EAS572, "The Atmospheric Boundary Layer" Final Exam 6 Dec. 2012

Professor: J.D. Wilson Time available: 150 mins Value: 35%

Notes and Suggestions: Take time to read the whole exam first. A few equations are given below. Undefined symbols have their usual meaning. Before beginning to write, plan your response. It is more important to be coherent than to be exhaustive. Point form is acceptable. Make sure your terminology is clear. Use diagrams if they help.

Mandatory Questions

1. Your colleague, a soil scientist studying wind erosion, decides to correlate observed aerial concentration of dust against the 15-min mean magnitude

$$\tau = \overline{\sqrt{\tau_x^2 + \tau_y^2}} \ \ [\mathrm{N\,m^{-2}}]$$

of the wind drag on an eroding bare soil surface. She asks you, as a micrometeorologist, to provide and operate equipment to determine τ, but you have available only a pair of cup anemometers (providing the "cup" wind speed $s = \sqrt{u^2 + v^2}$) and a single thermocouple (allowing to measure a temperature difference ΔT). How would you proceed? Describe how you would use the signals from these instruments to evaluate (or estimate) τ, noting any approximations or uncertainties. Hint: you may assume the site of the experiments is ideal, such that Monin-Obukhov similarity theory applies.

Suppose (further) that after a first campaign along the above lines, you were to write an equipment grant proposal — hoping for funding to refine your measurements of τ, by providing one or more alternative estimates. What new equipment would you request, and what additional perspective on τ might it add?

Ingredients: *The mean shear stress* defines *the friction velocity, viz.*

$$\tau = \rho_R \, u_*^2 \ = \rho_R \sqrt{\overline{u'w'}^2 + \overline{v'w'}^2} \ . \tag{1}$$

Thus in effect, you need to measure u_*. A nuance is that the momentum flux τ varies with height, and erosion is presumably related to the surface value τ_0. Deal with this by the standard line – the surface layer is a (approximately) a constant stress layer, and the height change of τ is small enough to be overlooked. A second nuance is whether τ is formed by averaging the magnitude of the instantaneous stress, or (as in Eq. 1, by taking the resultant of the two component mean stresses.

So you want u_*. Consider the connection with this year's Assignment 2, in which you had multiple mean velocity differences and mean temperature differences from which you were able to extract u_*, L by best-fitting MOST profiles. In the problem posed, you have instruments that can provide a single $\Delta\overline{u}$ and a single $\Delta\overline{T}$ over some finite height interval Δz. You search the u_*, T_* parameter space to find best fit values.

To refine the epxeriment for a further year, you might request a 3-dimensional sonic anemometer, which (Assignment 1) provides signals (u, v, w, T) permitting to measure τ directly (the *question ought to have stated you also have a barometer and thermometer allowing to deduce* ρ_R*, but this is rather a trivial aspect). If you wanted to focus on height variation of* τ *you might have requested several sonics, giving a profile* $\tau(z)$*.*

Finally, you might also have asked for a drag plate – to measure τ_0 *directly.*

2. Illustrating in terms either of *windbreak flow* or in terms of *local advection* caused by surface inhomogeneity (e.g. bare dry soil upwind of irrigated turf, or a step change in surface temperature), outline the approach of micrometeorology in dealing with "disturbed flows." Adopt the perspective that disturbed flows can be said to be "understood" only to the extent that they can be *computed* satisfactorily, using a closed set of (Reynolds-averaged) governing equations — and that this computability needs to be proven. Focus your response around the design of an experiment intended to provide a basis to judge a theory or model of the flow.

