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Monin-Obukhov Functions for Standard Deviations of Velocity

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Abstract The origins of Monin-Obukhov similarity theory (MOST) are briefly reviewed, as a context for the analysis of signals from sonic anemometers operating in the surface layer over a Utah salt flat. At this site (over the interval of these measurements) the neutral limit for the normalized vertical velocity standard deviation (σ_w/u_*) deviates markedly from what has generally been regarded as the standard value (i.e. about 1.3), suggesting (since others have also reported such deviations) that this Monin-Obukhov constant is not, in fact, universal. New (but tentative) formulae are suggested for σ_w and for the longitudinal standard deviation σ_u.

Keywords Atmospheric surface layer · Monin-Obukhov similarity theory · Turbulent velocity variance

1 Introduction

From a pragmatic point of view, a similarity theory is justified by its utility. The Monin-Obukhov similarity theory (MOST) of the atmospheric surface layer (Monin and Obukhov 1954) has certainly proven useful: having been tested in several major (e.g. the Kansas experiment, Businger et al. 1971; the International Turbulence Comparison Experiment, Dyer and Bradley 1982) and many lesser campaigns, MOST has been widely adopted and serves as the basis for treatment of the surface layer in a vast array of environmental models. In terms of parsimony, accessibility and convenience MOST is far more appealing than any alternative strategy yet offered, e.g. numerical solution of a truncated set of governing equations supplemented by empirical closure relations (Mellor 1973; Lewellen and Teske 1973).

Yet according to McNaughton (2006), the assumptions of Monin-Obukhov similarity theory "are no longer tenable." That suggestion serves as the context and motivation for the present paper. We shall review MOST, and agree with McNaughton that the proposition of

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universality (of the MOST functions) appears to have been unduly optimistic. Some recent observations of daytime velocity standard deviations in the inertial sublayer over a salt flat (i.e. dry lakebed, or 'playa') at the Dugway Proving Grounds, Utah, U.S.A. serve to illustrate the point.

2 Background on Monin-Obukhov Similarity Theory

From the perspective of the well-known conservation equations for velocity statistics (mean $\overline{u}_i$, covariance $\overline{u_i' u_j'}, \ldots$) and coupled properties (e.g. temperature $\overline{T}$, temperature variance $\overline{T'^2}$, heat flux $\overline{u_i' T'} \ldots$), it would seem miraculous that dimensionless ratios, such as (for example) the ratio $\sigma_w^2 / \overline{u'w'}$ of vertical velocity variance to the mean shear stress ($\overline{u'w'} \equiv -u_*^2$) in a frame aligned with the mean wind, should present a *universal* behaviour, across even a severely restricted class of turbulent flows (e.g. the inertial sublayer of the horizontally-homogeneous wall shear layer). Nevertheless the existence of an *approximate* universality of surface-layer profiles was the essential proposition of MOST, which offered a means of organizing and summarizing observations in the surface layer that eclipsed earlier suggestions. The degree of acceptance of MOST for the scaling of surface-layer velocity statistics is measured by the prevalence of textbook treatments (e.g. Haugen 1973; Stull 1988; Garratt 1992; Kaimal and Finnigan 1994) and in the view of Foken (2006) "twenty to thirty years ago, the Monin-Obukhov similarity theory was the accepted dogma and it was nearly impossible to publish results in disagreement… (especially in the former Soviet Union)." Latterly, however, a number of authors have identified deficiencies or suggested there are processes or dependencies in the surface layer that lie outside the scope of (or must tend to confound) MOST (e.g. Wyngaard 1985; Hill 1989; Kader and Yaglom 1990; Hogstrom 1990; Laubach et al. 2000; Mahrt et al. 2001; McNaughton and Brunet 2002).

It is important to preface what follows by noting that Monin-Obukhov similarity theory was never *expected* to be perfect: in the words of Monin and Obukhov (1954), their use of the "methods of the theory of similitude" gave a "satisfactory qualitative description of the process" (of "ground-layer physics"). Monin and Obukhov were careful to emphasize that their theory, while an improvement on those that preceded it, was to be taken as approximate. It is also useful to recall that its invention was more or less simultaneous with (and made possible by) the emergence of new and explicit idealizations, such as that of an "atmospheric surface layer" that was synonymous with an (approximately) constant flux layer.[1] Furthermore the very notion of 'perfection' of a theory of atmospheric statistics needs to be qualified: by definition, 'statistics' are subject to sampling error, whose magnitude is related to averaging duration (see Lumley and Panofsky 1964, who state: "We wish to achieve the same results by averaging a single realization as we would achieve by averaging the whole ensemble. For this to be true, a single realization must itself be an ensemble, that is, different sections of the record must be regardable as independent experiments"; see also Wyngaard 1973, p. 135; Yaglom 1977). Thus we would not expect that every (MOST) normalized surface-layer statistic ϕ that had been measured by a perfect instrument in the inertial sublayer of a surface layer that perfectly ascribed to the restriction of horizontal homogeneity, should plot against z/L exactly in conformity with the (putatively universal) $\phi(z/L)$ curve. Observations will always scatter about theoretical predictions. In the context of whether MOST can be refined,

[1] "The condition that the fluxes τ and q are constant (within the given tolerance) can serve to determine the actual concept of the ground layer" (quoted from Monin and Obukhov 1954; see also Obukhov 1971, and Businger 1955).

the question is not that of eliminating scatter, but that of *reducing* the scatter, and/or reducing bias (of normalized statistics emerging from different sites or campaigns) by choosing different (or more numerous) scaling parameters than suggested by MOST.

