EAS 471 3rd Scored Assignment (20%) Due: 7 Apr. 2016

Option A: Lagrangian Simulation of Project Prairie Grass

In the Project Prairie Grass tracer gas dispersion trials (Barad, 1958; Haugen, 1959), sulphur dioxide was released continuously from a nozzle at height $z = h_s = 0.46$ m over a flat prairie, and the resulting 10-min mean concentrations of gas were observed on downstream arcs at radii $x = (50, 100, 200, 400, 800)$ m. Here we will focus on the concentrations observed at $x = 100$ m, where six 20 m towers sampled mean concentration at multiple heights, providing sufficient information to compute the vertical profile of crosswind-integrated mean concentration

$$\chi(100, z) = \int_{\theta=-\pi}^{\pi} \bar{c}(x, \theta, z)\; r\, d\theta \; . \tag{1}$$

Earlier analyses suggest that absorption of SO_2 by the dry prairie grass can be considered negligible. We shall assume the wind statistics to be consistent with Monin-Obukhov Similarity Theory (MOST), and that the underlying probability density function $g_a(w)$ for the Eulerian vertical velocity is a Gaussian, with zero mean and a known standard deviation σ_w.

Let $\mathbf{X}(t) \equiv (X, Z)$ represent the coordinates of a fluid element, and $\mathbf{U} \equiv (U, W)$ its velocity on the radial (i.e. downstream, x) and vertical axes. We shall perform *two-dimensional* simulations of these experiments, making the approximations that (i) radial motion occurs at the local mean cup wind speed; and (ii) the Lagrangian vertical velocity can be simulated using the (unique) one-dimensional, first-order Lagrangian stochastic (LS) trajectory model for Gaussian inhomogeneous turbulence.

By computing trajectories in the (x, z) plane, compute the vertical profiles of crosswind-integrated concentration at $x = 100$ m for each of the Project Prairie Grass dispersion experiments documented in Tables (1, 2). Compare the simulated and measured concentration profiles graphically. More specifically, please perform a total of 7 simulations:

- Simulate Run 57 repeatedly, comparing outcomes for three values of the universal constant C_0, viz. $C_0 = (1, 3.1, 10)$ with the parameter $\mu = 0.1$ (showing the impact of choice of C_0

- Run a fourth simulation of Run 57 with $C_0 = 3.1$, $\mu = 0.02$ (showing the impact of the choice of time step, if any)

- Simulate each of Runs (33, 50, 59) with $C_0 = 3.1$, $\mu = 0.1$

Each simulation should use a large enough ensemble (particle count N_P) to give statistically reliable outcome. Before embarking on your "final" simulations, you should experiment with increasing the particle count. A fourfold increase in N_P will *halve* the statistical uncertainty (irregularity) in your computed concentration profiles. (Note: Figure (2) of Wilson (2015) gives comparable simulations of PPG runs 57 & 50.)

Further details of the LS model

Assume particle position (X, Z) evolves in time with velocity (U, W) where

$$U = \overline{u}(Z(t)) \tag{2}$$

and where W evolves in time according to the unique, 1-d, first-order, well-mixed model for Gaussian inhomogeneous turbulence, ie.

$$dW = a\ dt + \ b\ d\xi\ . \tag{3}$$

Here dW is the increment in particle velocity over timestep dt (computed as indicated below), and $d\xi$ is a Gaussian random variate with $\overline{d\xi} = 0$, $\overline{(d\xi)^2} = dt$. The conditional mean acceleration and the coefficient b of the random forcing are:

$$\begin{aligned} a &= -\frac{C_0\ \epsilon(z)}{2{\sigma_w}^2(z)}\, W + \ \frac{1}{2}\frac{\partial {\sigma_w}^2}{\partial z}\ \left(\frac{W^2}{{\sigma_w}^2} + \ 1\right)\ , \\ b &= \sqrt{C_0\ \epsilon(z)}\ , \end{aligned} \tag{4}$$

where C_0 is a universal coefficient introduced by Kolmogorov, and ϵ is the turbulent kinetic energy (TKE) dissipation.

The MO profile for the mean wind speed $\overline{u}$ is

$$\overline{u}(z) = \frac{u_*}{k_v}\ \left[\ln\ \frac{z}{z_0} - \psi_m\left(\frac{z}{L}\right) + \psi_m\left(\frac{z_0}{L}\right)\ \right] \tag{5}$$

where the function $\psi_m(z/L)$ is

$$\psi_m = \begin{cases} 2\ \ln\left(\frac{1+\phi_m{}^{-1}}{2}\right) + \ln\left(\frac{1+\phi_m{}^{-2}}{2}\right) - 2\ \mathrm{atan}\left(\phi_m{}^{-1}\right) + \frac{\pi}{2} & , L \leq 0 \\ -5z/L & , L > 0\ . \end{cases} \tag{6}$$

