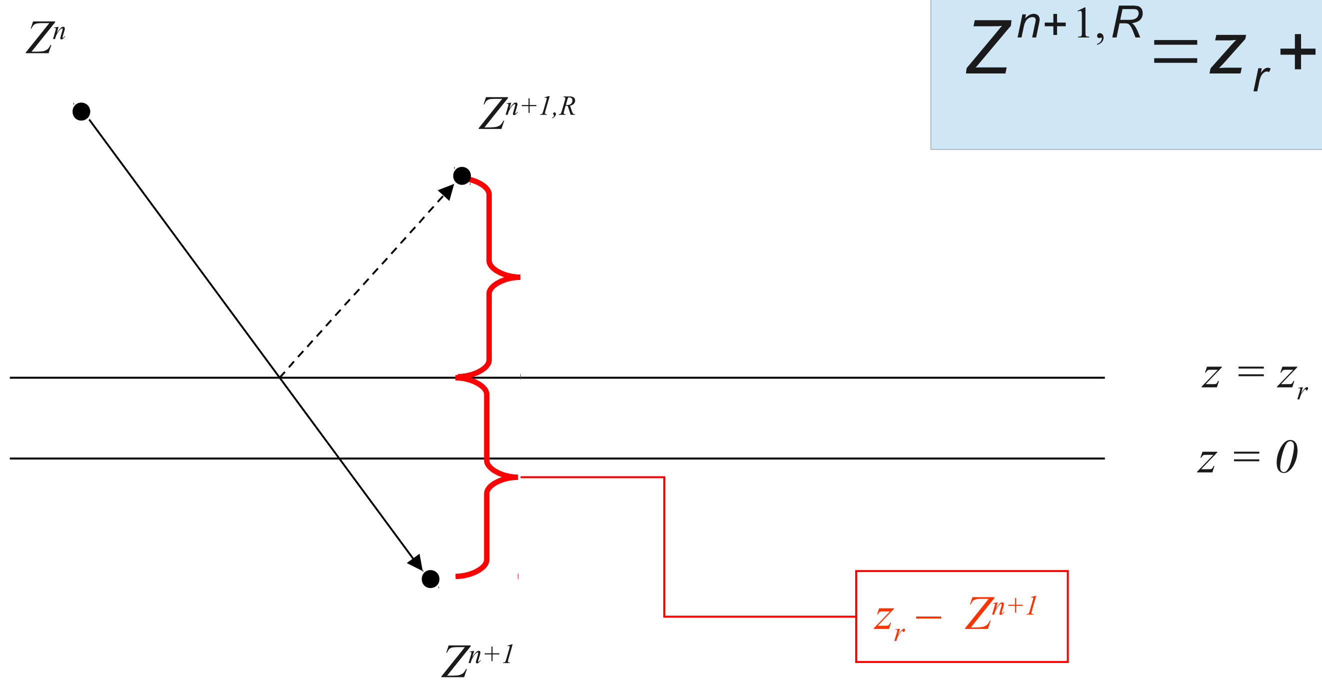
Reflection of Trajectories in LS models

• a reflection algorithm tacked to a well-mixed LS model can cause violation of the w.m.c. (e.g. Wilson & Flesch, 1993, “Flow boundaries in random flight dispersion models: enforcing the well-mixed condition," J. Appl. Meteorol. 32, 1695-1707)

• however “perfect reflection” (“smooth wall reflection”) at an artificial boundary (reflection height z_r) is acceptable in Gaussian turbulence provided $\partial \sigma_w / \partial z \rightarrow 0$ as $z \rightarrow z_r$

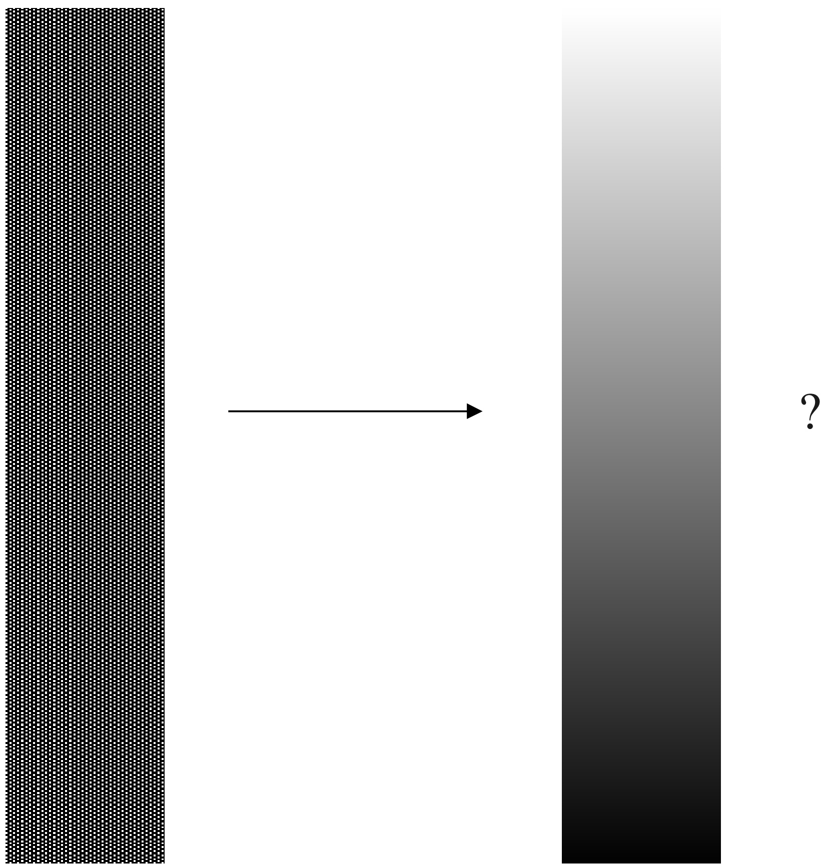
$$Z^{n+1,R} = z_r + \left[z_r - Z^{n+1} \right]$$



• in practise acceptable to set z_r much larger than z_0 to reduce computation time
• if simulating whole ABL may need reflection at $z = \delta$ as well

How to judge if reflection is problematic?