   Elements: Define the meaning of "disturbed flow" (in general) and identify the manner in which your chosen flow is disturbed (i.e. what "drives" the disturbance). Elaborate on assumptions or restrictions that simplify the disturbance you mean to study. Picking the field of primary interest, e.g. the mean streamwise velocity $\overline{u}$ or mean temperature $\overline{T}$, use an expression of the form

   $$\frac{\overline{g}(x,y,z,t)}{\mathcal{G}} = f\left(\frac{x}{\mathcal{X}}, ...\right)$$

   (with a reduced list of independent variables if appropriate) to convey the complexity of the flow ($\mathcal{G}$, $\mathcal{X}$ are scales that you should identify with specific and suitable quantities). Draw a schematic that defines (qualitatively) what instruments you would operate and where. Briefly explain your choices, making reference to the role of each measurement relative to running and evaluating a model (provision of an upwind boundary condition, for instance). Outline possible main elements (e.g. key equations) of the theoretical description (i.e. model).

   Ingredients: *A disturbed micrometeorological flow is simply a flow that is* not *horizontally homogeneous.*

   One would perhaps choose to study flows possessing a symmetry, notably crosswind symmetry (uniformity along one horizontal coordinate, say y). One would normally, too, assume stationarity.

   There is no such thing as a "complete" characterization of a disturbed flow – one could make an infinity of measurements. One chooses a manageable (and affordable) number of sensors to measure essential inputs for interpretive theories or models, and to sample the field of the variable(s) or paramount interest (e.g. perhaps mean wind speed in a windbeak flow) sufficiently well to pick out key features of the disturbance (focusing on upwind-downwind differences or ratios). Essential model inputs certainly include inflow boundary conditions, and the forcing change at the surface if applicable (e.g. windbreak drag properties; step change in surface temperature). Criteria for instrument placement? How many x-locations do you need upwind? – one. For windbreak flow, do you want to sample the quiet zone? If so, what height to choose for your transect(s)? If addressing local advection flow, how quickly does the disturbed layer deepen? – pick a figure (1/10?) and place your transect within that layer?

The "variable of interest" would have a dependency at least *as complex as*

$$\frac{\overline{u}(x,y,z,t)}{u_{*0}} = f\left(\frac{x}{H}, \frac{z}{H}, \frac{H}{z_0}, \frac{L}{H}, \frac{\delta}{H}...\right),$$
$$\frac{\overline{T}(x,y,z,t)}{T_{*0}} = f\left(\frac{x}{H}, \frac{z}{H}, \frac{H}{z_0}, \frac{L}{H}, \frac{\delta}{H}...\right)$$

where u_{*0}, T_{*0} are conventional MO scales measured in the upwind region (H is windbreak height).

As regards theory, one would identify one or more of the key equations (two salient examples were provided as data), and make appropriate simplifications – e.g. drop molecular transport and other terms from the $\overline{u}$-momentum equation. It would be appropriate to give your criteria for the modelling domain (height; upwind and downwind extent). How are the upper, the surface, and the downwind boundary conditions to be chosen and what data will your model need? (e.g. perhaps you will assume *the upper boundary is sufficiently high that the flow there is undisturbed, allowing you to apply your MOST values inplied by the upwind MO scales.*

A nice element of one student's response was to identify specific criteria to judge the performance of theory — e.g. correct placement (on the x/H axis) of the minimum mean relative wind speed.

Further Questions

Instructions: Please answer any **two** of the following questions.

- Compare and contrast the micrometeorology of a tall crop or forest canopy (depth h_c) with that of the layer described by Monin-Obukhov similarity theory (inertial sublayer). Assume flow properties have been averaged in the horizontal plane and are both stationary and horizontally homogeneous. Address qualitative generalities of the flow regimes (e.g. velocity PDFs are or are not Gaussian; the TKE budget is or is not in local equilibrium; etc.), and give indicative profiles of the following variables in and above the canopy to a height of about $2h_c$ during a fairweather summer afternoon: mean velocity U, kinematic shear stress τ/ρ_R, turbulent kinetic energy k, mean temperature Θ, and sensible and latent heat flux densities Q_H, Q_E.

