EAS 572: Assignment 3: Eulerian Simulation of Dispersion in the ASL

Write a program to calculate the mean concentration field $C = C(x, z)$ downwind from a continuous line source at $x = y = 0, z = h_s$ in the horizontally-uniform surface layer. Assume C is the solution of:

$$\frac{\partial}{\partial x} \left(U(z)\, C \right) = \frac{\partial}{\partial z} \left(K(z)\, \frac{\partial C}{\partial z} \right) \tag{1}$$

where the profiles $U(z), K(z)$ are those of Monin-Obukhov similarity theory. For the eddy diffusivity assume

$$K = \frac{k_{vc} u_* z}{\phi_c} = \frac{1}{S_c}\, \frac{k_v u_* z}{\phi_c} \tag{2}$$

where S_c is the turbulent Schmidt number.

Table 1: Normalized cross-wind integrated concentration $\frac{z_0 u_* \chi}{k_v Q}$ observed at distance $x = 100m$ from the source (height $h_s = 0.46m$) in Project Prairie Grass run 57.

$z[m]$	$\frac{z_0 u_* \chi}{k_v Q}$
17.5	1.5E-6
13.5	6.6E-6
10.5	1.56E-5
7.5	3.51E-5
4.5	7.9E-5
2.5	1.25E-4
1.5	1.53E-4
1.0	1.62E-4
0.5	1.70E-4

Table 2: Micrometeorological data for Project Prairie Grass run 57.

$z[m]$	$U, [m/s]$	$T, [C]$
16	9.89	33.54
8	8.79	33.76
4	8.24	33.91
2	7.2	34.11
1	6.42	34.19
0.5	5.56	34.33
0.25	4.69	34.52
0.12		34.61

Discretize using grid-lengths $\Delta x \sim 0.5m, \Delta z \sim 0.2m$. Compare your calculated solution at $x = 100m$ with the Project Prairie Grass run 57 (Tables 1, 2), with values $S_c = (1, 0.63)$. The algorithm suggested in class is:

$$A^C_{I,J}\, C_{I,J} = A^N_{I,J}\, C_{I,J+1} + A^S_{I,J}\, C_{I,J-1} \; + B_{I,J} \tag{3}$$

where the $A_{I,J}$ are the "neighbour coefficients", and $C_{I,J}$ is the concentration matrix. This is a marching problem ($C_{0,J} = 0 \;\forall J$), implicit along the J (vertical)-axis. You will need to use a Tridiagonal Matrix Inversion Algorithm (see *Numerical Recipes*).