Compute the evolution of an initially well mixed particle distribution… e.g. release a large set of N_P particles, with the initial height chosen $Z^0 \in U\left[z_r,\, \delta\right]$ which implies $p(z,0) = \dfrac{1}{\delta - z_r}$



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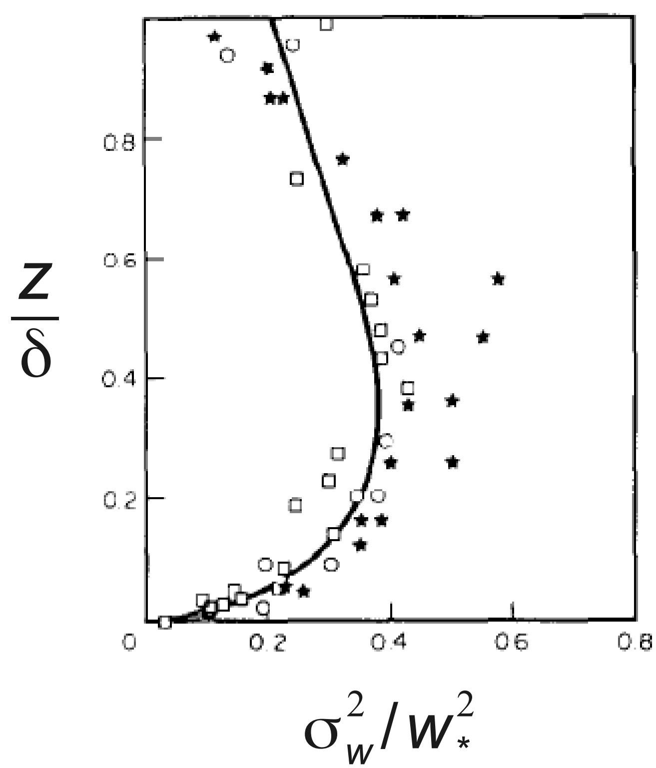
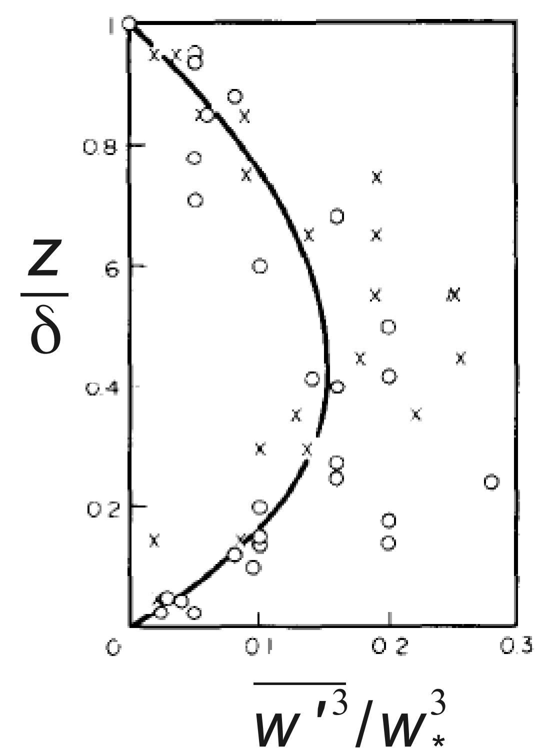
MONTE CARLO SIMULATION OF PLUME DISPERSION IN THE CONVECTIVE BOUNDARY LAYER

J. HØGNI BÆRENTSEN* and RUWIM BERKOWICZ

Atmospheric Environment Vol. 18, No. 4, pp. 701–712, 1984
Printed in Great Britain.





Particles remaining well mixed in position (CBL – Baerentsen-Berkowicz)

