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Lagrangian simulation of wind transport in the urban environment

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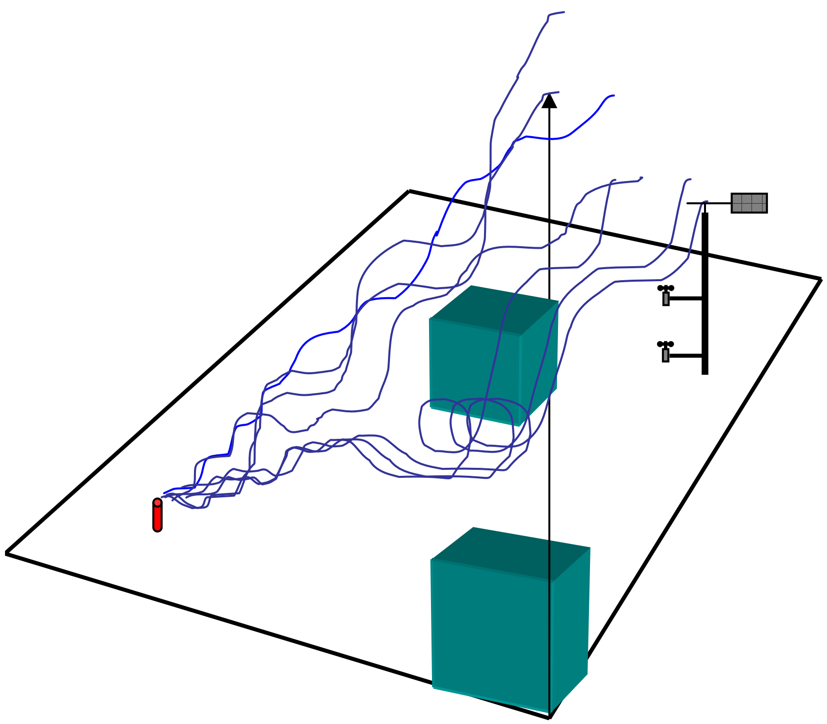


- an earlier class covered the unique *one-dimensional* well-mixed LS model for Gaussian inhomogeneous turbulence (used in assig. 3)

- today a quick look at a well-mixed *two-dimensional* LS model for horizontally-homogeneous Gaussian turbulence

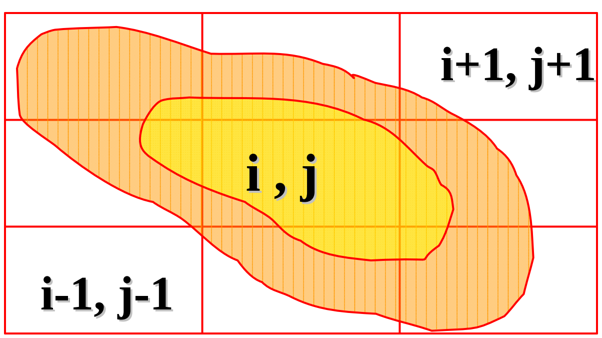
- then a look at the corresponding three-dimensional LS model for fully-inhomogeneous Gaussian turbulence

Eulerian approach:
"Mass is conserved..."

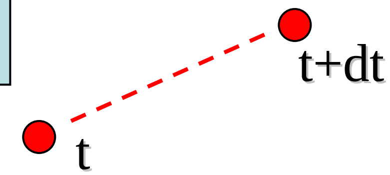
$$\frac{\partial \bar{c}}{\partial t} + \bar{u}\frac{\partial \bar{c}}{\partial x} + \bar{w}\frac{\partial \bar{c}}{\partial z} = -\frac{\partial}{\partial x}\overline{u'c'} - \frac{\partial}{\partial z}\overline{w'c'}$$

\+ closure $\overline{w'c'} = -K\frac{\partial C}{\partial z}$

"... during advection and 'diffusion' between control volumes... "



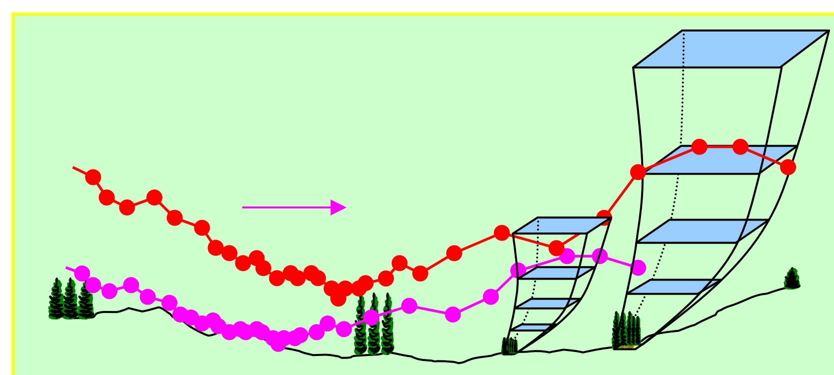
Lagrangian:
"Track motion..."