2.1 Usage of the Term 'Extended Monin Obukhov Similarity Theory'

Because the pertinent literature is substantial and spans six decades, an author who presumes to define what is meant by, implied by, and understood by "Monin-Obukhov similarity theory" is unlikely to satisfy all readers—particularly since the literature can never reveal what was merely "tacit" understanding (i.e. shared but unstated background). Different readers may disagree in their assessment of what constitutes the "physical content" of MOST, but if we accept MOST in the spirit of a similarity theory[2] we surely may say MOST is strikingly *agnostic* in regard to the *circulatory mechanisms* of transport, viz. characteristic eddy structures and hierarchies, etc. And that being the case, it is plausible that as more is learned about the circulations that co-exist in the ABL, it may be possible to improve on MOST.

The original Monin-Obukhov similarity theory explicitly limited its validity to a notional layer of the (horizontally-uniform) atmospheric boundary layer (ABL) within which velocity (and scalar) statistics are (hypothetically) independent of any length scales relating to the nature of the lower boundary (e.g. surface roughness length z_0, plant canopy height h_c, or depth h of the roughness sublayer) and simultaneously are (hypothetically) independent of (or insensitive to) the depth (δ) and/or condition of the overlying ABL.[3] The resulting set of governing scales for forming non-dimensional statistical properties was limited to the friction velocity u_* (definitions given later) and Obukhov's length scale (introduced 1946; English translation available as Obukhov 1971)

$$L = -\frac{u_*^3}{k_v \, \frac{g}{T_0} \, \left(\overline{w'T'}\right)_0} \tag{1}$$

where k_v the von Karman constant is included by convention, g is the gravitational acceleration, T_0 is the mean Kelvin temperature, $\left(\overline{w'T'}\right)_0$ is the surface value of the kinematic heat flux density.

Obukhov's (1946) paper (in translation, Obukhov 1971) and that of Monin and Obukhov (1954) addressed the profiles of mean wind speed, mean temperature, eddy viscosity and eddy diffusivity. The extension to velocity standard deviations (etc.) followed later, an example of particular interest to us here being Panofsky and McCormick (1960), who were among

[2] As noted by Businger and Yaglom (1971), Monin and Obukhov (1954) based their exposition "on purely dimensional considerations." Their interest was "ground layer physics," and they gave a definition of the "ground layer" as the layer within which momentum and heat fluxes lay within a certain tolerance of their surface values. They were very much aware that "in the case of unstable stratification at great height, large turbulent elements develop" but made no comment that would explain why they did not consider it warranted to account in their theory for processes *above* the "ground layer" (they did explicitly exclude the Coriolis effect, viz. assume constancy of wind direction in the "ground layer"). One may easily surmise why, given the superiority of their "ground layer" theory over alternatives of the time, they would have been content with the 'effectiveness-to-parsimony ratio' of their treatment. Note: Obukhov's earlier (1946) introduction (see Obukhov 1971) of the Obukhov (scaling) length, i.e. "the height of the sub-layer of dynamic turbulence," had been based on physical reasoning.

[3] Although this assumption appears not to have been identified by Monin and Obukhov, we may regard it as being implicit.

the earliest authors to evaluate the profile of σ_w near the surface.[4] From an assumption that

$$\sigma_w = A \; [z\,(P_s + \alpha P_b)]^{1/3} \tag{2}$$

where P_s, P_b are the rates of shear and buoyant production of turbulent kinetic energy and A, α are dimensionless constants, Panofsky and McCormick deduced that

$$\frac{\sigma_w}{u_*} = B \left[\phi_m\left(\frac{z}{L}\right) + \alpha\frac{z}{L}\right]^{1/3} \tag{3}$$

where $\phi_m(\,)$ is the MOST universal function for the mean wind shear (Panofsky and McCormick noted that the scaling of σ_w with u_* had been anticipated by Monin 1959, who primarily addressed turbulent dispersion, and enunciated the principle that "turbulent dispersion in a horizontally-homogeneous stationary surface layer of air obeys the similarity theory in which the values L and u_* are the only scales of length and velocity"). From observations, Panofsky and McCormick concluded that $B = 1.25$.

Thus we may loosely state that, according to original MOST, the vertical and longitudinal velocity standard deviations should scale as[5]

$$\frac{\sigma_w}{u_*} = \phi_w\left(\frac{z}{L}\right), \tag{4}$$

$$\frac{\sigma_u}{u_*} = \phi_u\left(\frac{z}{L}\right). \tag{5}$$

However it was soon noted that statistics of the horizontal velocity did not order well when normalized using (u_*, L). In reference to spectra of lateral velocity Lumley and Panofsky (1964) noted that "increasing instability greatly increases the low-frequency portion of the spectra but leaves the high frequency portions relatively unaffected" and that "the effect of increasing lapse rate is to superimpose long-period variations on top of the shorter mechanically produced variations." Thus it was recognized that (Calder 1966) "the classical form of (MOST) cannot be applied legitimately to the variances of the horizontal components of the wind velocity fluctuation" (see also Busch 1973; Wyngaard 1973; Panofsky 1973). Re-examining velocity spectra from the Minnesota experiments, Kaimal (1978) concluded that the energy-containing region of (daytime) horizontal velocity spectra "follows a mixed-layer similarity where (δ) is the only controlling length scale." Bradshaw (1978), commenting on Kaimal's paper, argued that even in neutral stratification ABL depth exerts an influence (on the energy containing range of horizontal velocity spectra), by virtue of the presence of large, so-called "inactive" scales of quasi-horizontal motion, as first spoken of by Townsend (1961)—the term 'inactive' describing the hypothetically small or vanishing contribution of this type of eddy to vertical transport, in particular to the shear stress and the shear production of turbulent kinetic energy.