Wherever needed the MO function ϕ_m (dimensionless mean wind shear), is given by

$$\phi_m = \begin{cases} (1-\ 28z/L)^{-1/4} & , L \leq 0 \\ 1 + 5\, z/L & , L > 0\ . \end{cases} \tag{7}$$

The turbulent kinetic energy dissipation rate ϵ should be obtained from

$$\frac{k_v z}{u_*^3}\ \epsilon \equiv \phi_\epsilon\left(\frac{z}{L}\right) = \phi_m\left(\frac{z}{L}\right) - \frac{z}{L} \tag{8}$$

and the velocity standard deviation should be computed as

$$\sigma_w = \begin{cases} 1.25\ u_*\ (1-3z/L)^{1/3} & , L \leq 0 \\ 1.25\ u_* & , L > 0\ . \end{cases} \tag{9}$$

The timestep should be set as a fixed proportion of the following turbulence timescale

$$T_L = \frac{2{\sigma_w}^2(z)}{C_0\ \epsilon(z)}\ , \tag{10}$$

ie. $dt = \mu\ T_L(z)$ where $\mu \ll 1$. The initial vertical velocity should be a random Gaussian number with zero mean and standard deviation σ_w.

Confining particles (surface reflection

Trajectories should be "reflected" about a surface $z_r \geq z_0$ (in practise it is probably acceptable to set $z_r \sim 10z_0$); each time a particle moves below z_r it should be "bounced" back to the same distance *above* z_r, and the sign of the vertical velocity that carried it below z_r must be reversed, viz.

```
if(Z.lt.zr) then
  Z=zr+(zr-Z)
  W=-W
endif
```

How is mean concentration derived from computed trajectories?

Imagine a mast or vertical axis standing at distance $x = 100$ m downwind from the source. Divide the vertical axis into layers of depth Δz, which will define the vertical resolution of your computed concentration profile. Label your layers with index J.

Each time a particle passes $x = 100$ m, increase the count $N(J)$ in the layer it occupies. When you have computed all N_P independent trajectories, the ratio $N(J)/N_P$ is clearly the probability that a single particle released at the source crosses $x = 100$ m in layer J. Therefore $N(J)/N_P$ is related to the mean horizontal flux $F_x(J)$ of particles in that layer, in fact

$$\frac{N(J)}{N_P} = \frac{F_x(J)\,\Delta z}{Q} \tag{11}$$

where Q is the real world (physical) source strength. And since we have no horizontal fluctuations u' in our treatment, we have $F_x(J) \equiv C(J)\,U(J)$ (the streamwise convective flux density is entirely due to the mean velocity), and so by rearrangement

$$\frac{C(J)}{Q} = \frac{N(J)}{N_P\,\Delta z\,U(J)}\,. \tag{12}$$

Don't make your bins too thin (Δz too small) or there will be a very small probability of any trajectory passing through your bins... with the result that unless you release an immense number (N_P) of trajectories, you will have a statistically unreliable (noisy, albeit high resolution) concentration profile. Probably $\Delta z \sim 0.1$ m is satisfactory.

References

Barad, M.L. 1958. *Project Prairie Grass, a Field Program in Diffusion (Vol. 2)*. Tech. rept. Geophysical Research Papers No. 59, TR-58-235(II). Air Force Cambridge Research Center.

Haugen, D.A. 1959. *Project Prairie Grass, a Field Program in Diffusion (Vol. 3)*. Tech. rept. Geophysical Research Papers No. 59, TR-58-235(III). Air Force Cambridge Research Center.

Table 1: Normalized cross-wind integrated concentration $u_* \chi / Q$ [m^{-1}] observed at distance $x = 100$ m from the source (height $h_s = 0.46$ m) in several Project Prairie Grass runs.

z [m]	Run 57	Run 33	Run 50	Run 59
17.5	1.1E-4	1.3E-4	2.3E-4	0
13.5	4.5E-4	4.8E-4	7.1E-4	0
10.5	1.08E-3	1.17E-3	1.72E-3	0
7.5	2.42E-3	2.80E-3	3.41E-3	0.07E-3
4.5	0.55E-2	0.58E-2	0.61E-2	0.21E-2
2.5	0.86E-2	0.92E-2	0.85E-2	1.05E-2
1.5	1.06E-2	1.08E-2	0.96E-2	1.75E-2
1.0	1.12E-2	1.16E-2	1.00E-2	2.14E-2
0.5	1.17E-2	1.22E-2	1.07E-2	2.40E-2

Table 2: Micro-meteorological parameters for the above Project Prairie Grass runs.

	Run 57	Run 33	Run 50	Run 59
u_* [$\mathrm{m\,s^{-1}}$]	0.5	0.55	0.44	0.14
L [m]	-239	-93	-26	7
z_0 [m]	0.0058	0.0075	0.0033	0.005

Wilson, J.D. 2015. Dispersion from an area source in the unstable surface layer: an approximate analytical solution. *Q.J.R. Meteorol. Soc.*, **141**(693), 3285–3296.