EAS 471, Optional 2^{nd} scored Cmpt'g Assignment, 2010

Lagrangian Stochastic Simulation of Short Range Dispersion

In an earlier computing assignment you wrote a Latlab program to calculate the mean concentration field $C = C(x, z)$ downwind from a continuous crosswind crosswind line source at located at $x = 0, z = h_s = 0.46$ m in the horizontally-uniform, neutrally-stratified atmospheric surface layer. As noted in that assignment, the field of C is the analog of the crosswind integrated concentration $\chi = \chi(x, z)$ due to a steady *point* source, and Project Prairie Grass provided field measurements of the latter.

In this assignment we implement an alternative, Lagrangian approach to simulate short range dispersion, in order to simulate atmospheric dispersion as observed in these two Project Prairie Grass dispersion trials:

- Run 57: $u_* = 0.50$ m s^{-1}, $z_0 = 0.0058$ m, $|L| = \infty$ (neutral stratification).
- Run 59: $u_* = 0.14$ m s^{-1}, $z_0 = 0.005$ m, $L = 7$ m (stable stratification).

L is the Obukhov length, the stratification parameter in the surface layer[1]. The PPG concentration profiles are given in Table 1.

[1] The numerical value of L is determined essentially by the friction velocity u_* and the vertical flux of heat $\overline{w'T'}$ across the layer, viz. $L = -u_*^3\, T_0\, \left(k_v\, g\, \overline{w'T'}\,\right)^{-1}$ where T_0 is the mean Kelvin temperature of the layer. However those details do not concern us here.

The LS model

Motion in a horizontally-homogeneous atmospheric surface layer can be adequately simulated by assuming the particle's horizontal velocity equal to the mean Eulerian velocity $\overline{u}(Z)$ wherever it happens to be located, while its vertical velocity is random. Specifically, we shall assume particle position (X, Z) evolves in finite steps

$$\begin{aligned} X(t+dt) &= X(t) + U(t)\ dt\ , & (1) \\ Z(t+dt) &= Z(t) + W(t)\ dt & (2) \end{aligned}$$

where dt is the timestep. The particle's horizontal velocity is

$$U(t) = \overline{u}(Z(t)) \qquad (3)$$

while the particle's vertical velocity W evolves in time according to [2]

$$\begin{aligned} W(t+dt) &= W(t) + dW\ , & (4) \\ dW &= -\frac{C_0\ \epsilon(Z)}{2\sigma_w^2}\ W\ dt + \sqrt{C_0\ \epsilon(Z)}\ d\xi & (5) \end{aligned}$$

For each timestep the random increment $d\xi$ is evaluated as $d\xi = r\ \sqrt{dt}$ where each successive value r is chosen at random from a standardized Gaussian distribution (i.e. normal distribution with zero mean and unit variance).

Where needed the micro-meteorological properties are determined from

[2] This model for the vertical velocity can be proven to be the uniquely correct choice under the approximation that vertical velocity fluctuations in the surface layer belong to a Gaussian distribution whose variance σ_w^2 is height independent. This is an acceptable approximation for the atmospheric surface layer during neutral or stable stratification.

the following relationships, by setting $z = Z$ (the particle's height):

$$\frac{\overline{u}(z)}{u_*} = \frac{1}{k_v}\left(\ln\frac{z}{z_0} + \beta\,\frac{z - z_0}{L}\right), \tag{6}$$

$$\frac{\sigma_w}{u_*} = 1.3\,, \tag{7}$$

$$\frac{k_v\, z\, \epsilon(z)}{u_*^3} = \left(1 + (\beta - 1)\,\frac{z}{L}\right), \tag{8}$$

where $\beta = 5$. The timestep must be set as a fixed proportion of the timescale, ie.

$$dt = \mu\, T_L(z) \tag{9}$$

where $\mu \leq 0.1$. The initial vertical velocity should be a random Gaussian number with zero mean and standard deviation σ_w.

Numerical details

- Compute an ensemble of $N_P \sim 10^5$ (or more) random paths from the source to a distance sufficiently far downstream from the point of observation that the trajectory can be abandoned, e.g. to $X = 105$ m. The profile of mean concentration is to be estimated from the mean particle residence times in a stack of sampling "volumes" centred on $x = 100$ m and having dimensions $\Delta x = 0.001$ m, $\Delta z = 0.2$ m. The detectors can be indexed J and effectively, when each particle passes $x = 100$ m within detector J we increment the residence time counter T_J for that detector by the amount $\Delta x / U(z_J)$. Then when all trajectories have been computed the mean concentration is obtained as

$$\left(\frac{z_0 u_* C}{k_v Q}\right)_J = \frac{z_0 u_*}{k_v}\,\frac{T_J}{N_P\,\Delta x\,\Delta z}\,. \tag{10}$$

- *Reflection boundary condition*: Wherever a particle attains a height $Z < z_0$, its trajectory must be reflected according to

$$
\begin{aligned}
Z &= 2z_0 - Z \ , \\
W &= -W \ .
\end{aligned}
\tag{11}
$$

- For each experiment, perform simulations with $C_0 = (1, 3.6, 10)$

- In the case of PPG57, compare your Lagrangian simulations with the outcome of assignment 3 (advection-diffusion equation)

Table 1: Normalized concentration $\frac{z_0 u_* C}{k_v Q}$ observed at distance $x = 100$ m from the source (height $h_s = 0.46$ m) in Project Prairie Grass runs 57, 59.

z [m]	Run 57	Run 59
17.5	1.5E-6	0
13.5	6.6E-6	0
10.5	1.56E-5	0
7.5	3.51E-5	0.09E-5
4.5	7.9E-5	0.26E-4
2.5	1.25E-4	1.31E-4
1.5	1.53E-4	2.19E-4
1.0	1.62E-4	2.68E-4
0.5	1.70E-4	3.00E-4