EAS 572: Assignment 1, Sept 2005   Value: 5%

For a certain range in position z, the mechanical system shown in Fig.(1) has a linear governing equation

$$m \frac{d^2 z}{dt^2} = -k\,z - \alpha \frac{dz}{dt} - m\,g \tag{1}$$

and will asymptote to the equilibrium state ($z = z_{eq} = -mg/k,\ dz/dt = 0,\ d^2z/dt^2 = 0$). Assume this system is slightly disturbed, such that at $t = 0$ the mass is stationary at a non-equilibrium position $z = z_{eq} + \Delta z$ that lies within the range for which eqn (1) applies.

- Perform a dimensional analysis to find a functional relationship for the period (T) of the oscillation and the timescale (Γ) for its decay. Your result will depend on how you interpret the implications of two points: (i) g only affects the equilibrium position; and (ii) in the context of oscillation (and its decay) z is not an interesting variable, for it offers detail we don't want.

- With respect to variations of this system (ie. differing parameters m, k, α, g, z_{eq}), what scale factors must be fixed to assure complete similarity?

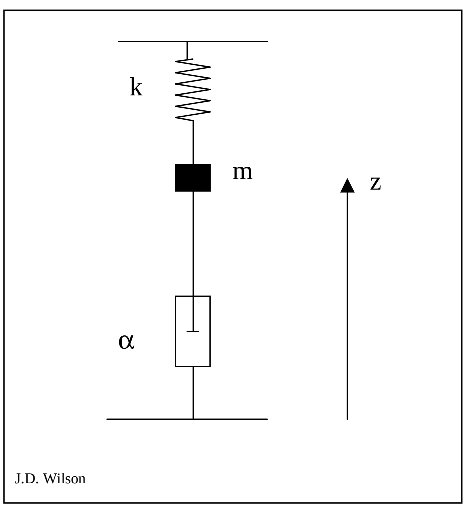

Figure 1: The mass m is connected to a fixed point on the ceiling via a spring (spring constant k) and to a fixed point on the floor via a damper (damping constant α).