EAS 471 "Optional Computing Task" Jan., 2010

## Advection in 1D – optional supplementary exercises

In the basic exercise you numerically integrated the linear advection equation

$$\frac{\partial \theta}{\partial t} + U \frac{\partial \theta}{\partial x} = 0 \tag{1}$$

(with velocity $U \equiv 1$) on domain $-1 \leq x \leq 1$, subject to the periodic boundary condition $\theta(-1,t) = \theta(1,t)$ and the initial condition

$$\theta(x,0) = \sin(\pi x) \ . \tag{2}$$

You used the algorithm

$$\frac{\theta_I^{n+1} - \theta_I^n}{\Delta t} + U \frac{\theta_I^n - \theta_{I-1}^n}{\Delta x} \ , \tag{3}$$

and found this "forward-in-time, backward-in-space" discretization is stable provided $0 \leq U\Delta t/\Delta x \leq 1$.

## Alternative discretization of the linear problem

- Try the Euler method (forward-in-time, central-in-space) or other methods referred to in our Course Booklet (note: as of January 22, 2010 we have not yet covered "implicit" methods)

## Non-linear advection

You might find it interesting to have a go at solving the *non*-linear advection equation,

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} = 0 . \tag{4}$$

Again, take domain $-1 \leq x \leq 1$, and use the periodic boundary condition $U(-1, t) = U(1, t)$ and the initial condition

$$U(x, 0) = U_0(x) = \sin(\pi x) . \tag{5}$$

Go back to the forward-time/backward-space algorithm. You should see your “wave” steepen — and your solution will probably go unstable.