EAS572, "The Atmospheric Boundary Layer" Mid-term Exam 25 Oct., 2012

Professor: J.D. Wilson Time available: 60 mins Value: 15%

Note: Indices $j = (1, 2, 3)$ are to be interpreted as denoting respectively the (x, y, z) components, e.g. $\mathbf{u} \equiv \vec{u} \equiv u_j \equiv (u_1, u_2, u_3) \equiv (u, v, w)$. In equations or symbols the summation convention applies for repeated alphabetic subscripts (e.g. $u_j u_j$). Symbols $p_R, \rho_R, T_R, \theta_R$ represent pressure, density, temperature and potential temperature of a hydrostatic and adiabatic reference state, in the context of equations simplified by adoption of the Boussinesq approximation. Velocity standard deviations are denoted σ_u (etc). Recall that in flows with unvarying mean wind direction it is convenient to orient the coordinate system such that $\overline{u}$ is the component in the direction (x) of the mean wind, and $\overline{v} = 0$. ***Data & eqns. given at back***

Multichoice ($10 \times \frac{1}{2}\% = 5\%$)

1. Assuming the velocity field u_j is non-divergent. Which of the following expressions for the term $u_j\, \partial\phi/\partial x_j$ (where ϕ is an arbitrary scalar) is **not** correct?

   (a) $u\, \partial\phi/\partial x + \; v\, \partial\phi/\partial y + \; w\, \partial\phi/\partial z$

   (b) $\partial\, u\phi/\partial x + \; \partial\, v\phi/\partial y + \; \partial\, w\phi/\partial z$

   (c) $\vec{u}\, \nabla \cdot \phi$

   (d) $\vec{u}\, \cdot \nabla\phi$

   (e) $\nabla \cdot (\vec{u}\, \phi)$

2. Suppose signals p, q have zero mean (i.e. $\overline{p} = \overline{q} = 0$) whereas the mean of a third signal r is arbitrary (may be non-zero). Reynolds averaging the product $p\,q\,r$ gives for $\overline{p\,q\,r}$ the value:

   (a) $\overline{p'q'r'}$

   (b) $\overline{p'q'}\; \overline{r}$

   (c) $\overline{p'q'}\; \overline{r} + \; \overline{p'q'r'}$

   (d) 0

   (e) $\overline{r}$

3. In the engineering and fluid mechanics literature the turbulent boundary layer on a solid boundary is often referred to as a wall shear layer. Which of the statements below is **false**?

   (a) the atmospheric boundary layer (ABL) has (broadly) the character of a wall shear layer

   (b) a wall layer partitions into an inner, strongly sheared layer and an outer layer in which a mean velocity deficit law applies

   (c) together the roughness sublayer and inertial sublayer comprise the inner layer of the ABL

   (d) the scope of Monin-Obukhov similarity theory is restricted to inertial sublayers that are horizontally homogeneous

   (e) turbulent shear stresses $\overline{u'w'}$, $\overline{v'w'}$ are height-independent in the outer layer

4. Consider a neutrally-stratified and horizontally-homogeneous wall shear layer and adopt the idealization that it is a constant stress layer ($\overline{u'w'}$ = const. = $-\,u_*^2$). Let $P_s = -\overline{u'w'}\;\partial\overline{u}/\partial z$ denote the rate of shear production and ϵ the rate of viscous dissipation of turbulent kinetic energy. If this flow is in 'local equilibrium' then:

   (a) $\epsilon =$ const.

   (b) $\overline{u'w'} = 0$

   (c) $P_s = \epsilon = 0$

   (d) $k_v\, z\, \epsilon / \, u_*^3 = 0$

   (e) $k_v\, z\, \epsilon / \, u_*^3 = 1$

5. In a 1915 paper on the potential temperature profile $\overline{\theta} = \overline{\theta}(z,t)$ in the atmospheric boundary layer, Sir G.I. Taylor developed the equation (here given in slightly revised notation)

$$\frac{\partial \overline{\theta}}{\partial t} = \mathcal{W}\mathcal{D}\,\frac{\partial^2\,\overline{\theta}}{\partial z^2}$$

   where $\mathcal{W}\mathcal{D}\,[\mathrm{m^2\,s^{-1}}]$ constitutes an effective eddy thermal diffusivity (a specific definition is given by Taylor). The right hand side should best be interpreted as being _____

   (a) the radiative flux divergence

   (b) the divergence of the vertical heat flux carried by molecular conduction

   (c) minus the vertical divergence ($-\partial/\partial z$) of the vertical eddy heat flux $-\mathcal{W}\mathcal{D}\,\partial\overline{\theta}/\partial z$

   (d) latent heating/cooling associated with change of phase of water

   (e) ϵ, the rate of conversion of turbulent kinetic energy to heat

6. The turbulent kinetic energy equation, in a unidirectional, steady state, horizontally-homogeneous flow is

$$\frac{\partial k}{\partial t} = 0 = -\overline{u'w'}\,\frac{\partial \overline{u}}{\partial z} + \Phi - \frac{\partial}{\partial z}\,\overline{w'\;(p'/\rho_R + \;e'\;)}\; - \;\epsilon$$

   where $e' \equiv \left(u'^2 + v'^2 + w'^2\right)/2$ and ϵ is the TKE dissipation rate. "Φ" must represent which of the quantities below that is (otherwise) missing from this TKE equation?