[4] According to Calder (1966), extension of MOST to cover statistical properties beyond those mentioned above, and in particular 'the higher moments of velocity and temperature,' was given by Monin (1962)." However, evidently the extension had been anticipated by Monin (1959), as it was alluded to by Panofsky and McCormick (1960).

[5] Making reference to (our) Eq. 4, Wyngaard (1973) noted: "This is a very succinct and powerful prediction. All $\overline{w^2}$ data from our ideal surface layer… follow a universal curve when treated as in (Eq. 4)." It is unclear whether by this statement Wyngaard was referring to *theoretical* universality of ϕ_w according to MOST, or expressing a conviction that this (putative) universality had been confirmed observationally.

Accordingly it became habitual to extend the set of MOST scaling parameters to (u_*, L, δ), that is, to include the ABL depth as a scaling parameter[6]—this will be referred to as "extended MO scaling." Extended MOST holds, in principal and at best, only well above the roughness sublayer, so that (for example) observed neutral values of σ_w/u_* just above a plant canopy need not prove universal when scaled as $(z/L, z/\delta)$. And as earlier noted, a number of investigators have cast doubt on the universality of the conventional MOST functions: for example Hogstrom (1990) found that $\phi_w(0)$ was not constant, and that its variability correlated well with $f(z-d)/u_*$ (where f is the Coriolis parameter, and d the displacement length).

2.2 Condition for 'Complete Similarity' of Physical Systems

A similarity theory may emerge from a heuristic dimensional analysis, wherein intuition serves a key role in the choice of governing 'external' scales, or it may emerge more ploddingly from an analysis of the governing equations (if known). Where the intuitive route is taken, there is a tension between the greater parsimony (thus potentially, efficacy) that emerges by eliminating factors suspected to be of marginal import, and the loss of generality and accuracy entailed if (in fact) the system responds tangibly to those neglected factors. It is very understandable that in the context in which MOST was developed, parsimony was a prime criterion.

We may ask, is it *plausible* that MOST scaling should render all members of the universe of horizontally-uniform atmospheric surface layers self-similar? "Complete similarity" between physical systems A and B is said to obtain if the governing equations (and boundary and initial conditions), expressed in non-dimensional form with the use of scale factors (dimensionless groupings of scales for the physical variables), can be made identical. It is easy to show that the incompressible Navier–Stokes equations, shed of Coriolis and viscous terms, *do* support pure MOST scaling: if made dimensionless on scales V, L, P, T_* of velocity, length, pressure, and temperature, the Navier–Stokes equations contain but two dimensionless scale factors, viz. $P/\rho_0 V^2$ and gT_*L/T_0V^2 (where ρ_0, T_0 are the bulk density and temperature of the layer). Choosing $V = u_*$, $T_* = -(\rho_0\, c_p\, u_*)^{-1}\, Q_{H0}$, $P = \rho_0 u_*^2$ and $L \propto u_*^3 T_0/(gT_*)$ renders these two scale factors constant, such that one arrives at an invariant (dimensionless) momentum equation.

But apart from the crucial oversight of having neglected the boundary conditions (which would admit salient parameters such as ABL depth), the above constitutes an inadequate 'proof' of validity for MOST, which concerns ASL *statistics*. The governing equations pertinent to discussion of "complete similarity" properly must comprise an infinite (and unclosed) set—the Reynolds (mean momentum) equations; the mean thermodynamic (and humidity) equations; and the infinite set of moment equations.[7] Proof of the "complete similarity" of (horizontally-homogeneous) surface layers would demand an examination, too, of the influence (within the nominated layer) of upper and lower boundary conditions, from which it is certain that further governing scales would arise. To conclude this brief review, in the author's (reluctant) opinion a robust universality of the MOST functions should not be expected. We turn now to field evidence.

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[6] The convective velocity scale $w_* \equiv \left[(g/T_0)\,(\overline{w'T'})_0\,\delta\right]^{1/3}$ is not an independent scale, for from the definition of L it can be written $w_* = \left[-k_v^{-1}\, u_*^3\, \delta/L\right]^{1/3}$.

[7] Those derived from the momentum equations (alone) cannot expand the set of scale factors to be included; however their associated boundary conditions could.

3 Measurements at Dugway Proving Grounds

In an experiment specifically intended to study the issues summarized above, during 24 May–2 June 2005 eighteen three-dimensional sonic anemometers (Campbell Scientific Inc. model CSAT3) were operated over a dry lakebed at the Dugway Proving Grounds, Utah, U.S.A. Nine sonics were mounted on a tower (heights $z \leq 26$ m), and the balance on a cross-wind transect at height[8] $z = 3$ m (all had been returned to the manufacturer for calibration immediately prior to the experiment). Although the salt flat was ideally uniform for winds from the north (Metzger and Holmes 2008), the tower and transect had been installed near a raised parking area, on which stood several large instrument trailers (see McNaughton et al. 2007, for a photograph of the set-up). Comparison of statistics from sonics along the transect (at $z = 3$ m) during northerly flow revealed an influence of these obstacles that could readily be detected at the 3-m sonic on the tower (viz. a 4% reduction in 30-min mean wind speed), but was insignificant at and beyond 20 m westward of the tower. The present analysis excludes instruments in the disturbed flow by restricting consideration to signals from the four westernmost sonics standing at $y = (30, 40, 50, 60)$ m west of the tower, and the four uppermost sonics on the tower at $z = 8.71, 12.52, 17.94, 25.69$ m (the lowest of the latter stood 8 m upwind from, and 5 m above the roof of, the nearest trailer).