- Illustrate the sequence of assumptions or approximations or simplifications that lead from the fundamental statement of mass conservation

  $$\frac{\partial c}{\partial t} = -\nabla \cdot (\mathbf{u}\, c - \mathcal{D}\, \nabla c) + Q$$

  (where $\mathcal{D}$ is the molecular diffusivity of c in air) to the Gaussian puff model (given below) for dispersion from an instantaneous point source in an unbounded regime of stationary, homogeneous turbulence.

  Steps:

  1. assume horizontal homogeneity of the velocity field
  2. neglect source term and neglect molecular transport

3. Reynolds average
4. assume velocity field non-divergent
5. close using K-theory
6. assume the mean velocity field is $(U, 0, 0)$ (unidirectional, no mean shear)
7. assume the Ks are constants (homogeneous turbulence)

- Discuss the content, significance and implications of G.I. Taylor's Lagrangian theory of passive scalar dispersion (key equations given below)

- Describe the hierarchy of first- and higher-order turbulence closures used in RANS (Reynolds-averaged Navier-Stokes) models of turbulent flows (*No-one chose to answer this question*)

- Discuss the insights provided by the turbulent kinetic energy equation (given below) and its role in theoretical or numerical models of the atmospheric surface layer

  The TKE equation was given as data (one would typically neglect the molecular transport term and assume stationarity). Points relevant to what was asked:

  1. TKE eqn explicitly shows the role of buoyancy (suppresses TKE if heat flux negative) and mean wind shear
  2. terms can be classified as local or otherwise, i.e. source terms or transport terms. The transport term shows TKE can be imported/exported along the z-axis
  3. the TKE is a central variable in many turbulence closures, e.g. its use as the turbulent velocity scale in the first-order closure $K = \sqrt{\alpha\, k}\, \lambda$
  4. to that end the TKE eqn would need to be closed, e.g. using an eddy viscosity K (and eddy diffusivity for heat, K_h) one has

  $$0 = K \left[\left(\frac{\partial \overline{u}}{\partial z} \right)^2 + \left(\frac{\partial \overline{v}}{\partial z} \right)^2 \right] - \frac{g}{T_R}\, K_h\, \frac{\partial \overline{\theta}}{\partial z} - \frac{\partial}{\partial z} \left(\frac{K}{\gamma}\, \frac{\partial k}{\partial z} \right) - \epsilon$$

  where α, γ are constants (γ allows the eddy diffusivity for vertical transport of TKE to differ from the eddy viscosity K). Typically it is assumed that $\epsilon = (\alpha\, k)^{3/2}/\lambda$.

Symbols, Definitions, Data, Equations

Symbols: reference density and reference (Kelvin) temperature ρ_R, T_R; kinematic viscosity ν.

- Reynolds' equation for the mean streamwise momentum:

$$\frac{\partial \overline{u}}{\partial t} + \overline{u}\,\frac{\partial \overline{u}}{\partial x} + \overline{v}\,\frac{\partial \overline{u}}{\partial y} + \overline{w}\,\frac{\partial \overline{u}}{\partial z} = -\,\frac{1}{\rho_R}\,\frac{\partial \overline{p}}{\partial x} - \frac{\partial \overline{u'^2}}{\partial x} - \frac{\partial \overline{u'v'}}{\partial y} - \frac{\partial \overline{u'w'}}{\partial z} + \nu \nabla^2 \overline{u} + S$$

  where the source term S parameterizes interaction with obstacles and is typically treated as

$$S \propto -\,\overline{u}\,\sqrt{\overline{u}^2 + \overline{v}^2 + \overline{w}^2}\ .$$

  For some problems it is appropriate to add $+f\overline{v}$ to the right hand side (f the Coriolis parameter) and interpret $\overline{u}$, $\overline{v}$ as specifically the zonal and meridional components.