   (a) $-\,\overline{v'w'}\;\;\partial\overline{v}/\partial z$

   (b) $(g/T_R)\;\overline{w'T'}$

   (c) $-\,(g/T_R)\;\overline{w'T'}$

   (d) $\overline{u}\;\;\partial k/\partial x$

   (e) $-\,\rho_R^{-1}\;\;\partial\overline{p}/\partial x$ ($\overline{p}$ the mean pressure)

7. In many numerical models of the atmospheric surface layer the eddy viscosity is parameterized $K = \lambda \sqrt{\alpha\, k}$, where k is the turbulent kinetic energy and the length scale λ is specified such that in the neutral limit $\lambda = k_v\, z$ (k_v the von Karman constant). The constant α is therefore

   (a) the value of k appropriate to a neutral surface layer
   (b) the value of $\left(\sigma_u^2 + \sigma_v^2 + \sigma_w^2\right)/2$ appropriate to a neutral surface layer
   (c) the value of u_*^2/k appropriate to a neutral surface layer
   (d) the value of k/u_*^2 appropriate to a neutral surface layer
   (e) the value of $K(u_*\lambda)^{-1}$ appropriate to a neutral surface layer

8. Vertical velocity spectra $S_{ww}(k)$ expressed in terms of wavenumber k can be transformed into the frequency representation $E_{ww}(f)$ given the wavenumber-frequency relation $k = 2\pi\, f/U$ (where U is the mean wind speed; k has the unit m^{-1}). The correct relationship is

   (a) $S_{ww}(k) = E_{ww}(f)$
   (b) $k\, S_{ww}(k) = f\, E_{ww}(f)$
   (c) $S_{ww}(k)\, dk \;=\; E_{ww}(f)\, df$
   (d) $dk \;=\; df$
   (e) $\int\limits_0^\infty S_{ww}(k)\, dk = 1$

9. In a K-closure the eddy viscosity/diffusivity is generically represented as $K = q\lambda$. The simplest schemes prescribe the length scale λ algebraically. Which of the formulations below seems most suitable for the length scale within a neutral or stable ABL? (L is the Obukhov length; β is a dimensionless constant; λ_∞ represents a prescribed upper limit for λ).

   (a) $\lambda = k_v z$
   (b) $\lambda = k_v(z - L) + \lambda_\infty$
   (c) $\lambda = k_v L\, (1 + z/\lambda_\infty)$
   (d) $\lambda^{-1} = (k_v z)^{-1} + (\beta L)^{-1}$
   (e) $\lambda^{-1} = (k_v z)^{-1} + (\beta L)^{-1} + \lambda_\infty^{-1}$

10. Mixed-layer scaling is suitable for

   (a) mean profiles ($\overline{u}$, $\overline{\theta}$, etc.) in the entire ABL
   (b) mean profiles ($\overline{u}$, $\overline{\theta}$, etc.) in a very unstable CBL, but only above the surface layer
   (c) turbulence statistics in the entire ABL
   (d) turbulence statistics in a very unstable CBL, but only above the surface layer and beneath the entrainment layer
   (e) both mean profiles and turbulence statistics, whatever the stratification

1 Short answer ($2 \times 5\% = 10\%$)

Instructions: Please **answer question A plus any other *one* question**. Use diagrams wherever they may be helpful. State assumptions or simplifications you make. ***No page limit applies.***

A. Compulsory question on MOST. Succinctly describe the Monin-Obukhov similarity theory, defining its domain of validity, stating its key heuristic postulates as to which 'external' variables 'control' or govern the turbulence in the ideal atmospheric surface layer, and giving some illustrative examples of the type of formulae it provides. Explain the significance of the Obukhov length L and the meaning of the limit $|z/L| \to 0$.

Using appropriate (ie. MO) scales[1], and assuming weakly unstable stratification, sketch *surface layer* profiles of mean wind speed $\overline{u}$, mean potential temperature $\overline{\theta}$, kinematic momentum flux $\overline{u'w'}$, mean vertical heat flux density $\overline{w'\theta'}$, and the rate of dissipation of turbulent kinetic energy ϵ. Note: these need not be quantitatively accurate sketches, but must convey the main character of the profiles.

B. An experimental design, exploiting the mass conservation principle. Suppose you were asked to determine the rate of evaporation per unit length $E\,[\mathrm{kg\,m^{-1}\,s^{-1}}]$ from a long straight canal whose width is W (order 10 m); this canal runs through a desert, and the dry air upstream of the canal has a small, constant absolute humidity ρ_{v0}. The canal runs parallel to the y axis, and you may assume the mean wind runs along the x-axis perpendicular to the canal. You have resources to mount a 10 m tower, and instrument it at several levels with anemometers and humidiometers suitable for determining mean values of wind velocity $\overline{u}$ and absolute humidity $\overline{\rho}_v$.