$$Z(t + dt) = Z(t) + W\ dt$$
$$W(t + dt) = W(t) + dW$$
$$dW = a(\mathbf{X}, \mathbf{U})\ dt + b\ d\xi$$

memory

random forcing



Thomson's well-mixed LS model for 2-D Gaussian, vertically-inhomogeneous turbulence

$$dU = -\frac{b^2}{2\sigma^2}\left[U\,{\sigma_w}^2 - W\,\overline{u'w'}\right]dt + \frac{\phi_u}{g_a}\,dt + b\,d\xi_u$$

$$dW = -\frac{b^2}{2\sigma^2}\left[W\,{\sigma_u}^2 - U\,\overline{u'w'}\right]dt + \frac{\phi_w}{g_a}\,dt + b\,d\xi_w$$

$$dX = [\,\overline{u}(Z) + U\,]\,dt$$

$$dZ = W\,dt$$

The well-mixed condition is a single equation constraining the vector coefficient $a_i = (a_u, a_w)$ of the generalized Langevin equation, and (in the case of multi-dimensional models) selects a *class* of models – but not a unique model.

where $\sigma^2 = {\sigma_u}^2{\sigma_w}^2 - {u_*}^4$; $b = \sqrt{C_0\,\epsilon}$; $g_a = g_a(u, w|z)$ is the Eulerian joint PDF for *u*, *w* (specifically, the joint Gaussian); and:

$$\frac{\phi_u}{g_a} = \frac{1}{2}\frac{\partial\overline{u'w'}}{\partial z} + W\frac{\partial\overline{u}}{\partial z} + \frac{1}{2\sigma^2}\left[\frac{\partial{\sigma_u}^2}{\partial z}\left({\sigma_w}^2\,U\,W - \overline{u'w'}\,W^2\right) + \frac{\partial\overline{u'w'}}{\partial z}\left({\sigma_u}^2\,W^2 - \overline{u'w'}\,U\,W\right)\right]$$

$$\frac{\phi_w}{g_a} = \frac{1}{2}\frac{\partial{\sigma_w}^2}{\partial z} + \frac{1}{2\sigma^2}\left[\frac{\partial{\sigma_w}^2}{\partial z}\left({\sigma_u}^2\,W^2 - \overline{u'w'}\,U\,W\right) + \frac{\partial\overline{u'w'}}{\partial z}\left({\sigma_w}^2\,U\,W - \overline{u'w'}\,W^2\right)\right]$$

A TWO-DIMENSIONAL TRAJECTORY-SIMULATION MODEL FOR NON-GAUSSIAN, INHOMOGENEOUS TURBULENCE WITHIN PLANT CANOPIES

T. K. FLESCH and J. D. WILSON

Boundary-Layer Meteorology **61**: 349–374, 1992.

- tested the Thomson 2-D LS model (of previous page) and an alternative based on non-Gaussian velocity PDF (an added hypothesis of the alternative model is that a_i must be anti-parallel to the Lagrangian velocity fluctuation)

