$$

so that right away, by equating the two expressions for K, we have a tentative identification:

$$c_e = \frac{u_*^2}{k} = \frac{2}{c_u^2 + c_v^2 + c_w^2} \ .$$

But does this result satisfy the TKE equation? The latter simplifies, as clearly k is height-independent for this stated (reference) flow. Substituting,

$$0 = \lambda \, \sqrt{c_e \, k} \, \left(\frac{u_*}{k_v \, z} \right)^2 - \frac{(\sqrt{c_e \, k})^3}{\lambda} \ ,$$

and if we multiply through by $\lambda / \sqrt{c_e \, k}$ we have

$$0 = \lambda^2 \, \frac{u_*^2}{(k_v \, z)^2} - (\sqrt{c_e \, k})^2 \ = \ u_*^2 - c_e \, k \ .$$

Thus the TKE equation is satisfied provided $c_e = u_*^2 / k$, the same result as obtained by the first-guess method. (This problem had been given as an exercise. Only the second paragraph was essential to obtain the desired "calibration").

4. List the sequence of procedures and assumptions (or restrictions or approximations) that lead from a rigorous statement

$$\frac{\partial c}{\partial t} = -\nabla \cdot \mathbf{F} + Q$$

of mass conservation (c being instantaneous, local concentration and $\mathbf{F}$ the vector mass flux density) to the Gaussian plume model

$$\chi(x,z) = \frac{Q}{\sqrt{2\pi}\,\sigma_z\,U}\left[\exp\left(-\frac{(z-h)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+h)^2}{2\sigma_z^2}\right)\right]$$

for the crosswind-integrated concentration due to a continuous point source of strength Q at $(x,z) = (0,h)$ over a non-absorbing surface ($U =$ const. is the wind speed; $\sigma_z = \sigma_z(x)$ is usually specified empirically).

- Assume steady state
- Assume horizontal homogeneity *of the meteorological state* (but not of concentration statistics))
- Neglect in-situ production/destruction ($Q = 0$)
- Reynolds average, to obtain

$$\frac{\partial \overline{c}}{\partial t} = 0 = -\frac{\partial \overline{F}_x}{\partial x} - \frac{\partial \overline{F}_y}{\partial y} - \frac{\partial \overline{F}_z}{\partial z}$$

- identify the fluxes:
  - neglect molecular diffusion, so that these fluxes are entirely convective
  - $\overline{F}_x = \overline{u}\,\overline{c} + \overline{u'c'}$ etc.
  - neglect $\overline{u'c'}$ relative to $\overline{u}\,\overline{c}$
  - recall (or assume) $\overline{w} = 0$ so that $\overline{F}_z = \overline{w'c'}$
  - drop the term $\partial \overline{F}_y / \partial y$ (it can't affect crosswind-integrated concentration)
- introduce eddy diffusion closure

$$\overline{w'c'} = -K\,\frac{\partial \overline{c}}{\partial z}$$

- neglect wind shear, $\overline{u}(z) \to U_0$
- assume $K =$ const.
- resulting advection-diffusion eqn. for crosswind-integrated concentration is

$$U_0\,\frac{\partial \chi}{\partial x} = K\,\frac{\partial^2 \chi}{\partial z^2}$$

- solution in unbounded space for a source of strength Q at $z = h$ is the first term in the given Gaussian plume model; the second term is an "image term" which is added to ensure there is no vertical flux of the gas across $z = 0$ (i.e. ensures a non-absorbing surface)

5. Describe the main features of Kuo's parameterization for cumulus convection.

- Kuo scheme is a "gridpoint computation," i.e. each column is treated independently of neighbouring columns.
- If the scheme is applied, it assigns for the column a fractional area μ that is covered by deep cumulus clouds
- For each layer within which the cumulus convection is diagnosed as operating, temperature is corrected as

  $$T(z) = \widetilde{T}(z) + \mu \left(T_c(z) - \widetilde{T}(z) \right)$$

  where $\widetilde{T}$ is the model temperature before application of the scheme, and $T_c(z)$ is the (moist adiabatic) temperature profile within the cloud
- Similar correction is made for humidity
- Sensible and latent heat, and water mass (all forms) conserved on a layer-integrated basis
- Criteria for application of the scheme
  - conditional instability of a deep layer
  - plus low level convergence
- Cloud location and state: lift a surface parcel along dry adiabat to LCL (= cloud base), continue along a moist-adiabat to level of non-buoyancy (= cloud top)
- Vapour supply evaluated as sum of large-scale convergent advection of vapour plus surface evaporation
- Vapour needed to form the cloud is partitioned into
  - part that is condensed to raise temperature from $\widetilde{T}$ to T_c
  - part that raises humidity to saturation at temperature T_c
- Cloud fraction μ evaluated as ratio of vapour needed to vapour supply
- Scheme takes no account of CAPE