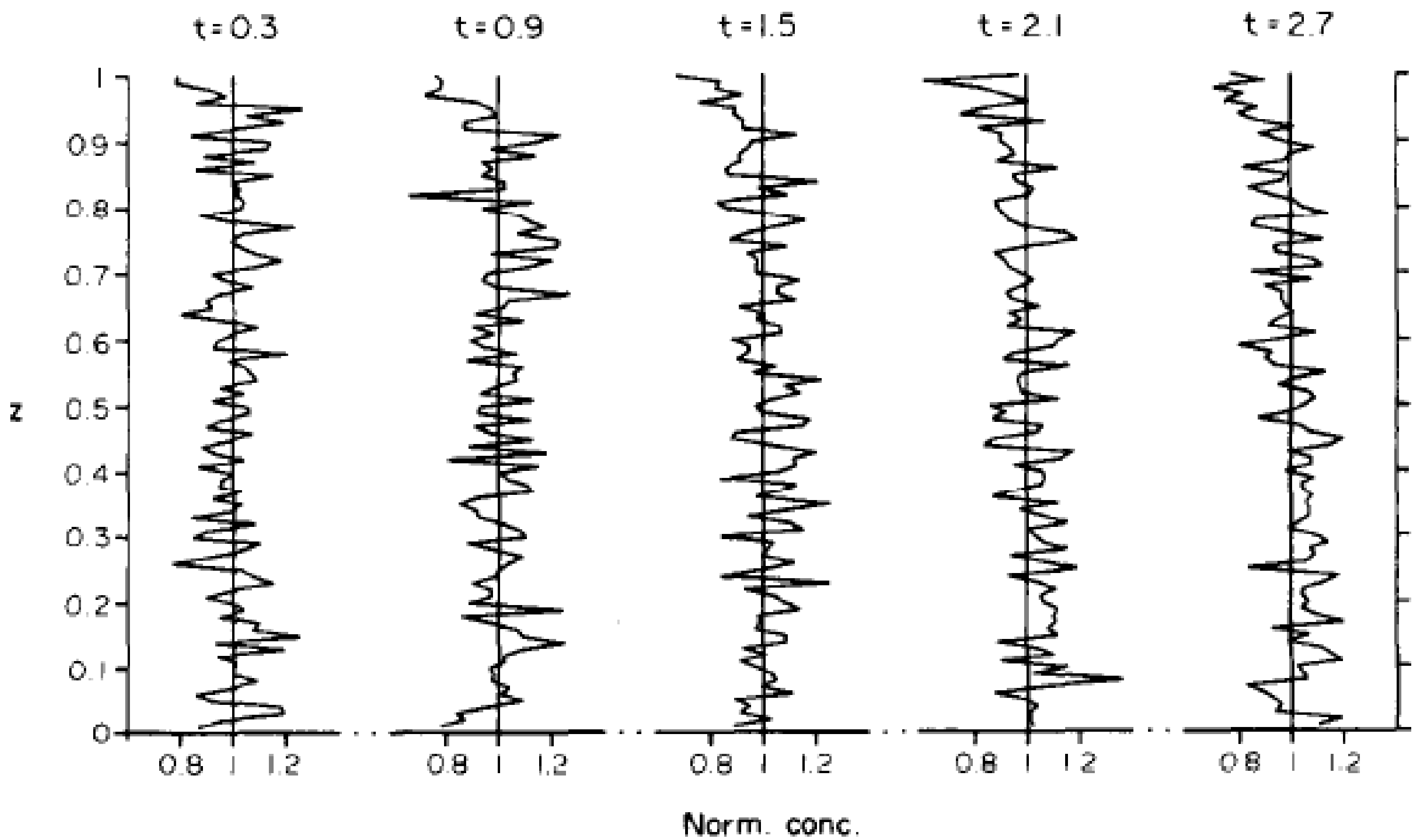
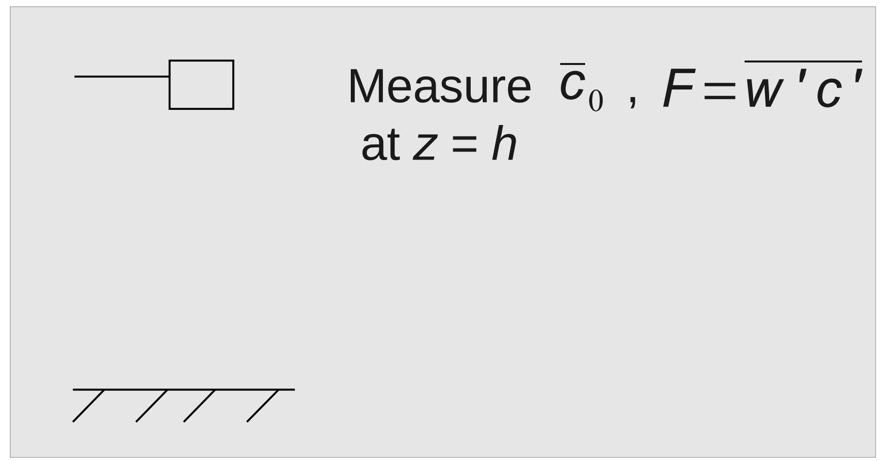


Fig. 4. Distribution of particles after different travel times as simulated by the Monte Carlo model. The source is distributed uniformly through the whole boundary layer. 10,000 particles were used in the simulation.

This work prior to, and in some measure stimulated, Thomson's (1987) well-mixed condition

Deposition to ground or canopy

Uptake at ground is often parameterised in terms of a "deposition velocity," defined as the ratio $w_d = F/\bar{c}_0$ of the magnitude of the flux density to the surface to a mean concentration $\bar{c}_0$ measured at an arbitrary reference location above the surface (this only makes sense if the flux and concentration are measured within the constant flux layer)



If deposition velocity known, can incorporate in LS model by performing *partial* reflection: a fraction *A* of particles contacting the surface is absorbed, and the complementary fraction *R=1-A* is reflected in the usual way. Wilson et al. (1989, Agric. Forest Meteo. Vol. 47) show *R* relates to w_d as:

$$\frac{1-R}{1+R} = \sqrt{\frac{\pi}{2}}\,\frac{w_d}{\sigma_w}$$

Thomson's LS model for 2-D Gaussian, vertically-inhomogeneous turbulence

$$dU = -\frac{b^2}{2\sigma^2}\left[U\,{\sigma_w}^2 - W\,\overline{u'w'}\right]dt + \frac{\phi_u}{g_a}\,dt + b\,d\xi_u$$

$$dW = -\frac{b^2}{2\sigma^2}\left[W\,{\sigma_u}^2 - U\,\overline{u'w'}\right]dt + \frac{\phi_w}{g_a}\,dt + b\,d\xi_w$$

$$dX = [\,\overline{u}(Z) + U\,]\,dt$$

$$dZ = W\,dt$$

where $b^2 = C_0\,\epsilon$, $\sigma^2 = {\sigma_u}^2{\sigma_w}^2 - {u_*}^4$, $g_a = g_a(u,w\,|\,z)$ is the joint PDF (specifically, the joint Gaussian), and:

$$\frac{\phi_u}{g_a} = \frac{1}{2}\frac{\partial\overline{u'w'}}{\partial z} + W\frac{\partial\overline{u}}{\partial z} + \frac{1}{2\sigma^2}\left[\frac{\partial{\sigma_u}^2}{\partial z}\left({\sigma_w}^2\,U\,W - \overline{u'w'}\,W^2\right) + \frac{\partial\overline{u'w'}}{\partial z}\left({\sigma_u}^2\,W^2 - \overline{u'w'}\,U\,W\right)\right]$$

$$\frac{\phi_w}{g_a} = \frac{1}{2}\frac{\partial{\sigma_w}^2}{\partial z} + \frac{1}{2\sigma^2}\left[\frac{\partial{\sigma_w}^2}{\partial z}\left({\sigma_u}^2\,W^2 - \overline{u'w'}\,U\,W\right) + \frac{\partial\overline{u'w'}}{\partial z}\left({\sigma_w}^2\,U\,W - \overline{u'w'}\,W^2\right)\right]$$

Dispersion in the CBL

• a peculiarity of observed dispersion in the CBL that Gaussian plume models had been unable to explain is that the (time-average) plume centreline from an elevated continuous point source initially descends towards ground (with increasing downwind distance), then ascends