- proved acceptable to aproximate the canopy velocity PDF as Gaussian

- inhomogeneity has greater influence than non-Gaussianity

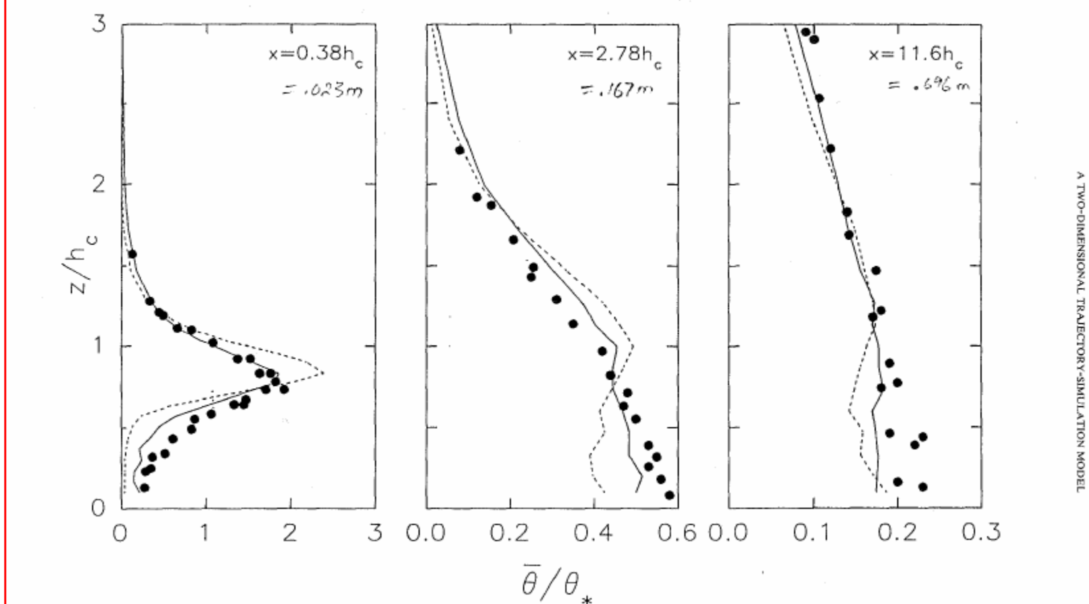
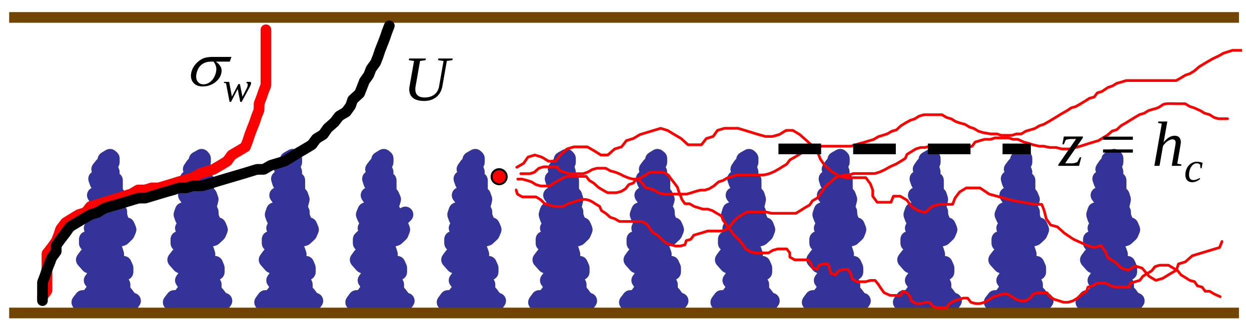
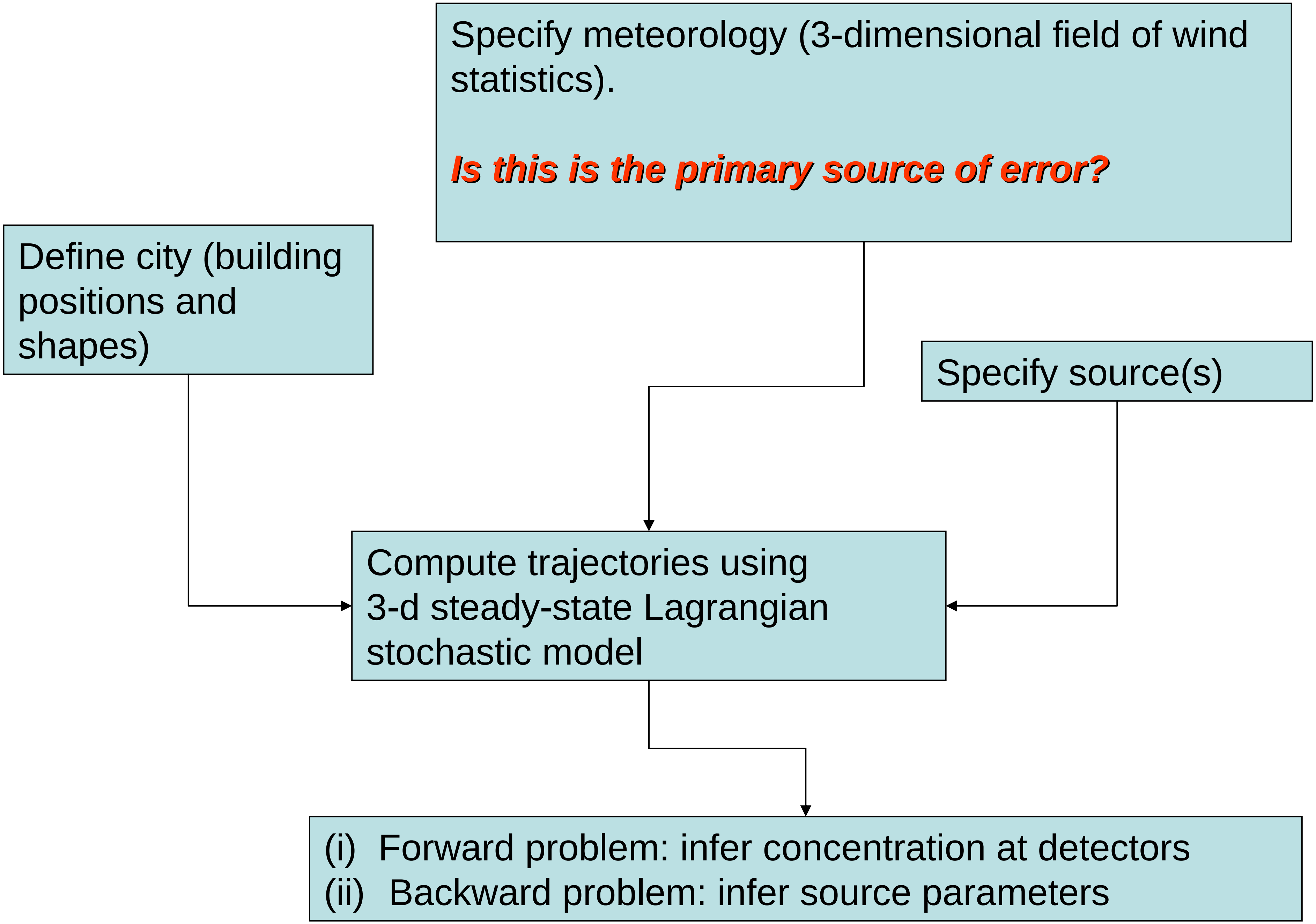


Fig. 6. Comparison of normalized temperature measurements from the LRC experiment (·) with predicted values using Gaussian (——) and non-Gaussian (-------) PDFs at three fetches (x).