Time series from these instruments were collected continuously at 20 Hz for about 9 days, during several of which the wind blew from the north over about 100 km of uniform desert surface (one selection criterion for data to be shown was that the mean wind direction $|\beta| \leq 25°$). Coordinate rotations, sequentially enforcing $\overline{v} = 0$ then $\overline{w} = 0$ (e.g. Wilczak et al. 2001) were performed individually for each anemometer,[9] run by run; then, diagnostic values were computed for the 30-min mean friction velocity u_* and Obukhov length L, based on the mean (post-rotation) vertical fluxes of heat and momentum at the uppermost four anemometers on the mast, viz.

$$u_*^4 = \left(\overline{\overline{u'w'}}\right)^2 + \left(\overline{\overline{v'w'}}\right)^2, \tag{6}$$

$$T_* = -\overline{\overline{w'T'}}/u_*, \tag{7}$$

$$L = \frac{u_*^2\, T_0}{k_v\, g\, T_*}, \tag{8}$$

where T_* is the turbulent temperature scale; $\overline{\overline{w'T'}}$ denotes the average value of the kinematic heat flux density $\overline{w'T'}$ over the four highest sonics, etc. Giving an indication of the quality of the data, Table 1 summarizes the coefficients of variation (CV) of (post-rotation) statistics across (a) the four sonics with common height $z = 3$ m, and (b) the four uppermost sonics on the mast.

We have seen that extended-MOST includes boundary-layer depth δ as a scaling parameter. Unfortunately, however, δ was not measured during these measurements at Dugway, so that an estimate has been made from an idealized heat budget

$$\frac{\partial \overline{T}}{\partial t} = \frac{\overline{\overline{w'T'}} + 0.2\,\overline{\overline{w'T'}}}{\delta} \tag{9}$$

[8] Late in the experiment the sonics on the transect were lowered; however all data considered here stem from periods when the transect was at 3 m.

[9] The rotation was mandatory because a precise leveling of the sonics had not been possible. Azimuthal orientation of the sonics, on the other hand, was excellent, such that pre-rotation azimuth angles were highly consistent along the tower, and therefore azimuthal rotation angles were highly consistent from one sonic to the next.

Table 1 Statistical summary of the coefficients of variation (CV) of σ_u, σ_w, $\overline{w'T'}$ and u_* (the latter computed for each sonic as $u_* = (\overline{u'w'}^2 + \overline{v'w'}^2)^{1/4}$)

	Mean	Median
Transect		
CV_{σ_u}	0.016	0.014
CV_{σ_w}	0.020	0.019
CV_{u_*}	0.154	0.151
$CV_{\overline{w'T'}}$	0.053	0.053
Mast		
CV_{u_*}	0.130	0.094
$CV_{\overline{w'T'}}$	0.039	0.034
σ_β	0.85	0.79

Under "Transect" are the coefficients of variation across the four western-most sonics on the transect at $z = 3$ m. Under "Mast" and pertaining to the four uppermost sonics on the tower, are the CV's (of nominally height-independent MOST scaling factors u_*, $\overline{w'T'}$); also listed are the mean and median of the standard deviation (across the top four sonics) of the mean wind direction (the small value of the latter implying mean wind direction varied negligibly with height). Selection criteria: time between 0900 MDT and 1930 MDT, $|\beta| \leq 25°$, $L < 0$, $\overline{\overline{w'T'}} \geq 0.05$ m s^{-1} K, $u_* \geq 0.15$ m s^{-1}. Resulting sample satisfying these criteria contained fifty-four 30-min runs

which assumes, among other things, a downward heat flux at the top of the ABL amounting to 20% of the surface heat flux; see Carson and Smith (1974). The 30-min temperature trend for each of the upper four sonics was obtained by linear regression, and averaged to evaluate the left-hand side. Figure 1 shows the resulting daily cycles of ABL depth. Metzger and Holmes (2008) deduced a single characteristic afternoon value ($\delta = 1.3$ km), by invoking a previously reported empirical relationship (Liu and Ohtaki 1997; Kaimal et al. 1976, Fig. 5) between δ and the position of the spectral peak in u'. Their estimate is broadly consistent with Fig. 1, but cannot be compared directly because the underlying u' time series from which it was deduced spanned 4 h, starting at noon.[10] The present simple-minded (but direct) diagnosis provides plausible, albeit noisy, time-resolved values[11] for daytime δ.

3.1 General Nature of the Flow

Owing to the small surface roughness length ($z_0 = 0.2$–0.5 mm; Metzger and Holmes 2008) ratios $u_*/\overline{u}$ of friction velocity to mean wind speed assumed categorically smaller values than usually encountered, in consequence of which the magnitude of the Obukhov length, varying with u_*^3, was often exceptionally small in magnitude. This in turn meant that a wide range on the z/L axis was sampled. Vertical profiles of (post rotation, 30-min mean) $\overline{w'T'}$, $\overline{u'w'}$ and friction velocity showed the irregularity that is normal due to the width of the sampling distribution for these second-order moments, while differences in mean wind direction from one to another sonic on the tower were small (see Table 1).

The top panel of Fig. 1 indicates that daytime values of the kinematic heat flux density $\overline{w'T'}$ peaked at around 0.2 m s^{-1} K, implying maximum surface heat flux densities Q_{H0} of around 200 W m^{-2}. This rather modest daytime heat flux owes to the fact of the salt flat

[10] The particular afternoon for which their estimate applies is not reported; it could not have been May 24, and had to be one of the three other afternoons represented on Fig. 1.