• fractional area (*A*) of "updrafts" (wherein mean velocity is upward, but instantaneous velocity need not be) and complementary fractional area (*B=1-A*), considered the (predominantly) subsiding environmental region. Obsv. give $A \sim 0.2 - 0.4$

• ignore specifics of thermals; consider ABL horiz. homog; then, the vertical velocity PDF is skewed $Sk_w(z) = \overline{w'^3}/{\sigma_w}^3 \sim 1/2$

• form a 1-D LS model: first choose a suitable PDF, e.g. Luhar & Britter (1989; Atmos. Env. Vol. 23) $g_a(w) = A\ P_A(w) + B\ P_B(w)$ where P_A, P_B are Gaussians (whose moments vary with *z*)

• setting aside the details of "fitting" the parameters of the PDF's, the 1D model is

$$dW = a\ dt + \sqrt{C_0\,\epsilon}\ d\xi$$

where the w.m.c. fixes the conditional mean acceleration as $a = -\dfrac{\sigma_w^2}{T_L}\dfrac{Q(W)}{g_a} + \dfrac{\phi(W)}{g_a}$

Atmospheric Environment Vol. 23, No. 9, pp. 1911–1924, 1989.
Printed in Great Britain.
0004–698

A RANDOM WALK MODEL FOR DISPERSION IN INHOMOGENEOUS TURBULENCE IN A CONVECTIVE BOUNDARY LAYER

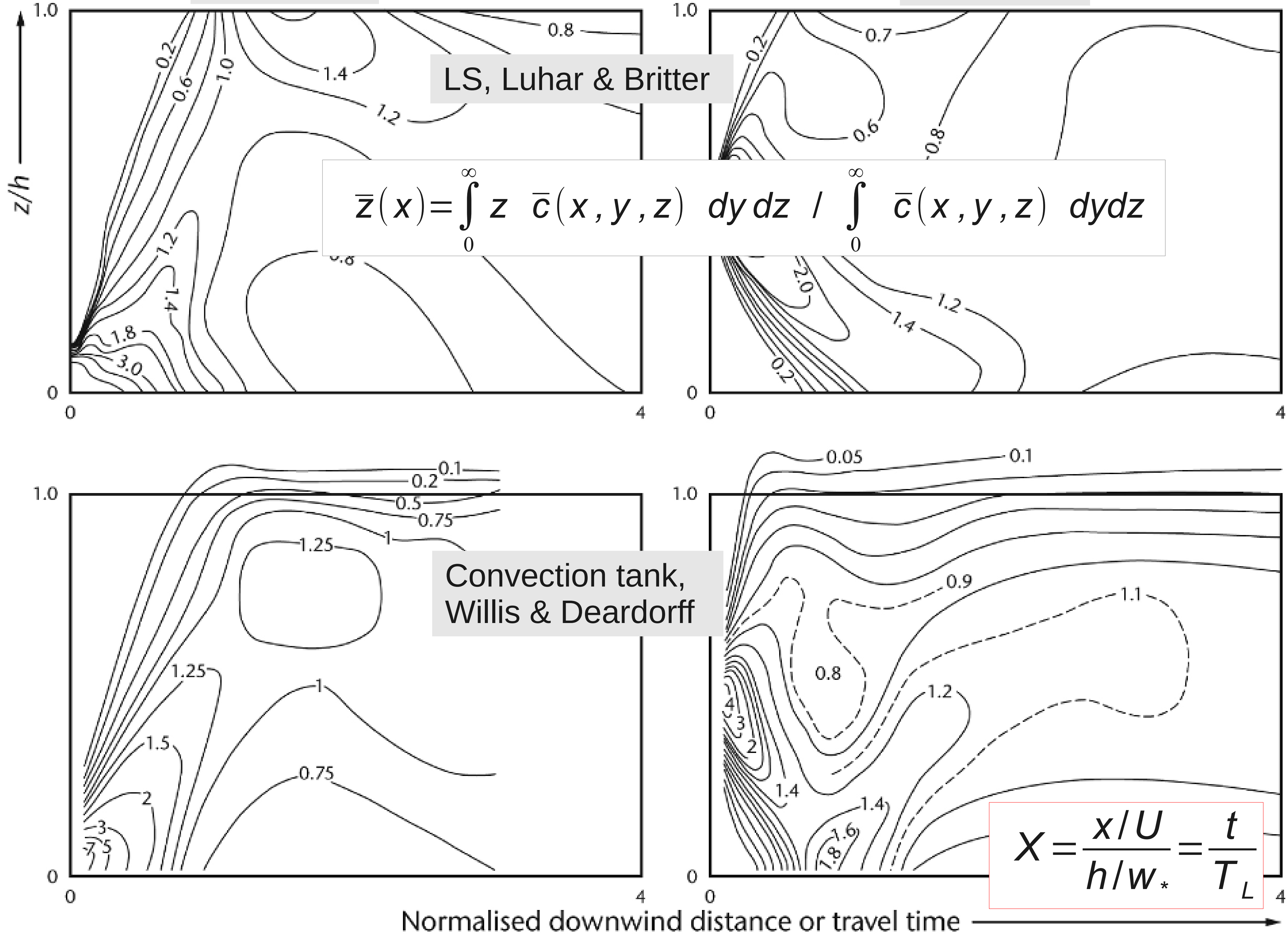
ASHOK K. LUHAR and REX E. BRITTER

$$Q(W) = \frac{A}{\sigma_{wA}^2}\,(W - \overline{w}_A)\;P_A(W) + \frac{1-A}{\sigma_{wB}^2}\,(W + \overline{w}_B)\;P_B(W)$$

$$\begin{aligned}
\phi(W) = \; & - \; \frac{1}{2}\left(A\,\frac{\partial \overline{w}_A}{\partial z} + \overline{w}_A\,\frac{\partial A}{\partial z} \right) erf\left(\frac{W - \overline{w}_A}{\sqrt{2}\,\sigma_{wA}} \right) \\
& + \; \frac{1}{2}\left(B\,\frac{\partial \overline{w}_B}{\partial z} + \overline{w}_B\,\frac{\partial B}{\partial z} \right) erf\left(\frac{W + \overline{w}_B}{\sqrt{2}\,\sigma_{wB}} \right) \\
& + \; \left[\frac{A}{2}\,\frac{\partial \sigma_{wA}^2}{\partial z}\left(\frac{W^2}{\sigma_{wA}^2} + 1 \right) + \sigma_{wA}^2\,\frac{\partial A}{\partial z} \right] P_A(W) \\
& + \; \left[\frac{B}{2}\,\frac{\partial \sigma_{wB}^2}{\partial z}\left(\frac{W^2}{\sigma_{wB}^2} + 1 \right) + \sigma_{wB}^2\,\frac{\partial B}{\partial z} \right] P_B(W)
\end{aligned}$$

