EAS 572: Assig. 1. Due 11 Oct 2012 Value: 10%

A. Dimensional Analysis

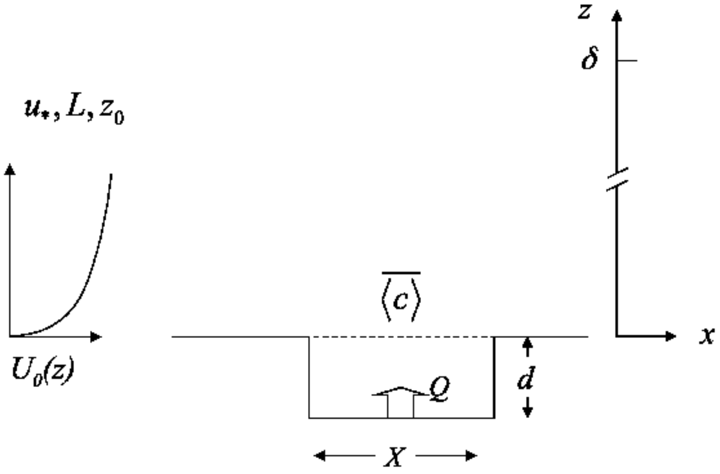

Figure 1: Surface layer winds blowing over a cavity: geometry for the dimensional analysis problem.

Consider an infinitely long, narrow, rectangular channel (depth $d \sim 1 - 10$ m, width $X \sim 1 - 10$ m) that is aligned perpendicular to a coordinate x (and so lies parallel with the y-axis). The mean wind direction relative to the x-axis is β, with $\beta = 0$ for a wind perpendicular to the axis of the channel[1]. At the bottom of the channel a liquid is evaporating at a uniform but unknown rate Q [$\mathrm{kg\,m^{-2}\,s^{-1}}$]. Upwind from the channel, where the surface roughness length is z_0, the atmospheric surface layer is undisturbed, and characterized by the friction velocity u_*, Obukhov length L and mean wind direction β (the depth δ of the atmospheric boundary layer may exert a minor influence on some surface layer velocity statistics). Except at the base of the channel, the source strength for this particular gas is zero: the "background" (or "upwind") concentration is zero.

Now suppose you have a laser gas detector which you have set up to measure the line average gas concentration $\langle c \rangle$ along a line normal to the channel (i.e. parallel to the y-axis) and tangent to the ground surface at a height $z_b \sim 1 - 3$ m ("beam height"), and that you

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[1] As an aside, wind flow over cavities has been studied by Raithby et al. (1978) and by Baratian-Ghorghi and Kaye (2012).

average this signal for (say) 30 minutes to obtain the time average $C \equiv \overline{\langle c \rangle}$. You would like to be able to infer Q from C and other relevant variables.

Perform a dimensional analysis suggesting (as specifically as you are able) the form of the relationship between C and Q.

B. Computation of micrometeorological statistics

The accompanying data file 'stalbert2011_timeseries13.csv' (1.2 MB) contains a time series of wind components, temperature, carbon dioxide concentration (ρ_c) and absolute humidity (ρ_v), registered at 10 Hz and covering 30 min, from a sonic anemometer at height $z = 2.55$ m over a wheat field at St. Albert (16 Aug. 20011, 13:30–14:00 MDT). The data are arranged in columns in the order $u, v, w, T, \rho_c, \rho_v$, where u is the westerly component, v the northerly component and w the vertical component. The number of entries (N) in each column is $N = 10 \times 1800$. The velocity components are in $\mathrm{m\,s^{-1}}$; the temperature in °C; carbon dioxide concentration is in $\mathrm{mg\,m^{-3}}$ *and is expressed as the deviation from* $600\,\mathrm{mg\,m^{-3}}$; and the absolute humidity is in $\mathrm{g\,m^{-3}}$.

Write a program to read the data file, compute, and write to a file the following statistics:

- mean velocity components U, V, W and mean wind direction $\beta = \arctan(V/U)$
- the evapotranspiration rate in the flux density unit $E = \overline{w'\rho_v'}$, and as a velocity $[\mathrm{mm\,dy^{-1}}]$
- the carbon dioxide flux density $F_c = \overline{w'\rho_c'}$
- Reynolds stress tensor

$$\mathbf{R} \equiv R_{ij} \equiv \overline{u_i' u_j'} = \begin{pmatrix} \sigma_u^2 & \overline{u'v'} & \overline{u'w'} \\ \overline{u'v'} & \sigma_v^2 & \overline{v'w'} \\ \overline{u'w'} & \overline{v'w'} & \sigma_w^2 \end{pmatrix}$$

- the friction velocity u_*, defined by

$$u_*^4 = \left(\overline{u'w'}\right)^2 + \left(\overline{v'w'}\right)^2$$

- the turbulent temperature scale $T_* = -\overline{w'\,T'}/u_*$
- the Obukhov length

  $$L = -\frac{u_*^3\,T_0}{k_v\,g\,\overline{w'T'}}$$

  where $k_v = 0.4$ is the von Karman constant and T_0 [K] is the mean air temperature. (Note: L is negative in unstable stratification)
- heat flux density $Q_H = \rho c_p\,\overline{w'T'}$ (to compute the density, assume the local pressure was $p = 91$ kPa).

Plot a scatter diagram of w' versus ρ_v'

References

Baratian-Ghorghi, Z., & Kaye, N.B. 2012. Flushing a finite volume of dense fluid from a square street canyon by a turbulent overflow. *Atmos. Env.*, **60**, 392–402.

Raithby, G.D., Hallett, W.L., Crawford, T.L., & Slawson, P.R. 1978. Measurements and Predictions of Turbulent Recirculating Flow over a Rectangular Depression. *Boundary-Layer Meteorol*, **15**, 181–194.