[11] D. Charuchittipan has computed hourly values for δ for the present experiment, by best-fitting empirical spectral curves to computed u' spectra, which approach is conceptually similar to using a correlation between spectral peak position and ABL depth. These values proved comparably noisy, but broadly consistent with values stemming from Eq. 9 applied to the same 1-h intervals (pers. comm. 2008).

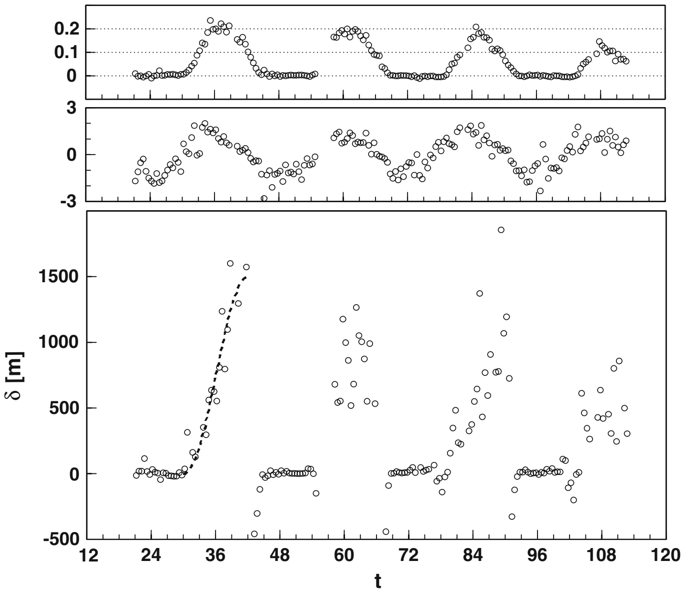

Fig. 1 Diurnal cycle in kinematic heat flux density (top panel, m s^{-1} K), temperature tendency (middle panel, K h^{-1}), and ABL depth as computed from Eq. 9. Time t = 30 corresponds to 0600 Mountain Daylight Time May 24, 2005. The dashed line is the smooth fitted curve $\delta = 750[1 - \cos(\pi(t - 30)/12)]$. (Negative values for δ can arise from Eq. 9, if the trend in mean temperature does not have the same sign as the surface heat flux. They are displayed here only to convey the limitations of using that equation to estimate δ.)

being exceptionally wet that May. Although it may appear that sensible heat flux density was "zero" at night (which would be incomprehensible), closer examination shows it was merely (and as expected) much smaller in magnitude than during the afternoons: nocturnal values were $\overline{w'T'} \approx -10\,\mathrm{W\,m^{-2}}$.

4 Standard Deviation of Vertical Velocity

Figure 2 gives the 'daytime' (0900MDT to 1930MDT, MDT being Mountain Daylight Time) observations of σ_w/u_* plotted as function of z/L (selection criteria: $L < 0$, $|\beta| \leq 25^{\mathrm{o}}$, $u_* \geq 0.15\,\mathrm{m\,s^{-1}}$). A solid line gives the widely-used relation

$$\frac{\sigma_w}{u_*} = \phi_w\left(\frac{z}{L}\right) = 1.25\left(1 - 3\,\frac{z}{L}\right)^{1/3}, \tag{10}$$

(Kaimal and Finnigan 1994, Eq. 1.33) which originated with Panofsky et al. (1977), while the dashed lines are least squares fits

$$\frac{\sigma_w}{u_*} = 0.8\left(1 - 9.5\,\frac{z}{L}\right)^{1/3}, \tag{11}$$

$$\frac{\sigma_w}{u_*} = 1.0\left(1 - 4.5\,\frac{z}{L}\right)^{1/3}, \tag{12}$$

to all data. Both the above equations presuppose the power-law index to be 1/3 (which is necessary for consistency with the free convection limit), while Eq. 12 also presumes a value

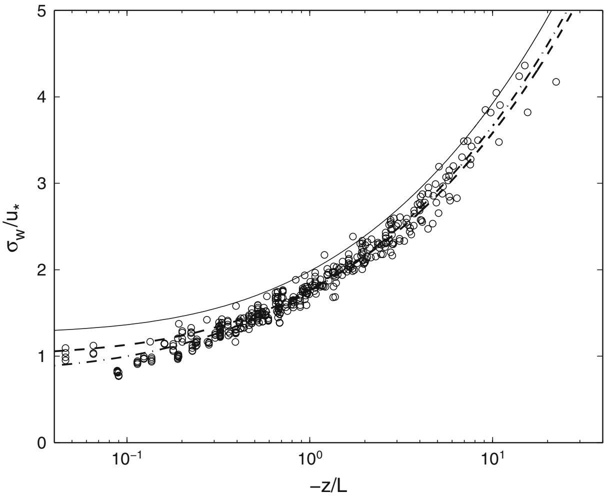

Fig. 2 Observed daytime values of σ_w/u_*, with friction velocity evaluated using Eq. 6. Solid curve is $\sigma_w/u_* = 1.25(1-3z/L)^{1/3}$ (Kaimal and Finnigan 1994 Eq. 1.33), dash-dotted line is Eq. 11, and dashed line is Eq. 12

of unity for the neutral limit of σ_w/u_*, which in view of the paucity of neutral data, may be more broadly representative than 0.8. As indicated by Table 1, coefficients of variation across the four sonics at $z = 3$ m on the transect were about 0.02 for σ_w, and categorically larger (about 0.15) for u_*, implying most of the statistical scatter of σ_w/u_* at fixed z/L originates from the stress measurement. (As an aside, this implies it is advantageous, e.g. in dispersion modelling, to use directly measured values for σ_w rather than apply an idealized MOST relationship to a measured u_*).