<u>Wind tunnel canopy</u> – experiment by Legg, Raupach & Coppin
– crosswind line source of tracer heat



Salient property of wind in a city: short term (order one hour) wind statistics in a city are ***extremely*** inhomogeneous on all three axes


Specify meteorology (3-dimensional field of wind statistics).
Is this is the primary source of error?
Define city (building positions and shapes)
Specify source(s)
Compute trajectories using 3-d steady-state Lagrangian stochastic model
(i) Forward problem: infer concentration at detectors
(ii) Backward problem: infer source parameters

Using a Lagrangian stochastic model to compute the concentration field due to a gas source in urban winds

High resolution weather analysis/prediction: "Urban GEM-LAM"

↓ **Provides upwind and upper boundary conditions**

Building-resolving *k*-ε turbulence model: "**urbanSTREAM**" (steady state, no thermodynamic equation, control volumes congruent with walls)

↓ **Provides computational mesh over flow domain and these gridded fields:**

$$\overline{u_j}\ ,\ \overline{u_i'u_j'}\ ,\ \partial\overline{u_i'u_j'}/\partial x_k\ ,\ \varepsilon$$

Lagrangian stochastic model "**urbanLS**" to compute ensemble of paths from source(s)

Thomson's 3D well-mixed LS trajectory model

- assumes probability density function for velocity is Gaussian
- $\overline{u_i}$, R_{ij} , ε $\Rightarrow$ coefficients (***T*** 's) determining paths

$$dU_i = a_i \; dt + \sqrt{C_0 \varepsilon \, dt} \quad r$$

(*r* is a standardized Gaussian random variate: mean is zero, variance is1)

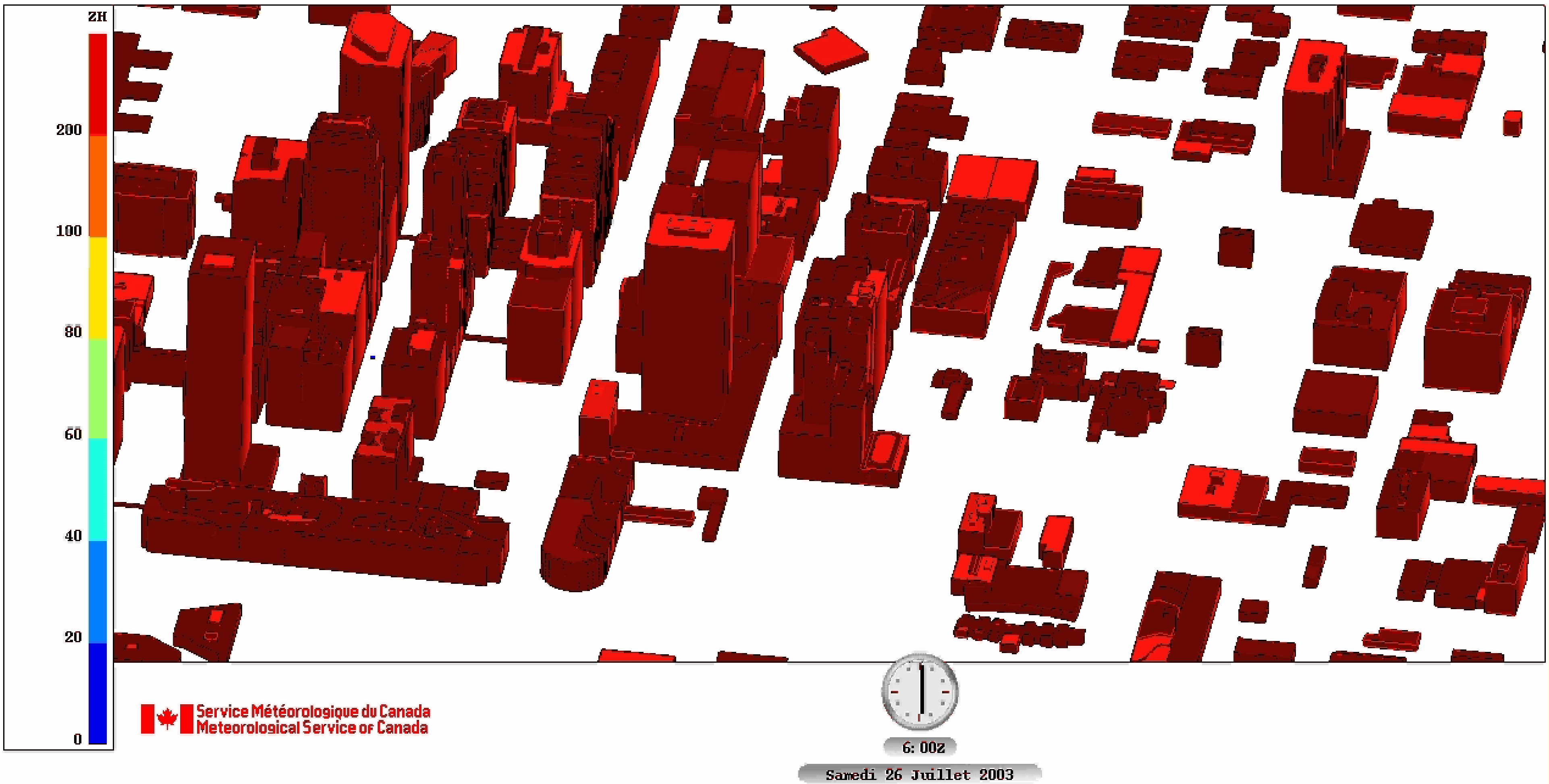
$$a_i = \tfrac{1}{2} \frac{\partial R_{i\ell}}{\partial x_\ell} - \tfrac{1}{2} C_0 \varepsilon R_{ij}^{-1} U_j + \frac{1}{2} R_{\ell j}^{-1} \frac{\partial R_{i\ell}}{\partial x_k} \left(U_j \overline{u_k} + U_j U_k \right)$$
$$= T_i^0 + T_{ij}^1 U_j + T_{ijk}^2 \; U_j U_k$$