The moments $\overline{w}_A(z),\ \sigma_{wA}(z)...$ of the component Gaussians vary with height and are related to suitable empirical profiles of $\overline{w'^2},\ \overline{w'^3}, \overline{w'^4}$ for the CBL

$z_s/h = 0.07$ $z_s/h = 0.07$



LS, Luhar & Britter

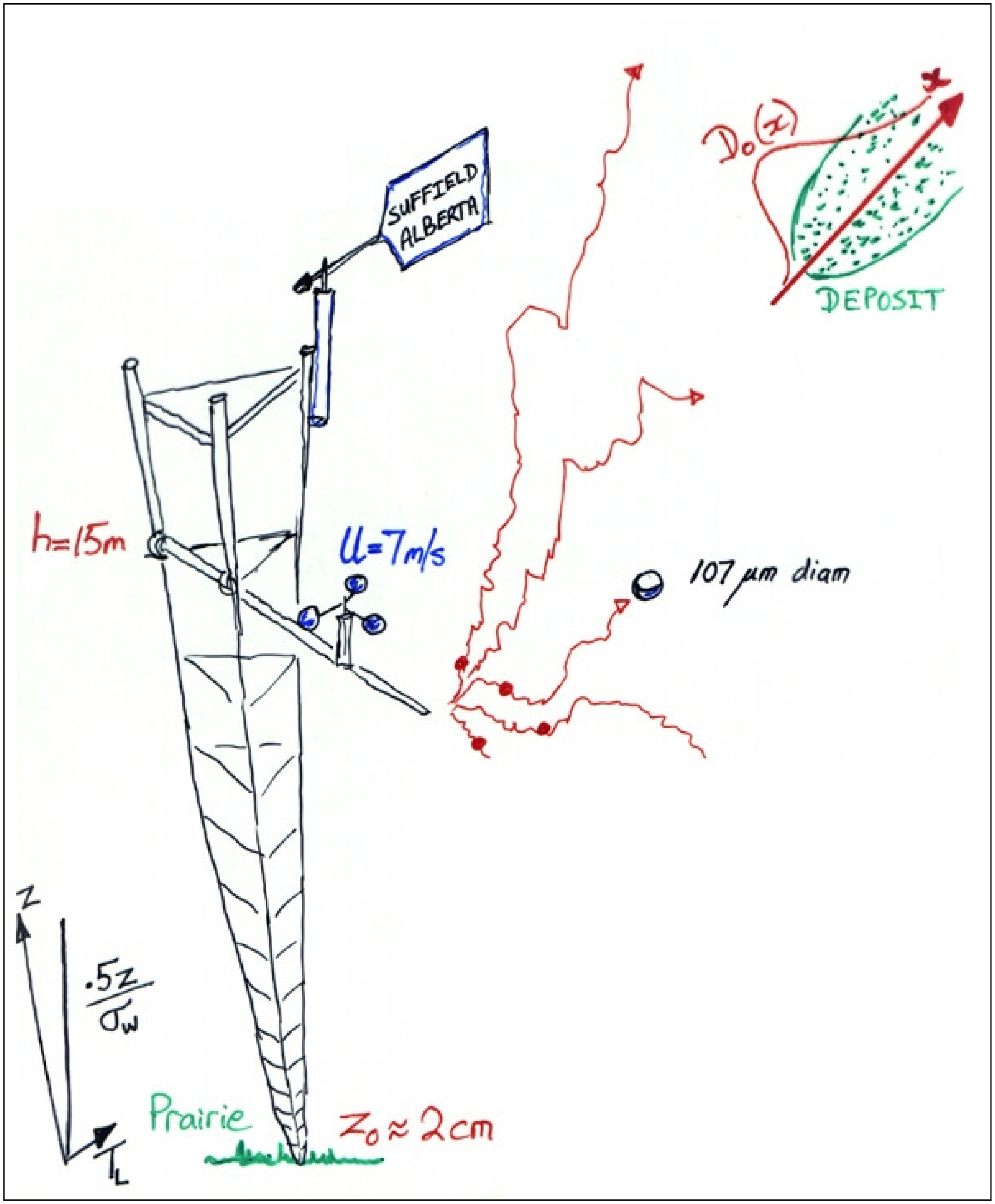
$$\bar{z}(x) = \int_0^\infty z \;\; \bar{c}(x, y, z) \;\; dy\,dz \;\; / \int_0^\infty \bar{c}(x, y, z) \;\; dydz$$

Convection tank, Willis & Deardorff

$$X = \frac{x/U}{h/w_*} = \frac{t}{T_L}$$

Heavy particle dispersion

- inertia
- gravitational settling
- deposition on ground
- what is the “well-mixed state”?
- lack rigorous criteria for LS models
- path of a heavy particle is not a fluid trajectory