Returning to Fig. 2, this is the crux of our study and the motivation for the above (Sect. 2) review of MOST. For according to these observations—and others, e.g. Hogstrom (1990), Moraes (2000), Pahlow et al. (2001), Flesch et al. (2004)—not always and not everywhere does σ_w/u_* approach, in the neutral limit, the 'universal' value $\phi_w(0) = 1.25$ (or 1.3) that has been widely assumed 'normal' for the neutrally-stratified inertial sublayer (e.g. Merry and Panofsky 1976). Thus while on the one hand it is tempting to make the statement that the orderliness of Fig. 2 *confirms* the validity of pure MO scaling for vertical velocity statistics in the ideal inertial sublayer, on the other hand since the limiting neutral value is in practise *not* universal, such a claim would be spurious.

4.1 Interpretation

To help with the interpretation of Fig. 2 and to give some feeling for how it could be that a definite value for a parameter as omni-relevant and familiar as the neutral limit "$\phi_w(0)$" for σ_w/u_* could remain elusive, it is necessary to deepen the discussion by at least alluding to confounding factors. We shall continue to assume sites and flows that are (truly) horizontally-uniform.

4.1.1 Ambiguity Owing to the Influence of Averaging Time?

For given "external" conditions (u_*, L, δ, and geostrophic wind speed $\vec{U}_G$) a velocity variance such as ${\sigma_w}^2$ can be expected to depend on the averaging period T. As $T \to 0$ the variance will vanish, while at the other extreme, if we could sustain stationary conditions (above-listed external scales constant for a very long time) we could define an asymptotic (maximal) variance for the given external conditions, say $\sigma^2_{w,\infty}$. Then it is of interest to know the (expected) variance $\sigma^2_{w,T}$ over an arbitrary, finite averaging time T.

This has been considered by Pasquill (1974), by Wollenweber and Panofsky (1989), and by Kaimal et al. (1989). If $S_{w,\infty}(f)$ is the asymptotic (long-time) power spectrum, then on average the variance seen over interval T is:

$$\sigma^2_{w,T} = \int_0^\infty S_{w,\infty}(f) \left(1 - \frac{\sin^2(\pi f T)}{(\pi f T)^2}\right) df. \tag{13}$$

For specific choices of $S_{w,\infty}(f)$ one may deduce the (expected) variance fraction $\sigma^2_{w,T}/\sigma^2_{w,\infty}$. Taking the empirical spectral curves of Hojstrup (1982), Wollenweber and Panofsky deduced that $\sigma^2_{w,T}/\sigma^2_{w,\infty}$ should have no systematic sensitivity to averaging time in the range $1\,\text{min} \leq T \leq 1\,\text{h}$ (this result pertains to stably stratified flow). Analogous calculations for σ^2_u performed by the author (with $\delta = 2,000\,\text{m}$, $L = -30\,\text{m}$, $z_0 = 0.02\,\text{m}$) indicate that even with a sampling duration of only 2 min, over 60% of the u-variance is expected to be seen (i.e. the expected velocity standard deviation will be about 80% of the long-term value; obviously the 60% of $\sigma^2_{u,\infty}$ captured in only 2 min must be rapid variability). It is safe to assume that the w signal is even more forgiving, since the w spectrum lacks the low frequency power seen in the u, v spectra. Thus we may tentatively assume that our inability to pin down "the" value of $\phi_w(0)$, i.e. the neutral limit for σ_w/u_*, cannot be blamed on varying choices of averaging duration.

4.1.2 Ambiguity Owing to the Definition and Estimation of u_*

Across the population of estimates of neutral σ_w/u_*, there exist inconsistencies in the estimation of the friction velocity—most obviously because some early measurement campaigns lacked an instrument capable of measuring the shear stress directly. The two common *definitions* for u_*, following the terminology of Weber (1999), are

$$u^4_{*A} = \left(\overline{u'w'}\right)^2 + \left(\overline{v'w'}\right)^2, \tag{14}$$

$$u_{*B} = \sqrt{|\overline{u'w'}|}, \tag{15}$$

where in the context of the latter definition u' is the velocity fluctuation along an x-axis aligned with the local (instrument) mean wind vector (please note: in this paper the momentum fluxes appearing in Eqs. 14, 15 were evaluated as $\overline{\overline{u'w'}}$, $\overline{\overline{v'w'}}$, the double overbar designating the average of the post-rotation quantity over the four uppermost sonics on the tower). Obviously $u_{*B} \leq u_{*A}$, and u_{*B} if chosen for normalization must boost σ_w/u_* relative to the alternative choice. Furthermore, to the extent that shear stress is height-dependent[12] obviously both properties depend on the height of measurement, ie. $u_{*A} = u_{*A}(z)$, $u_{*B} = u_{*B}(z)$. In the cases where u_{*A} (or u_{*B}) has been measured directly, by multi-axis hot film or propeller or sonic anemometers, not all instruments have had equal quality (the shear stress is a particularly demanding type of measurement). Finally some workers were forced or have chosen to estimate the friction velocity *indirectly*, from measured profiles of mean wind speed and

[12] In theory it is, although measuring that variation is only marginally feasible, in view of experimental uncertainties and possible site imperfections.