The ***T*** 's are computed and stored on the grid prior to computing the ensemble of paths. At each timestep, use ***T*** 's from gridpoint closest to particle (ie. no interpolation to particle position). Note that the cond'tl mean accel'n in Thomson's model comprises a constant term, a term linear in the velocity fluctuation, and a term quadratic in the velocity fluctuation

Thomson LS trajectory model to compute paths in urban flow – modifications:

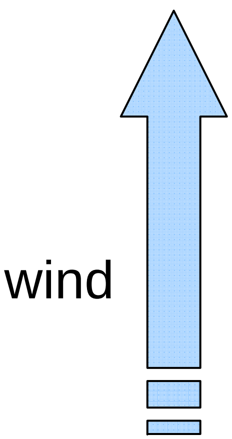
• when particle moves out of cell (I,J,K), check for encounter with building wall: perform perfect reflection off walls

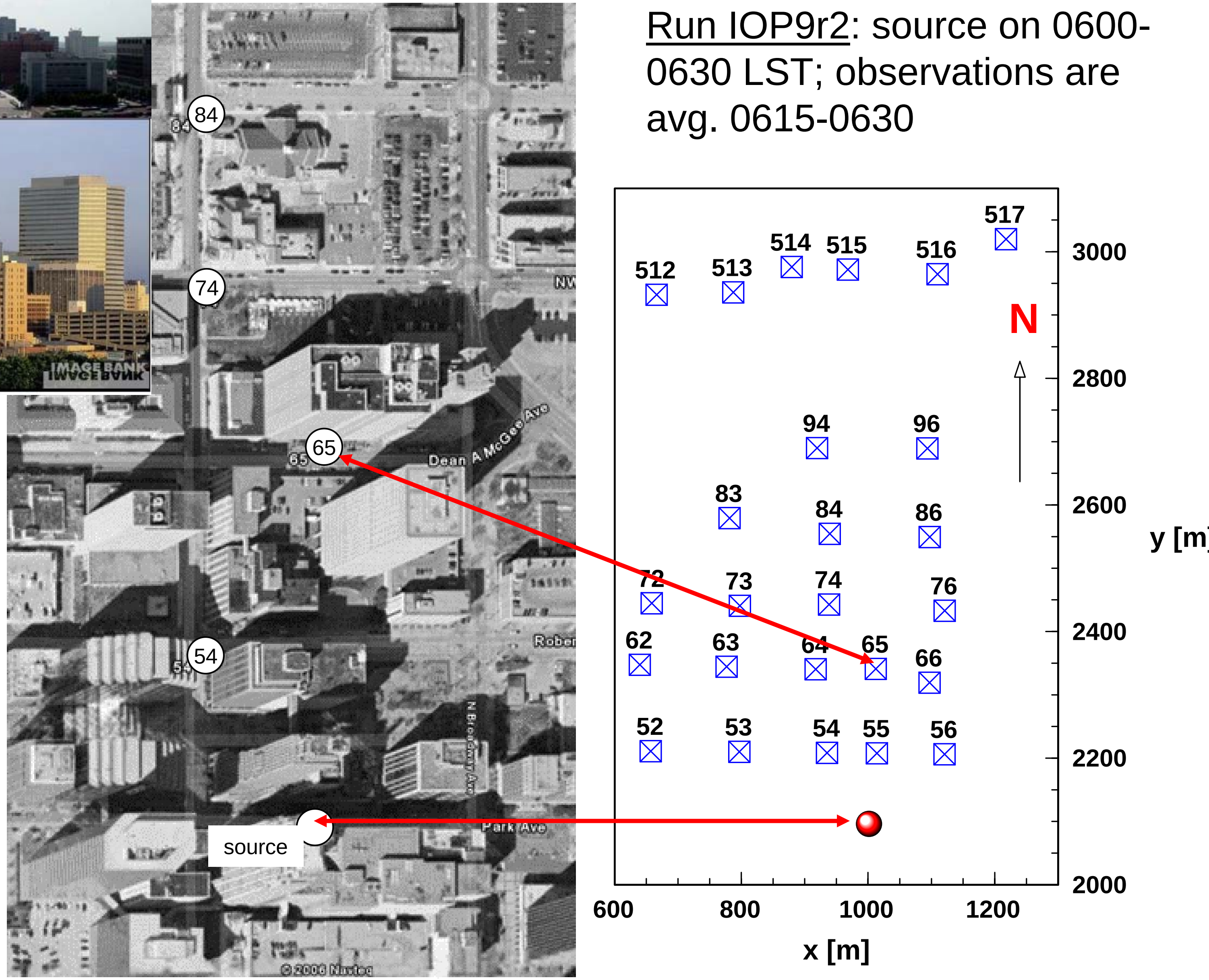
• prohibit particle velocities that differ from the local mean by more than (arbitrarily) 6 standard deviations



