EAS 471 “Optional Computing Task” Jan., 2010

Linear 1D advection

Numerically integrate the linear advection equation

$$\frac{\partial \theta}{\partial t} + U \frac{\partial \theta}{\partial x} = 0 \tag{1}$$

on domain $-1 \leq x \leq 1$, subject to the periodic boundary condition $\theta(-1,t) = \theta(1,t)$. As initial condition assume

$$\theta(x,0) = \sin(\pi x) \ . \tag{2}$$

Specify the advecting velocity as $U = 1$. Use the following explicit, upstream differencing method, viz. use the algorithm

$$\frac{\theta_I^{n+1} - \theta_I^n}{\Delta t} + U \frac{\theta_I^n - \theta_{I-1}^n}{\Delta x} \ . \tag{3}$$

This is expected to be stable provided $0 \leq C \leq 1$, where the numerical parameter $C = U\Delta t/\Delta x$ is the Courant number.

Note that the natural timescale for this problem is $\tau = 2/U = 2$, so that presumably we need a timestep $\Delta t \ll 2$. A second, artificial timescale will enter the treatment by virtue of your choice of a gridlength Δx, where as Δx is refined we have finer resolution. You should be able to examine the consequence of violating the Courant condition (which as stated above, requires $C = U\Delta t/\Delta x \leq 1$). Convey the behaviour of your solutions by presenting plots of $\theta(x)$ for several values of t that cover $t \ll \tau$, $t \sim \tau$, and $t \gg \tau$. The analytical solution valid at those times should also be shown.