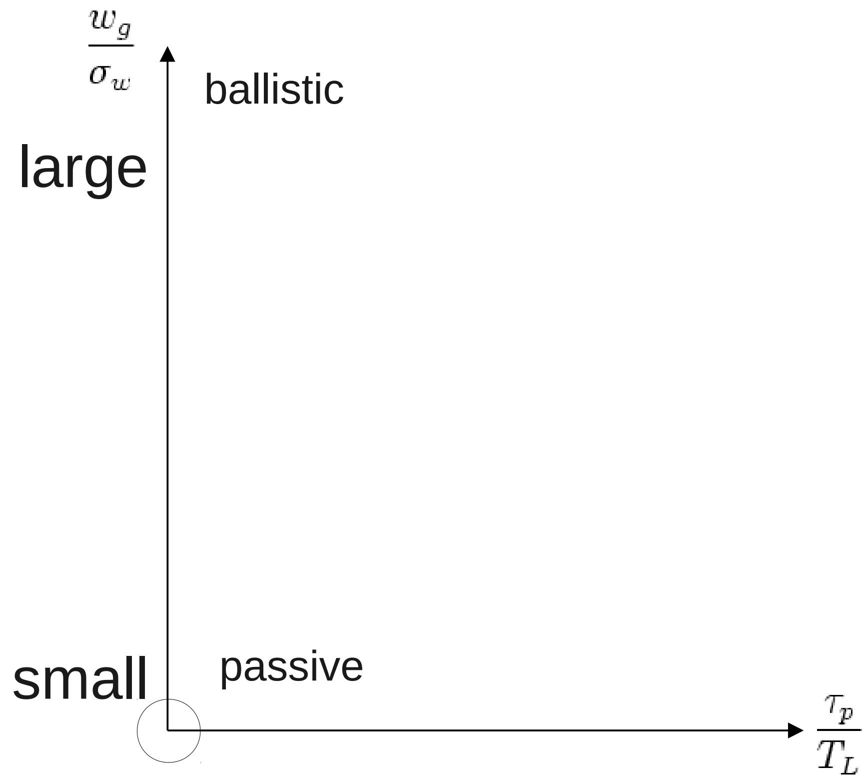
For spherical particles (diam. *d*) at low slip Reynolds number

$$R = \frac{|\vec{U}_p - \vec{u}|\ d}{\nu}$$

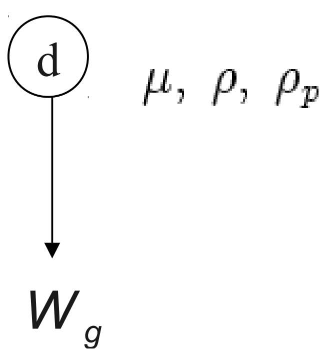
the eqn of motion is

$$\frac{d\,W_p}{dt} = \frac{w(t) - W_p}{\tau_p} - g$$

(drag depends linearly on relative velocity)



At steady state with Eulerian velocity
w = const., the terminal velocity is $W_g = \tau_p\ g$

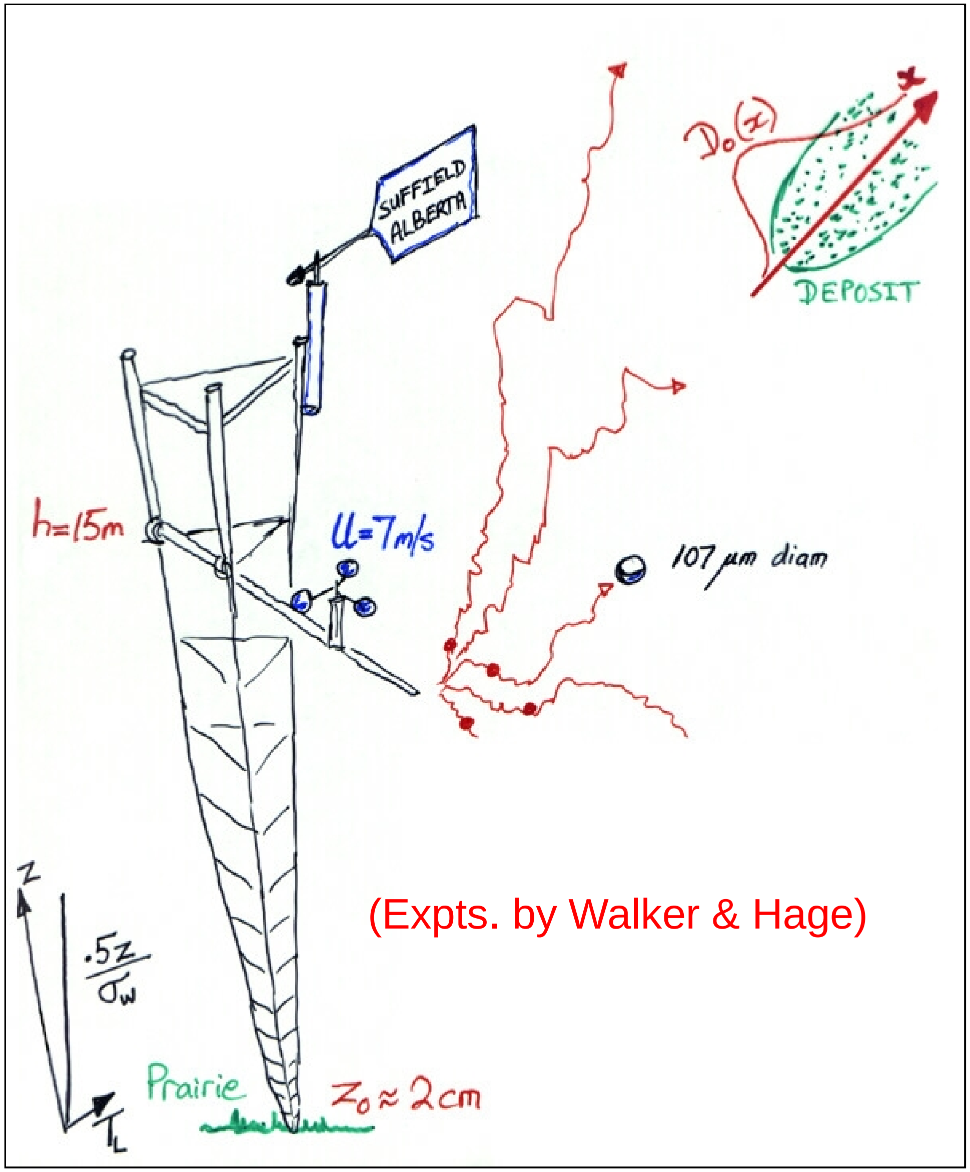


What does a dimensional analysis suggest for W_g?