Explain how you might determine E, and what approximation(s) would be implicit in the method. Be as specific as possible about instrument placement, i.e. what would be your criteria. Draw schematic mean profiles $\overline{u}(z)$, $\overline{\rho}_v(z)$, $\overline{u}\,\overline{\rho}_v$ representing what one would expect the measurements to show (no need to show actual numerical values nor even their order of magnitude).

C. Application of MOST. Derive an equation for the profile of the mean absolute humidity $\overline{\rho}_v = \overline{\rho}_v(z)$ in the stably-stratified, horizontally-homogeneous atmospheric surface layer (ASL), assuming Monin-Obukhov similarity applies. Recall that the scale ρ_{v*} for humidity fluctuations and gradients will be formed from the humidity flux and friction velocity, as $\rho_{v*} = -\overline{w'\rho_v'}/u_*$. Assume that the MO universal function for mean humidity in the stable ASL is $\phi_v(z/L) = 1 + \beta z/L$ (where $\beta =$ const.), and introduce as your boundary condition the prescription that $\overline{\rho}_v(z_{0v}) = \rho_{vs}$, i.e. at the "roughness height for water vapour" (z_{0v}) the mean absolute humidity is ρ_{vs}.

D. Application of MOST. Adopting Monin-Obukhov similarity theory for the vertical gradients in mean velocity and mean temperature, prove the relationship

$$R_i^f = \frac{\phi_m}{\phi_h}\, R_i^g$$

between the flux and gradient Richardson numbers (ϕ_m, ϕ_h are MO functions of z/L.)

[1] For example, one might plot a normalized profile of ϵ as $k_v z\,\epsilon/u_*^3$ versus z/L or $(z/-L)$ or $(z/|L|)$.

Data

Note: Below ρ_R, T_R are the reference density and reference (Kelvin) temperature.

- The conservation equation for an admixture whose concentration is ϕ (and whose mixing ratio is ϕ/ρ, where ρ is the fluid density) can be written

$$\frac{\partial \phi}{\partial t} = -\nabla \cdot \left(\mathbf{u}\,\phi - \rho\, D\, \nabla \frac{\phi}{\rho} \right) + Q\ .$$

- The height-gradients in mean windspeed and mean potential temperature, according to the Monin-Obukhov similarity theory (MOST), are

$$\begin{aligned} \frac{k_{vm} z}{u_*} \frac{\partial \overline{u}}{\partial z} &= \phi_m\left(\frac{z}{L}\right) \\ \frac{k_{vh} z}{\theta_*} \frac{\partial \overline{\theta}}{\partial z} &= \phi_h\left(\frac{z}{L}\right) \end{aligned}$$

where the temperature scale is $\theta_* \equiv -\overline{w'\theta'}/u_*$. It is acceptable to assume equality of the von Karman constants ($k_{vm} = k_{vh} = k_v = 0.4$), and that in the neutral limit, defined by $z/L \to 0$, the universal functions evaluate as $\phi_m(0) = \phi_h(0) = 1$.

- The flux Richardson number

$$R_i{}^f = \frac{\frac{g}{T_R}\,\overline{w'T'}}{\overline{u'w'}\ \partial \overline{u}/\partial z}$$

- The gradient Richardson number

$$R_i{}^g = \frac{g}{T_R} \frac{\partial \overline{T}/\partial z}{\left(\partial \overline{u}/\partial z\right)^2}$$

- The Obukhov length

$$L = -\frac{u_*^3\, T_R}{k_v\, g\, \overline{w'T'}}$$

- The mixed-layer scales are

$$\delta, \qquad w_* = \left[\frac{g}{\theta_R}\, \delta\, \frac{Q_{H0}}{\rho_R c_p} \right]^{1/3}, \qquad \theta_* = \frac{-Q_{H0}}{\rho_R c_p w_*}$$

where δ is the ABL depth and $Q_{H0} \equiv \rho_R c_p \left(\overline{w'\theta'}\right)_0$ is the heat flux density at the surface.

- Assuming an eddy viscosity/diffusivity closure the governing equations for the mean horizontal velocities and the mean potential temperature in a horizontally-homogeneous ABL are

$$\begin{aligned} \frac{\partial \overline{u}}{\partial t} &= \frac{\partial}{\partial z}\left(K \frac{\partial \overline{u}}{\partial z} \right) + f(\overline{v} - V_g)\ , \\ \frac{\partial \overline{v}}{\partial t} &= \frac{\partial}{\partial z}\left(K \frac{\partial \overline{v}}{\partial z} \right) - f(\overline{u} - U_g)\ , \\ \frac{\partial \overline{\theta}}{\partial t} &= \frac{\partial}{\partial z}\left(\frac{K}{Pr} \frac{\partial \overline{\theta}}{\partial z} \right)\ , \end{aligned}$$

where U_g, V_g are the (constant) components of the Geostrophic wind, f is the Coriolis parameter and $Pr = K/K_h$ is the turbulent Prandtl number.