Data

- Under eddy diffusion closure the horizontal momentum equations in the horizontally-uniform ABL simplify to

$$\begin{aligned}
\frac{\partial U}{\partial t} &= \frac{\partial}{\partial z}\left[K\,\frac{\partial U}{\partial z}\right] + f\,(V - V_g)\ , \\
\frac{\partial V}{\partial t} &= \frac{\partial}{\partial z}\left[K\,\frac{\partial V}{\partial z}\right] - f\,(U - U_g)\ ,
\end{aligned}$$

  where the geostrophic wind components U_g, V_g parametrize the synoptic scale pressure gradient (and can be regarded as constants). The corresponding equation for the mean potential temperature is

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[P_r^{-1} K\,\frac{\partial \theta}{\partial z}\right]\ ,$$

  (where P_r, of order 1, is the turbulent Prandtl number) while the turbulent kinetic energy (TKE, k) equation is

$$\frac{\partial k}{\partial t} = K\left[\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2\right] - \frac{g}{\theta_0}\,P_r^{-1} K\,\frac{\partial \theta}{\partial z} + \frac{\partial}{\partial z}\left[\gamma^{-1} K \frac{\partial k}{\partial z}\right] - \epsilon,$$

  ($\gamma \sim 1$ is analogous to the Prandtl number). A popular model for the eddy viscosity and TKE dissipation rate is

$$\begin{aligned}
K &= \lambda\,\sqrt{c_e\,k}\ , \\
\epsilon &= \frac{(c_e\,k)^{3/2}}{\lambda}\ ,
\end{aligned}$$

  where $\lambda = \lambda(z)$ is a suitable length scale (whose asymptotic value near ground is $k_v z$) and c_e is a tunable dimensionless coefficient.

- The micrometeorology of ‘disturbed atmopheric surface layer flows” is often investigated in the context of two-dimensional cases, such that the symmetry (constancy) along one horizontal axis (habitually designated y) is preserved. In that case the mean streamwise momentum equation, assuming the x-coordinate is aligned with the mean wind direction, is

$$U\,\frac{\partial U}{\partial x} + W\,\frac{\partial U}{\partial z} = -\,\frac{1}{\rho_0}\,\frac{\partial P}{\partial x} - \frac{\partial \overline{u'^2}}{\partial x} - \frac{\partial \overline{u'w'}}{\partial z}$$

  where ρ_0 is the mean density (in most cases the Coriolis term can be neglected for microscale flows). This is an example of the Reynolds-averaged Navier-Stokes (RANS) equations.

- Dimensionless mean wind shear in the horizontally-uniform atmospheric surface layer

$$\frac{k_v\, z}{u_*}\,\frac{\partial \overline{u}}{\partial z} = \phi_m\left(\frac{z}{L}\right)$$

where $\phi_m(z/L)$ is a universal dimensionless Monin-Obukhov. As $|z/L| \to 0$ (i.e. in the neutral limit), $\phi_m \to 1$.

- The log wind profile for a neutrally stratified, horizontally homogeneous surface layer

$$\overline{u} = \frac{u_*}{k_v}\,\ln\frac{z}{z_0}$$

where the von Karman constant $k_v = 0.4$, and z_0 is the surface roughness length.

- Dimensionless mean temperature gradient in the horizontally-uniform atmospheric surface layer

$$\frac{k_v\, z}{T_*}\,\frac{\partial \overline{T}}{\partial z} = \phi_h\left(\frac{z}{L}\right)$$

where $\phi_h(z/L)$ is a universal dimensionless Monin-Obukhov. As $|z/L| \to 0$ (i.e. in the neutral limit), $\phi_h \to 1$.

- The Obukhov length

$$L = \frac{-\,u_*^3}{k_v\,\frac{g}{T_0}\,\overline{w'T'}}$$

where u_* is the friction velocity, k_v $(= 0.4)$ is the von Karman constant, T_0 [K] is the mean temperature of the atmospheric surface layer, and $\overline{w'T'}$ $[\equiv Q_H/(\rho c_p)]$ is the mean vertical (kinematic) flux density of sensible heat.