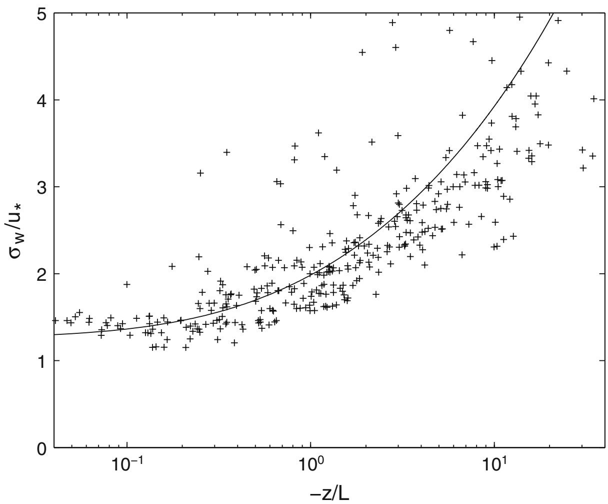

Fig. 3 Observed daytime values of σ_w/u_*, with friction velocity evaluated using Eq. 15. Solid curve is again Kaimal and Finnigan (1994)'s Eq. 1.33

temperature. The resultant friction velocity (say, "u_{*T}") is not height-dependent, but in principle should depend on the height of the tower, the distribution of sensors, and assumptions made in the analysis.

Indicating the potential significance of this ambiguity, Fig. 3 plots the same measurements of σ_w as does Fig. 2, with the distinction that here the friction velocity ("u_{*B}") defined by the simpler Eq. 15 has been used both to normalize σ_w and to define L. Although the data are more widely dispersed about the underlying mean pattern, the textbook relation without a doubt now *fits* with much less bias than it does in the case when u_* stems from Eq. 6. Obviously another alternative (not demonstrated here) would be to use the *individual* values of u_* (A or B) reported by each sonic, to form σ_w/u_* and z/L.

4.1.3 Historic Estimates of Neutral σ_w/u_*

As a precursor for what follows it is useful to extend the argument list for $\phi_w()$, viz.

$$\phi_w = \phi_w\left(\frac{z}{L}, \frac{\delta}{L}, \frac{z}{h}\right) \tag{16}$$

where the second argument takes us to 'extended MOST' and the third, involving depth (h) of the roughness sublayer, accommodates (in the most simplistic and perhaps naive spirit) proximity to the surface (where below only one argument is listed, it is to be interpreted as quantifying z/L). As noted elsewhere some authors would regard even this list of arguments as inadequate.

An interesting early discussion of σ_w/u_* in relation to MOST is given by Panofsky (1973), who cites data from Yamamoto (Fig. 4.7). What is most interesting is the considerable scatter of observations of this ratio: individual neutral values $\phi_w(0)$ scatter from a minimum of about 0.5, up to to over 3.

Probably the most influential experiment bearing on the acceptance of MOST was the Kansas experiment.[13] Haugen et al. (1971) reported that height variation (between 5.66 and 22.6 m) of (post-rotation) $\overline{u'w'}$ measured by the sonic anemometers was small, averaging about 6%, and comparable with experimental uncertainty in the sonics side-by-side. Temporal evolution of surface drag ($\vec{\tau}_0$), directly measured by two drag plates, was found to be more regular than that of the sonics, but the drag plates were found to indicate a substantially larger drag. This discrepancy Haugen et al. attributed to a mismatch of the drag-plate surface relative to that of the environment (wheat stubble), and a blanket correction factor (0.67) was applied to the drag-plate stresses. From 1-h averages (built from four consecutive 15-min sub-averages), and using the (corrected) drag plate shear stress vector, Haugen et al. found

$$\lim_{(1/L \to 0)} \frac{\sigma_w}{u_{*A}} = \phi_w(0) = 1.3 \tag{17}$$

while if they instead formed u_{*A} from the local sonic measuring σ_w, then $\phi_w(0) = 1.35$. Statistical scatter in σ_w/u_{*A} at the neutral intercept, to the extent that it can be judged from the few points available, was very small (about ±0.05, or less).

Pasquill (1974, Table 2.VI) reviewed a wide range of estimates of $\phi_w(0)$ dating from numerous experiments from 1960 to 1971. He found "average values of ($\phi_w(0)$) fall within a fairly narrow range 1.2 to 1.4 and suggested an overall mean value of near 1.3," but stressed "values for ($\phi_w(0)$) for individual short samples of data may be expected to deviate appreciably from the overall mean," that is, one expects considerable scatter of single short term period ratios about the expected value. More than a decade later Stull (1988) reviewed the gamut of suggested dimensionless relationships for the entire PBL, and gave three different values for the neutral surface layer (hh-NSL), namely $\phi_w(0) = (1.0, 1.3, 1.6)$.

Earliest atmospheric data on $\phi_w()$ may have concentrated on rather large instrument heights, i.e. large z/h, which in any case is what is needed if one is to omit z/h from the argument list (e.g. analysis of the Kansas data focused on sonics at 5.66 m and 22.6 m). As stated above a number of relatively recent authors, using anemometers quite near the ground or just above tall plant canopies, have reported smaller values of $\phi_w(0)$ than the 1.3 that has come to be regarded as 'normal' for the inertial sublayer, e.g. Raupach et al. (1986) reported $\phi_w(0) = 1.13$ based on observations just above a tall model "crop" (a staggered array of metal plates in a wind-tunnel wall shear layer). These findings are (qualitatively) in accordance with what is observed as the wall is approached in laboratory wall shear layers.