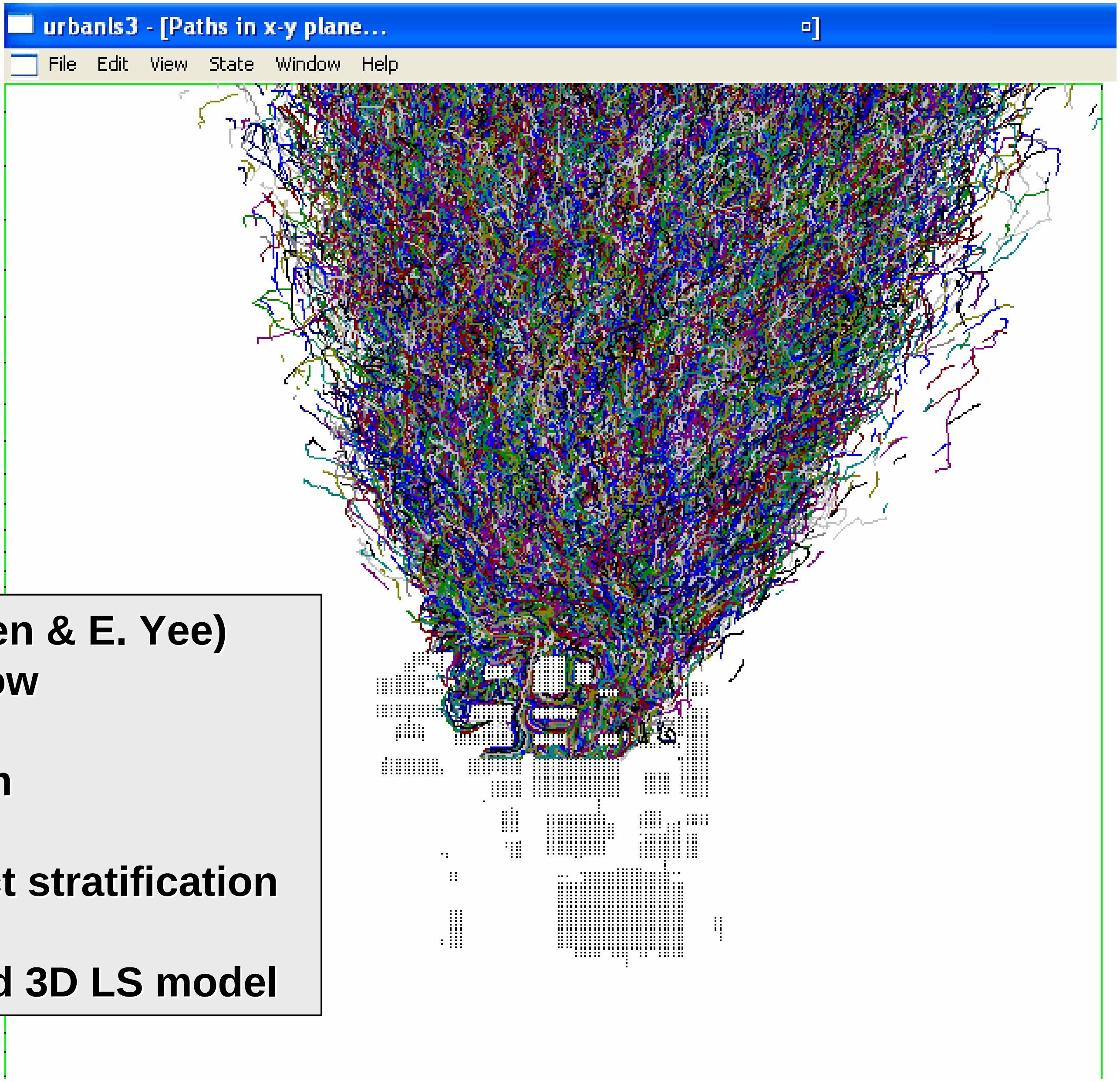
Joint Urban 2003 – tracer expt. in Oklahoma City

