EAS 471 "Optional Computing Task" Jan., 2010

Calculation of CAPE (file update: January 24, 2010)

Task: Compute the Convectively Available Potential Energy (CAPE, units being $\mathrm{J\,kg^{-1} \equiv m^2\,s^{-2}}$) for the Edmonton sounding (Fig. 1) for 12Z July 31, 1987 (day of the Edmonton tornado; 12Z would be 06MDT). The sounding data are in file edited_wse_19870731@12Z.txt.

Method: Let L_{fc}, L_{eq} be respectively the level of free convection, and the equilibrium level. CAPE is computed as

$$\mathrm{CAPE} = -R \int_{L_{fc}}^{L_{eq}} \left[\, T_p(p) - T_a(p) \,\right] \, d\ln p \tag{1}$$

where $R = 287\ \mathrm{J\,kg^{-1}\,K^{-1}}$ is the gas constant for dry air and $(T_p - T_a)$ is the difference between the parcel temperature (which evolves with height along a moist adiabat) and the ambient temperature (the latter is given by the sounding).

A variety of methods exist to determine L_{fc}. We'll content ourselves to take $L_{fc} = 730$ hPa at which level we'll assume $T_p = T_a = 10.6^{\circ}$C. Then one must evaluate CAPE using a numerical approximation to the integral of Eq. (1), i.e. a summation of the integrand over finite layers. Suppose the interval (L_{fc}, L_{eq}) is divided into $i = 1...N$ layers and that p_{1i}, p_{2i} represent respectively the base and the top of the i^{th} layer, while T_{pi}, T_{ai} are the parcel temperature and the environmental temperature at the midpoint of the layer. Then the simplest approximation to Eq. (1) is

$$\text{CAPE} \approx -\,R \sum_{i=1}^{N} \left(T_{pi} - T_{ai} \right) \ln \left(\frac{p_{2i}}{p_{1i}} \right) \, . \tag{2}$$

T_{ai} is read from the sounding file (interpolations will be needed). The more complicated operation is that we need a means to specify the parcel temperature T_{pi}. Graphically this is easy, for parcel temperature follows the moist adiabat that runs through (730 hPa, 10.6°C). However we need a sequence of numerical values, thus, we need to pick off specific values, working up the moist adiabat. According to the AMS Glossary of Meteorology,

$$\frac{dT_p}{dz} = -\,g \, \frac{1 + \frac{L_v\, r}{R\, T_p}}{c_{pd} + \frac{0.622\, L_v^2\, r}{R\, T_p^2}} \tag{3}$$

where $L_v \approx 2.5 \times 10^6 \ \mathrm{J\,kg^{-1}}$ is the latent heat of vapourization; $R = 287\,\mathrm{J\,kg^{-1}\,K^{-1}}$ is the dry gas constant; $r = 0.622\, e/p$ is the mixing ratio (and since the parcel is saturated, this is the saturation mixing ratio at the parcel's temperature); and $c_{pd} \approx 10^3 \ \mathrm{J\,kg^{-1}\,K^{-1}}$ is the specific heat capacity of dry air at constant pressure. Of course

$$\frac{dT_p}{dp} = \frac{dT_p}{dz} \left(\frac{dp}{dz} \right)^{-1} \tag{4}$$

where $dp/dz = -\rho\, g$.

The above description outlines a framework for computing CAPE, but does not spell out all the choices needing to be made. Recall this is an unscored exercise, and feel free to present your own solution for feedback.

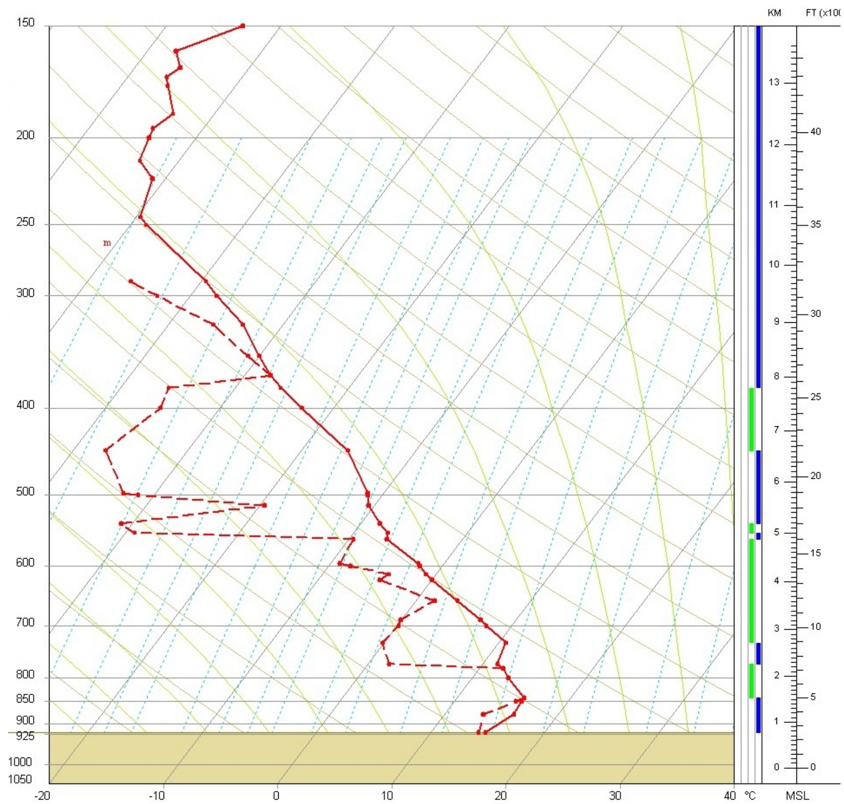

Figure 1: Edmonton sounding at 06 MDT on the day of the Edmonton tornado (i.e. sounding at 12Z July 31, 1987). The sounding has been plotted using the package RAOB, and the data are given in wse_19870731@12Z.txt.