EAS 471 Second Lab (unscored) 2016

RDM: dispersion in the atmospheric surface layer

Background

In the previous lab, we tracked particle displacement on the z axis, versus time. The flow properties (σ_w^2, τ and thus K) were independent of position. There were no boundaries. For the second lab, you will modify your program to simulate — rather realistically, as we will later see — dispersion of a pollutant within a neutrally stratified and horizontally-homogeneous[1] atmospheric surface layer[2], whose properties may be stated roughly as:

- about 50-100 m deep; turbulent, with velocity statistics that vary with height z. Suitable averaging time for statistics circa. 15 to 60 min

- mean wind direction independent of height; choose x axis parallel to mean wind; no mean vertical wind, i.e. $\overline{w} = 0$

- mean horizontal wind speed given by

$$\overline{u} = \frac{u_*}{k_v} \ln \frac{z}{z_0}$$

  where $k_v = 0.4$ is the von Karman constant, z_0 is the surface roughness length and u_* is the friction velocity[3]

- standard deviation of the vertical wind speed is $\sigma_w = b\, u_*$ where $b \approx 1.3$

- eddy diffusivity for a trace gas species is $K = (k_v/S_c)\, u_* z$ where S_c is the turbulent "Schmidt number" for this species. Assume the latter has the value $S_c = 0.64$ (Wilson 2013).

Task

Modify your RDM to track particles in x, z. Perform a simulation of vertical dispersion during Project Prairie Grass run 57 (Barad 1958): gas was released continuously at $z_{\rm src} = 0.46$ m, and the surface layer was neutrally stratified with $u_* = 0.5\,\mathrm{m\,s^{-1}}$ and $z_0 = 0.0058$ m. Determine the mean height $\langle Z \rangle$ and the root-mean-square height[4] $\sqrt{\overline{Z^2}}$ as a function of x from the source, over the range $0 \le x \le 1000$ m.

[1] In a "horizontally-homogeneous" flow, velocity statistics do not depend on the horizontal coordinates (x, y) but may vary with height z.

[2] Strictly speaking, the description to follow applies only under certain restrictions, most notiably the absence of tall vegetation.

[3] So called because $\tau = \rho u_*^2$ $[\mathrm{N\,m^{-2}}]$ is the mean shearing stress of the wind on the ground.

[4] It is a common convention to use upper case for Lagrangian variables, i.e. particle position.

Recall the core of the algorithm is

$$dX = \overline{u}(Z)\, dt\,, \tag{1}$$

$$dZ = \sqrt{2\,K\,dt}\; r\,, \tag{2}$$

where r is a standardized Gaussian random number (i.e. a normally distributed random variate with mean value 0 and variance 1). If a height step dZ should move a particle below the surface roughness length such that $Z < z_0$, which would be unphysical and cause an error in the needed $\ln(Z/z_0)$ when computing $\overline{u}$, you must impose "perfect reflection," viz.

$$Z^{\text{corrected}} \leftarrow z_0 + (z_0 - Z) \equiv 2z_0 - Z\ . \tag{3}$$

Compute your steps with a constant timestep[5] $dt = 0.1$ s.

Briefly document your lab exercise (PDF format), and submit for feedback. You should submit at least two figures, including your curves of $\langle Z \rangle$ and $\sqrt{\overline{Z^2}}$ versus x (you may wish to use a $\ln x$ axis). These graphs should be based on an ensemble of N_P paths, with N_P sufficiently large to yield reasonably smooth curves.

References

Barad, M.L. 1958. *Project Prairie Grass, a Field Program in Diffusion (Vol. 1).* Tech. rept. Geophysical Research Papers No. 59, TR-58-235(I). Air Force Cambridge Research Center. 280pp.

Wilson, J.D. 2013. Turbulent Schmidt numbers above a wheat crop. *Boundary-Layer Meteorol.*, **148**, 255–268.

[5] A more sophisticated treatment will be given later. A constant time step is somewhat an oversimplification because the "turbulence time scale" in the surface layer is related to the eddy diffusivity according to $\sigma_w^2 \tau = K$: in the present case this implies $\tau \propto z$. Ideally, one would choose $dt \ll \tau$.