<u>Run IOP9r2</u>: source on 0600-0630 LST; observations are avg. 0615-0630



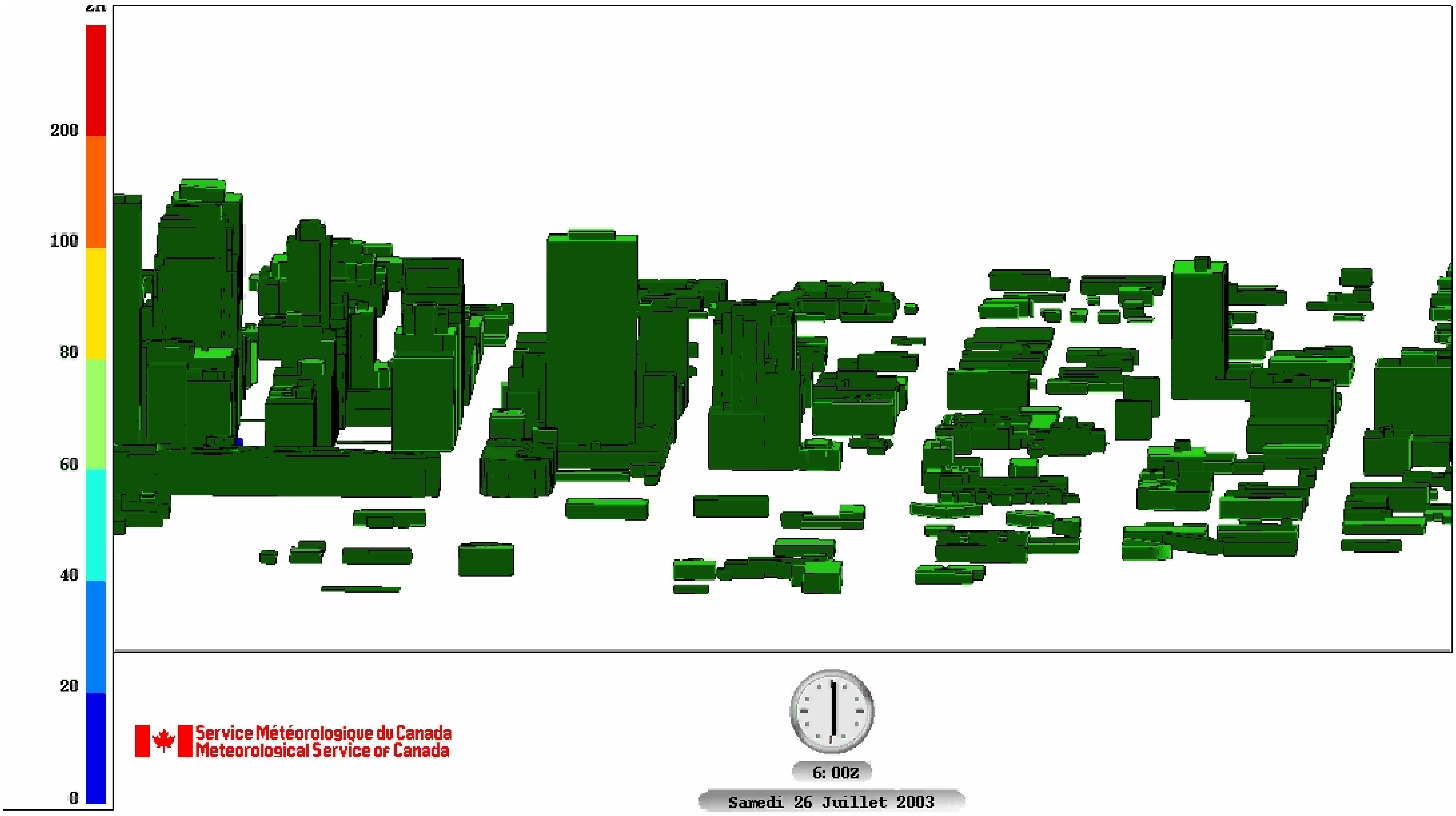


Forward paths computed for IOP9r2

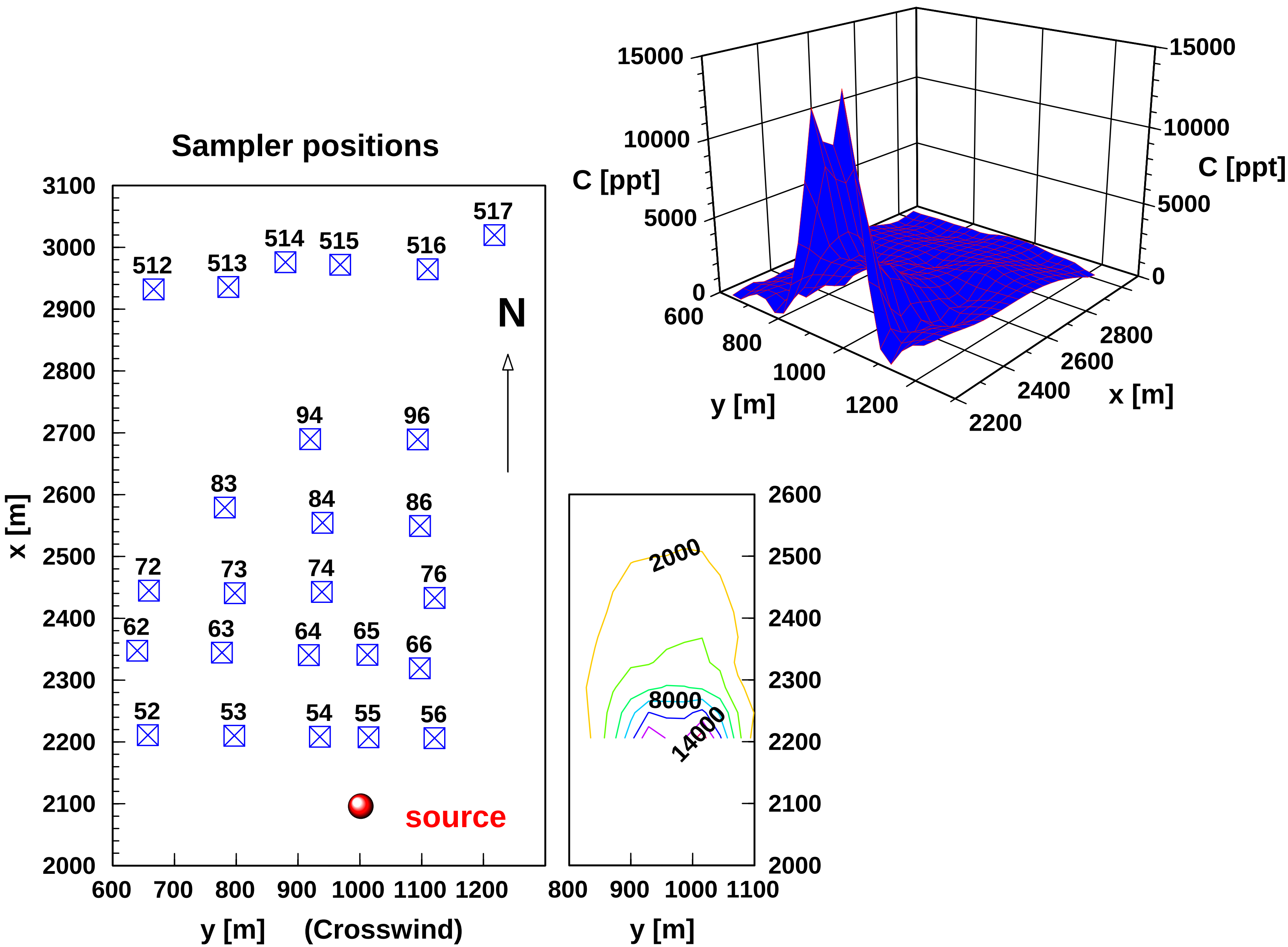


- *k*-epsilon model (F.-S. Lien & E. Yee) provides hi-res flow
- gridlength ~ 10 x 10 x 3 m
- resolve buildings, neglect stratification
- trajectories by well-mixed 3D LS model

