Tables (1-4)[1] give various micrometeorological statistics observed during a period of unstable stratification (22 May 2003) at Ellerslie (Alberta)[2]. Cup anemometers and shielded, ventilated thermocouples measured the vertical profiles of mean cup windspeed ("S") and mean temperature[3] on a mast standing in horizontally-homogeneous flow. The mean profile data (Tables 1, 2) suffice to determine the MOST scaling parameters for each interval, however in addition a three-dimensional sonic anemometer at $z = 2$ m on the mast provides an *independent* and direct estimate of the MOST scales, by manipulation of the statistics it provides (Tables 3, 4). A wind vane on the mast provided the mean wind direction ("θ") in the compass convention.

Task

For each 15 min interval, analyse the mean profile data on the mast to provide the friction velocity u_*, the temperature scale T_*, the Monin-Obukhov length L, and the sensible heat flux density[4] Q_H (you will need to write a computer program to do this: the general outline of a method is given below). Plot at least two of the given mean profiles alongside your corresponding theoretical profiles (i.e. those implied by your derived u_*, T_*).

From the sonic data compute alternative estimates u_*^s, T_*^s, L^s and mean wind direction $\beta_s = \arctan(V/U)$ (correct for the orientation of the sonic frame, ie. add 90^o).

Appendix: Profile Fitting Method

Create the set of measured differences $\Delta {S_z}^m = S_z - S_{ref}$, $\Delta {T_z}^m = T_z - T_{ref}$ (etc.) where S_{ref} is the windspeed at a reference height, such as $z = 0.62$ m. To each of these differences there correspond (for any guess of the scales u_*, T_*) theoretical differences $\Delta {S_z}^t = (S_z - S_{ref})^t$

[1] These data are also available in electronic form by downloading a file from the class web site.

[2] For details, see Wilson (2004; J. Applied Meteorol. Vol. 43, 1149-1167)

[3] Or more precisely, mean temperature *differences* ("$\overline{T} - \overline{T}_{ref}$") relative to a reference temperature at $z = 0.34$ m.

[4] The mean temperature T_0 during this period was about 20^o C, and for the purpose of calculating the density ρ_0 you may assume the atmospheric pressure $p = 93$ kPa.

(etc.) that may be calculated from the Monin-Obukhov profiles[5]. Your scales should be optimal in the sense that they minimize the dimensionless residual:

$$\begin{aligned} R &= \frac{\sum_1^{N_S} (\Delta S^m - \Delta S^t)^2}{\delta S^2} \\ &+ \frac{\sum_1^{N_T} (\Delta T^m - \Delta T^t)^2}{\delta T^2} \end{aligned}$$

Here $\delta S, \delta T$ are the estimated characteristic uncertainties in windspeed and temperature difference; assume values $\delta_S = 0.05$ m s^{-1}, $\delta_T = 0.1$ °C. In the present case the number of velocity differences is $N_S = 3$ and $N_T = 2$. The simplest computational approach is to use a nested loop: scan through all combinations of u_*, T_* covering a physically reasonable range, say, $0.05 \le u_* \le 0.5$ m s^{-1} (with interval 0.01) and -5 K $\le T_* \le 0$ (with interval 0.01).

In unstable stratification the mean wind and temperature profiles can be represented as:

Wind

$$\overline{u}(z) = \frac{u_*}{k_v} \left[\ln \frac{z}{z_0} - \psi_m \left(\frac{z}{L} \right) + \psi_m \left(\frac{z_0}{L} \right) \right]$$

where ψ_m is given in terms of the dimensionless mean shear (ϕ_m) as:

$$\psi_m = 2 \ln \left(\frac{1 + {\phi_m}^{-1}}{2} \right) + \ln \left(\frac{1 + {\phi_m}^{-2}}{2} \right) - 2 \,\mathrm{atan} \left({\phi_m}^{-1} \right) + \frac{\pi}{2} \,.$$

Temperature

$$\overline{T}(z) - \overline{T}(z_0) = \frac{T_*}{k_v} \left[\ln \frac{z}{z_0} - \psi_h \left(\frac{z}{L} \right) + \psi_h \left(\frac{z_0}{L} \right) \right]$$

where

$$\psi_h = 2 \ln \left[\frac{1}{2} \left(1 + {\phi_h}^{-1} \right) \right] \,.$$

For the dimensionless gradients ϕ_m, ϕ_h in mean velocity and temperature Dyer and Bradley (1982; BLM Vol. 22, 3-19) recommended

$$\begin{aligned} \phi_m(z/L) &= (1 - 28\, z/L)^{-1/4} \,, \\ \phi_h(z/L) &= (1 - 14\, z/L)^{-1/2} \,. \end{aligned}$$

[5] When you take *differences* in wind speed or temperature the roughness length will disappear from your MO formulae.