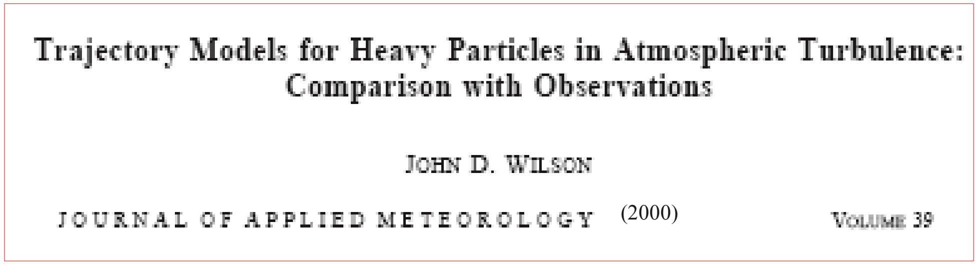
Stokes' analysis (linearized treatment) $$\frac{\tau_p}{d^2/\nu} = \frac{1}{18}\ \frac{\rho_p}{\rho}$$

Heavy particle dispersion – Deposition of Glass Beads

$$\begin{aligned} d &= 107\ \mu\mathrm{m} \\ \tau_p &= 0.06\ \mathrm{s} \\ w_g &= 0.6\ \mathrm{m\ s^{-1}} \end{aligned}$$



(Expts. by Walker & Hage)

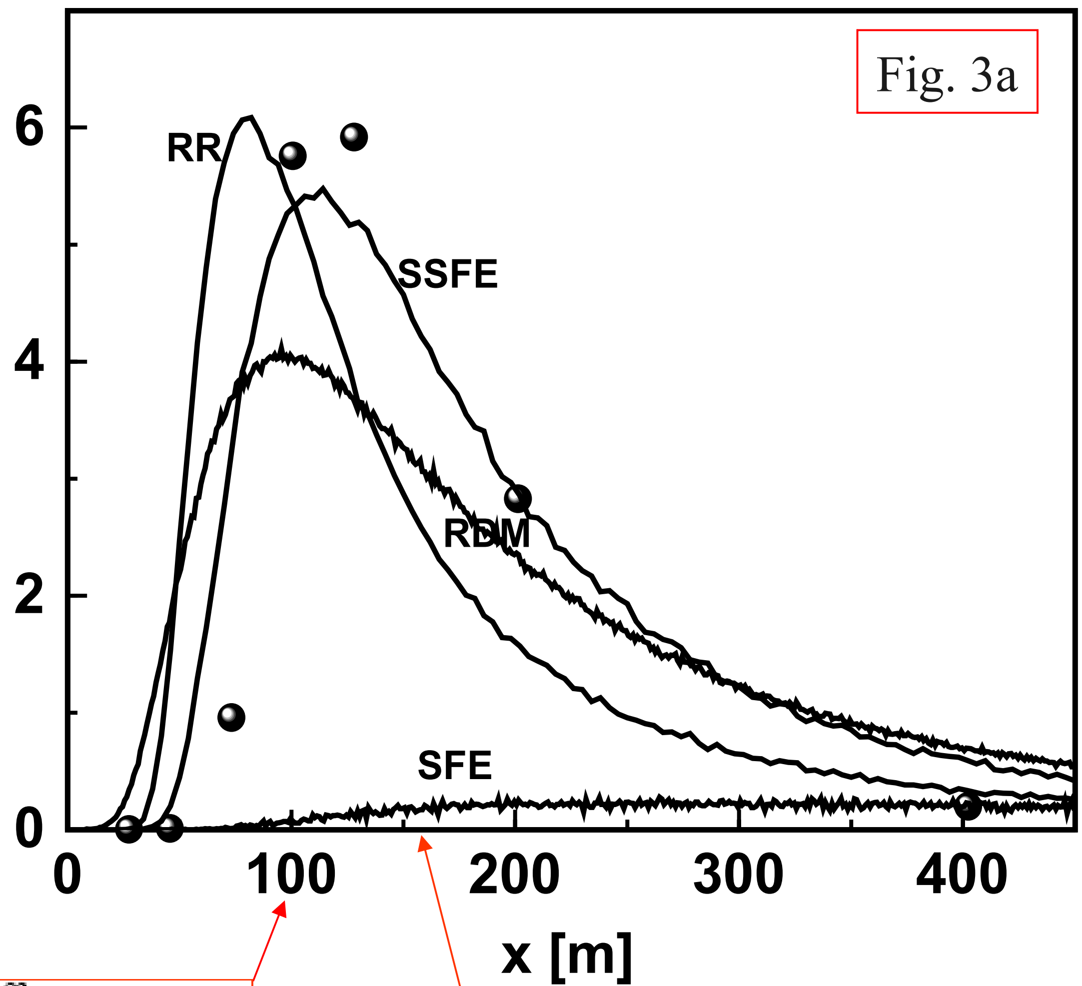


Trajectory Models for Heavy Particles in Atmospheric Turbulence: Comparison with Observations

JOHN D. WILSON

JOURNAL OF APPLIED METEOROLOGY (2000) VOLUME 39

Run C: neutral stratification, L = 341 m



D_0 mg $g^{-1}m^{-1}$

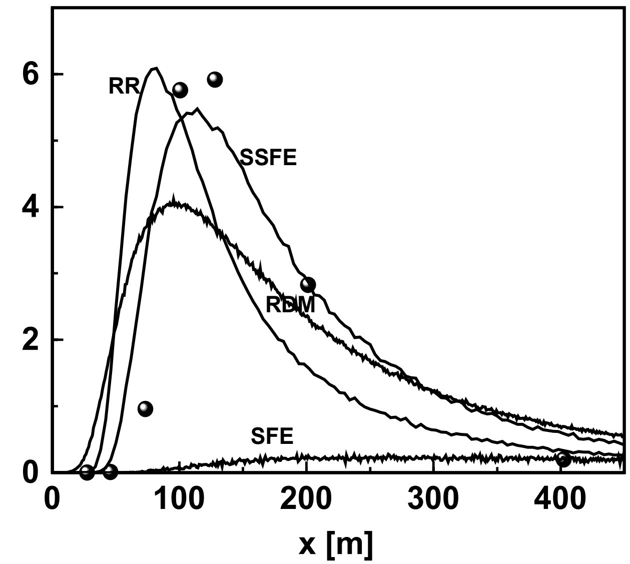
$$\frac{x}{\overline{u}(h)\,T_L(h)} \approx 1$$