- For an arbitrary scalar property ϕ the conservation equation is

$$\frac{\partial \phi}{\partial t} = -\,\nabla \cdot \left(\mathbf{u}\,\phi - \mathcal{D}\,\nabla \phi\right) + Q$$

  where $\mathcal{D}$ is the molecular diffusivity of ϕ in air.

- Assuming steady state and that there is symmetry along the y axis, the distribution of mean temperature is given by

$$\overline{u}\,\frac{\partial \overline{T}}{\partial x} + \overline{w}\,\frac{\partial \overline{T}}{\partial z} = -\,\frac{\partial \overline{u'T'}}{\partial x} - \frac{\partial \overline{w'T'}}{\partial z} + Q_T\ .$$

  The source term Q_T accounts for condensation/evaporation and radiative divergence (for an unsaturated layer with a negligible burden of aerosols, $Q_T = 0$). Within a plant canopy, Q_T is non-zero due to heat exchange with leaves. An analogous equation applies for the absolute (or specific) humidity.

- The turbulent kinetic energy (TKE, "k") equation, assuming horizontal uniformity, is:

$$\frac{\partial k}{\partial t} = -\overline{u'w'}\,\frac{\partial \overline{u}}{\partial z} - \overline{v'w'}\,\frac{\partial \overline{v}}{\partial z} + \frac{g}{T_R}\,\overline{w'T'} - \frac{\partial}{\partial z}\,\overline{w'\,\left(p'/\rho_R + e'\right)} - \epsilon + \nu\frac{\partial^2 k}{\partial z^2}\ ,$$

  where $e' \equiv \left(u'^2 + v'^2 + w'^2\right)/2$ and ϵ is the TKE dissipation rate.

- The Obukhov length $L = -\,u_*^3\,T_R\,\left(k_v\,g\,\overline{w'T'}\right)^{-1}$.

- The flux Richardson number

$$R_i{}^f = \frac{\frac{g}{T_R}\,\overline{w'T'}}{\overline{u'w'}\,\partial\overline{u}/\partial z + \overline{v'w'}\,\partial\overline{v}/\partial z}$$

- The gradient Richardson number

$$R_i{}^g = \frac{g}{T_R}\frac{\partial \overline{T}/\partial z}{\left(\partial \overline{u}/\partial z\right)^2}$$

- The Gaussian puff model for the mean concentration resulting from release of q units at $x = y = z = 0$ into unbounded homogeneous turbulence with uniform mean velocity U is

$$\overline{c}(x,y,z,t) = \frac{q}{(2\pi)^{3/2}\,\sigma_x\,\sigma_y\,\sigma_z}\ \exp\left[-\frac{(x-Ut)^2}{2{\sigma_x}^2} - \frac{y^2}{2{\sigma_y}^2} - \frac{z^2}{2{\sigma_z}^2}\right] ,$$

  where

$$\begin{aligned} \sigma_x &= \sqrt{2\ K_x\ t}\ , \\ \sigma_y &= \sqrt{2\ K_y\ t}\ , \\ \sigma_z &= \sqrt{2\ K_z\ t}\ , \end{aligned}$$

  (the K's being eddy diffusivities).

- G.I. Taylor's expression for the rate of increase of the mean square displacement of particles released at $z = t = 0$ into stationary, homogeneous, unbounded turbulence is

$$\frac{d\sigma_z^2}{dt} = 2\ {\sigma_w}^2 \int_0^t R_{ww}(\tau)\ d\tau\ . \tag{2}$$

  where

$$R_{ww}(\tau) = \frac{\overline{W(t')\ W(t'+\tau)}}{{\sigma_w}^2}$$

  is the Lagrangian velocity autocorrelation function. Integration gives

$$\sigma_z^2(t) = 2\ {\sigma_w}^2 \int_0^t (t-\tau)\ R_{ww}(\tau)\ d\tau\ .$$

  The Lagrangian time scale is defined

$$T_L = \int_0^\infty R_{ww}(\tau)\,d\tau\ .$$