Table 1: Profiles of (uncorrected) 15 min mean cup windspeed [$\mathrm{m\,s^{-1}}$] in an undisturbed ASL, Ellerslie (AB), 22 May, 2003 (on all tables, end times are given in Local Standard Time). Measurements have been rounded to nearest 0.01 $\mathrm{m\,s^{-1}}$. The cup anemometers should be assumed to have overestimated the mean speed by 8%, and each value should be corrected accordingly.

			HEIGHT	
t_{end}	0.62 m	1.57 m	3.07 m	5.02 m
1615	3.24	3.98	4.47	4.83
1630	2.83	3.43	3.80	4.10
1645	3.31	4.09	4.66	5.07
1700	2.52	3.11	3.46	3.73
1715	3.48	4.37	4.95	5.38
1730	2.28	2.82	3.16	3.43
1745	2.94	3.62	4.07	4.43
1800	3.17	3.92	4.46	4.84

Table 2: Profiles of 15 min mean temperature difference [K] in an undisturbed ASL, Ellerslie (AB), 22 May, 2003. Negative entries imply the upper level is cooler than the lower (reference) level, implying unstable stratification.

t_{end}	1.31 m (-) 0.34 m	4.25 m (-) 0.34 m
1615	-0.92	-1.53
1630	-0.85	-1.34
1645	-0.67	-1.14
1700	-0.80	-1.18
1715	-0.57	-0.92
1730	-0.23	-0.33
1745	-0.22	-0.34
1800	-0.15	-0.20

Table 3: Velocity statistics (MKS units) from the sonic anemometer at $z = 2.00$ m. The sonic was 'facing' west, thus when $v = 0$ wind direction is 270°. In principle, the statistics should be rotated into a frame for which $\overline{w} = 0$, but we shall neglect this step. All components of the Reynolds stress tensor $R_{ij} \equiv \overline{u_i' u_j'}$ can be computed from the given data.

t_{end}	$\overline{\sqrt{u^2+v^2}}$	$\overline{u}$	$\overline{v}$	$\overline{w}$	$\overline{uu}$	$\overline{vv}$	$\overline{ww}$	$\overline{uv}$	$\overline{uw}$	$\overline{vw}$
1600	3.8911	3.0302	1.8369	0.02797	11.091	5.3169	0.13858	5.0236	0.00715	0.00728
1615	3.8344	3.1779	1.7024	0.01642	11.959	5.1048	0.13379	5.9387	-0.00238	-0.00505
1630	3.3915	2.8322	0.87041	-0.00192	10.261	3.3805	0.11158	3.2606	-0.02209	-0.0007
1645	4.062	3.4954	0.80817	0.02067	13.94	3.9946	0.14443	2.6813	0.00212	-0.00099
1700	3.1466	2.8657	-0.24558	0.01233	10.099	1.5816	0.12456	-0.95182	0.00265	-0.00165
1715	4.2673	3.9732	0.889	0.08001	17.588	2.7315	0.18221	3.7178	0.18194	0.05048
1730	2.8478	2.2395	1.5841	0.00487	5.6528	3.1505	0.06604	3.596	-0.03078	-0.00674
1745	3.4946	2.8851	1.5797	0.00889	9.6414	3.6841	0.09758	4.4309	-0.00306	-0.03642
1800	3.8828	3.5494	1.0493	0.00491	13.572	2.4648	0.10391	3.7716	-0.0565	-0.01844

Table 4: Temperature and heat flux statistics (MKS units) from the sonic anemometer at z = 2.00 m. From these data the eddy heat fluxes can formed as, for example, $\overline{u'T'} = \overline{u\,T} - \overline{u}\,\overline{T}$.

t_{end}	$\overline{T}$	$\overline{T\,T}$	$\overline{u\,T}$	$\overline{v\,T}$	$\overline{w\,T}$
1615	19.97	399.2	63.45	33.994	0.42466
1630	19.837	393.96	56.439	17.35	0.054
1645	20.231	409.62	70.755	16.312	0.49237
1700	20.062	402.79	57.626	-5.0379	0.31351
1715	20.499	420.5	81.243	18.17	1.7131
1730	19.9	396.24	44.506	31.641	0.12985
1745	20.043	401.8	57.865	31.548	0.19873
1800	20.189	407.65	71.591	21.223	0.12048