Animations courtesy CMC (esp. Nils Ek & Jean-Philippe Gauthier)

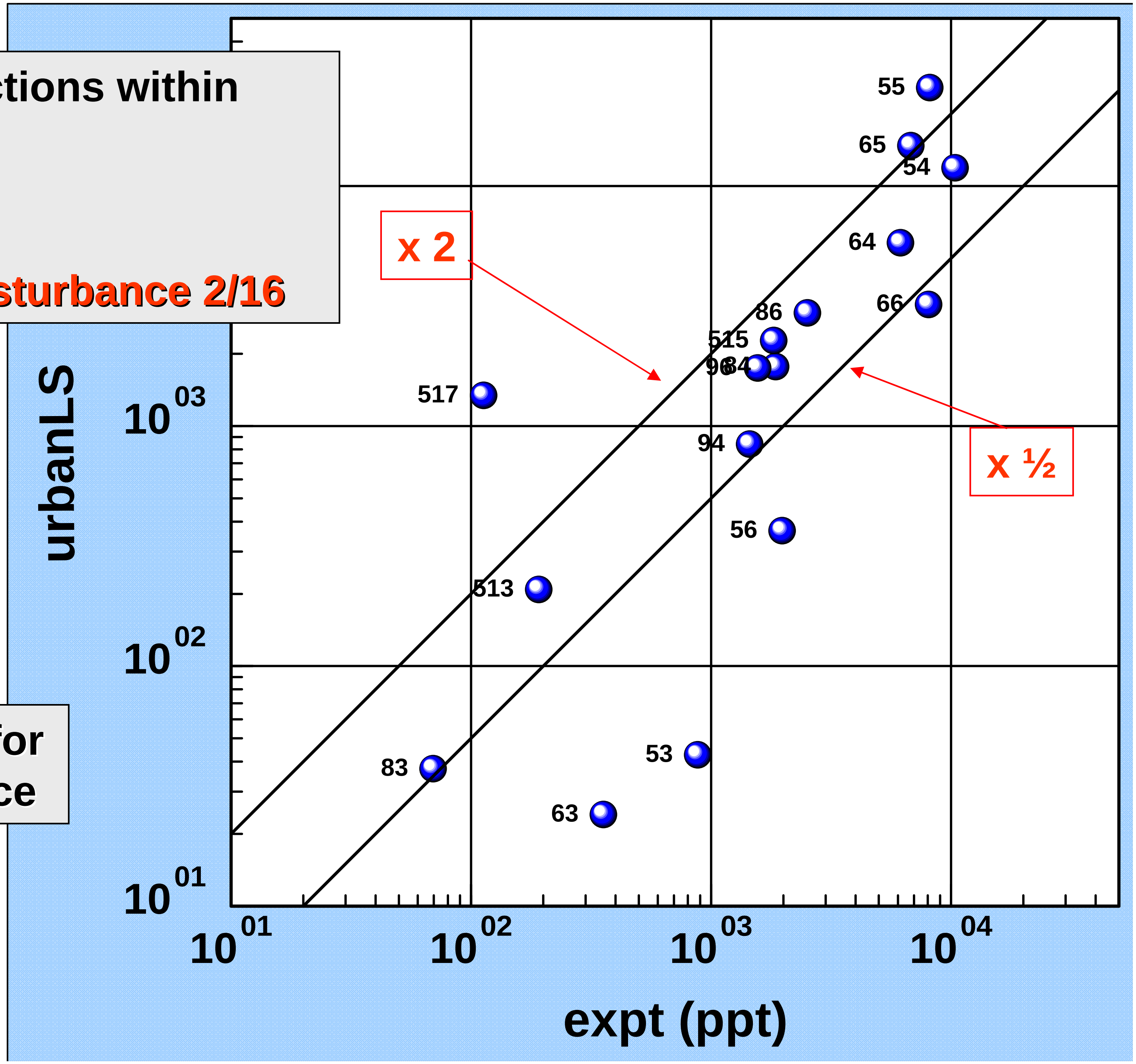


Mean ground-level concentration [parts per trillion] from forward LS simulation



Comparing LS model with observed concentration

Fraction of predictions within factor of two:
- forwards 9/16
- backwards 8/16
- ignoring flow disturbance 2/16



<u>must</u> account for flow disturbance

Conclusion

- CRTI urban dispersion project hinges on wind modelling from the global down to street scale

- prototype modelling system runs at CMC - more realistic than, say, re-tuning Gaussian puff/plume model

- time permitting, we'll later look at another application of Thomson's 3D LS model, used to infer strength *Q* of a gas source enclosed by a windbreak (i.e. emitting into a very disturbed surface layer) from the measured downwind concentration