Ballistic impact point x=159 m

$$\mathrm{RR}: \;\; W = \text{const.} = \sigma_w(h)\; r \;\; - w_g, \quad r \in \mathrm{N}(0,1)$$

$$\mathrm{RDM}: \;\; dZ = \frac{\partial(\sigma_w^2\, T_L)}{\partial z}\, dt + \sqrt{\frac{2\,\sigma_w^2\, dt}{T_L}}\; r \;\; - w_g\, dt, \quad r \in \mathrm{N}(0,1)$$

SSFE ("Settling sticky fluid element" model):



$$
\begin{aligned}
dW &= a\ dt + \sqrt{C_0\,\epsilon}\ d\xi \\
W &= W + dW \\
\boxed{W_p = W - w_g} \\
dZ_p &= W_p\ dt
\end{aligned}
$$

where $a(W) = -\dfrac{W}{T_L} + \dfrac{1}{2}\dfrac{\partial {\sigma_w}^2}{\partial z}\left(\dfrac{W^2}{{\sigma_w}^2} + 1\right)$ and $T_L = \dfrac{2{\sigma_w}^2(z)}{C_0\ \epsilon(z)}$

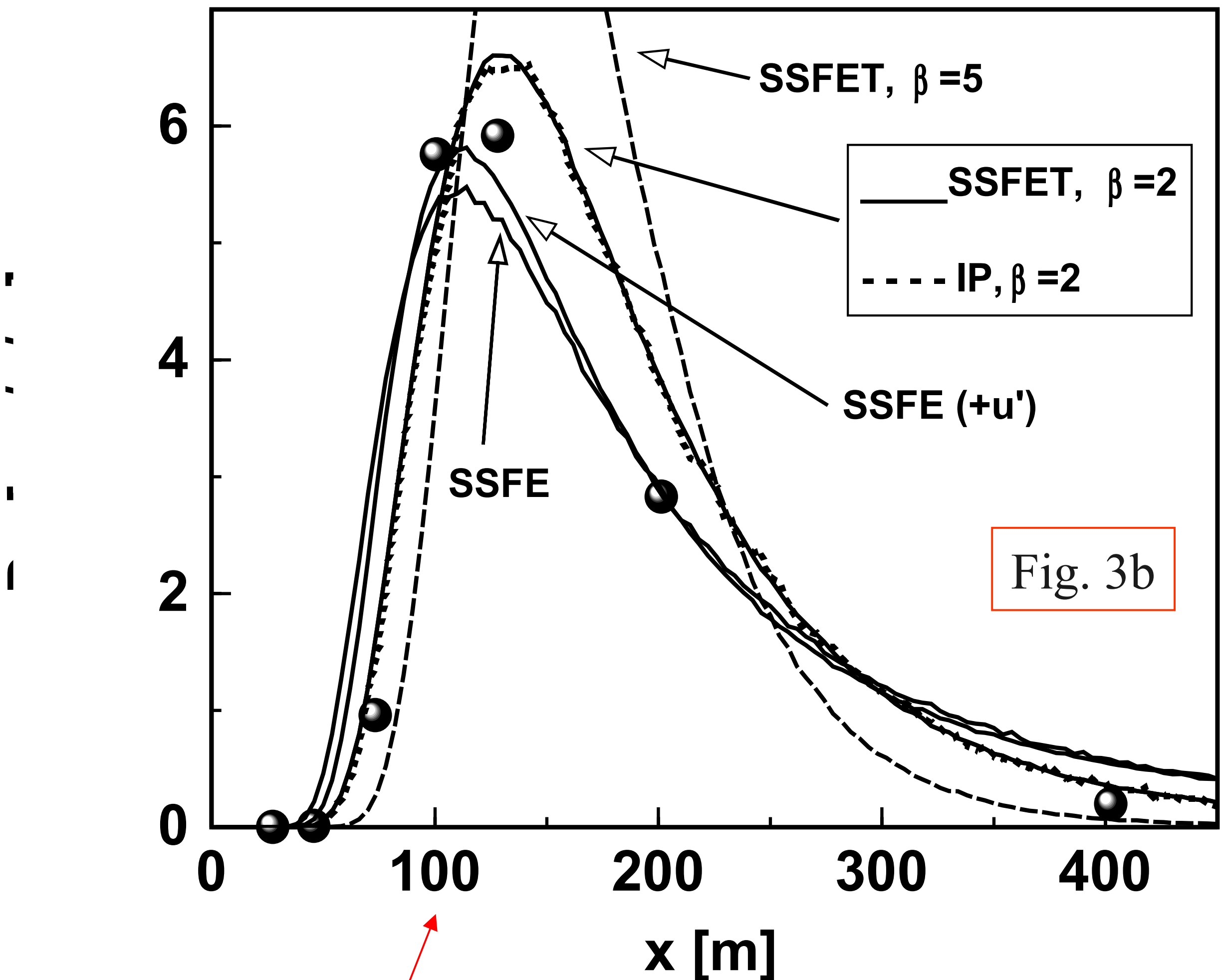
(*a* as for unique 1-D model for Gaussian inhomogeneous turbulence).

SSFET ("Settling sticky fluid element, reduced T" model):

Replace T_L in the above by T_p, a reduced timescale for the fluid velocity along the particle's path – this accounts empirically for the "crossing trajectories effect"

$$
T_p = \frac{T_L}{\sqrt{1 + \left(\beta\ w_g / \sigma_w\right)^2}}
$$

Run C: neutral stratification, L = 341 m



$$\boxed{\frac{x}{\overline{u}(h)\,T_L(h)} \approx 1}$$

Neither an Eulerian nor a Lagrangian sequence. Computed using Thomson's well-mixed 1st – order model, with timescale reduced relative to T_L

contrast with "IP" model

$$\textbf{IP:} \qquad \frac{dW_p}{dt} = \frac{w(\mathbf{X}(t)) - W_p}{\tau_p} - g$$

$$dZ = W_p\, dt$$