4.1.4 Summary

Although several factors can confound its evaluation, it does not appear that (the ensemble mean value of) $\phi_w(0)$ actually *is* universal, in the ideal inertial sublayer.[14] If this is true then where needed as input (e.g. for dispersion models), σ_w ought where feasible to be measured directly rather than deduced from u_* by application of a MOST function.

[13] There has been a subsequent discussion as to the possibility of flow distortion by large boxes on the tower near the sonic anemometer probes: see Wieringa (1980), Wyngaard et al. (1981), Gill (1982).

[14] An alternative point of view is that $\phi_w(0)$ *is* universal but with a distinctly smaller value, say around unity, than had been generally believed. However this would amount to suggesting many earlier estimates, by well-equipped and highly regarded investigators, had been wrong.

5 Streamwise Velocity

In this section we lay aside the above qualms in relation to extended-MOST, and on the basis of the very limited Dugway measurements[15] offer a tentative refinement of the universal function appropriate to σ_u/u_* during unstable stratification.

With shear production of $\overline{u'^2}$ maximal near the surface, and assuming the contribution to horizontal variance by the large (boundary-layer scale) quasi-horizontal eddies should vary only slowly with height, one might expect σ_u (and σ_v, fed by redistribution) to decrease with increasing z in the unstable surface layer. Panofsky (1973) stated that "Various authors have attempted to relate the ratios σ_u/u_* and σ_v/u_* to z/L, but with no consistent result." He then noted that (only) *some* observations had shown a decrease of the horizontal variances with increasing height. A commonly cited formulation for the surface layer, see Panofsky et al. (1977), is

$$\frac{\sigma_u^2}{u_*^2} = 4 + 0.6\left(\frac{\delta}{-L}\right)^{2/3}. \tag{18}$$

Kader and Yaglom (1990), noting Eq. 18 is based on observations at "several tens or even hundreds of metres," expressed reservations about its validity for small heights, and considered that there had been "no reliable measurements" of horizontal velocity variances above the dynamic sublayer of the ASL.

Figure 4 gives the values of σ_u/u_* observed on the afternoon of 24 May, the dominant variation from run to run correlating well with Eq. 18 as is indicated by the alternative view afforded by Fig. 5. Legends on these figures give the values assigned for $\delta/|L|$ in each run, based on a smooth fit to the values of δ shown on Fig. 1, and it should be borne in mind that these values are inexact. What is of most interest is the definite height-dependence shown in every run.[16] The fitted lines on Fig. 4 are from

$$\frac{\sigma_u^2}{u_*^2} = \left[4 + b\left(\frac{\delta}{-L}\right)^{2/3}\right]\left[1 - \left(\frac{z}{\delta}\right)^c\right] \tag{19}$$

evaluated with $b = 3/4,\ c = 1/4$, which values give the optimal least-squares fit of Eq. 19 to the data (rounded from $b = 0.73,\ c = 0.253$; if one prefers to retain $b = 0.6$, i.e. the Panofsky et al. value, an optimal fit requires $c = 0.38$, and the resulting curves are barely distinguishable from those given on the figure). Although in some cases the fitted curve is offset on the σ_u/u_* axis relative to the observations, if one may optimistically assume this owes to an inaccurate specification of δ then Eq. (19) represents an extension of the Panofsky et al. formula that (conservatively) is expected to apply in the range $h_c \ll z \ll \delta$, where h_c is the height of (any) canopy.

The σ_u relation given by Rodean (1996), presumably intended to apply in the bulk of the convective boundary layer, was found to severely underestimate the z-dependence observed here (this is the reason for suggesting Eq. 19 may not apply unless $z \ll \delta$). Raupach et al.

[15] By inspection of Fig. 1 the reader will note that only on the first afternoon, i.e. 24 May, did the procedure for estimating ABL depth yield a time series sufficiently unambiguous to permit an assessment of the scaling of σ_u/u_* with z/δ.

[16] Admittedly the z-dependence is of secondary importance, in agreement with the comment of Sorbjan (1989, p. 77): "even in the surface layer one would expect σ_u/u_* and σ_v/u_* to be more closely related to δ/L than to the local stability parameter z/L." Please note that σ_v/u_* was found to organize in broadly the same manner as σ_u/u_*, however ratios σ_v/σ_u plotted against z/L displayed no organization at all, and, pooling all daytime runs, spanned $0.5 \leq \sigma_v/\sigma_u \lesssim 1.7$.

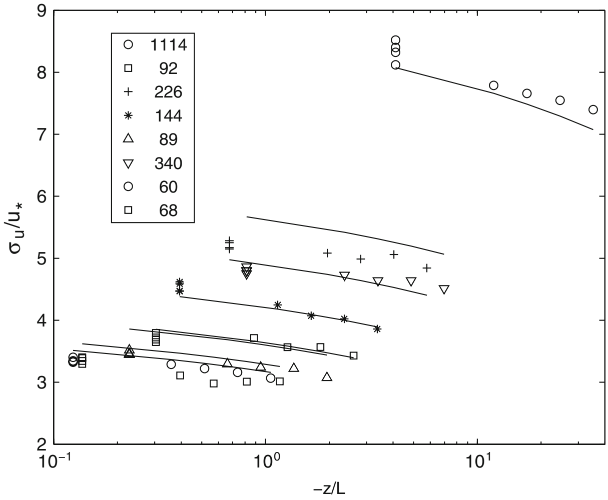

Fig. 4 σ_u/u_* vs. z/L on the afternoon of 24 May. Legend gives $\delta/|L|$, and the solid lines are Eq. 19

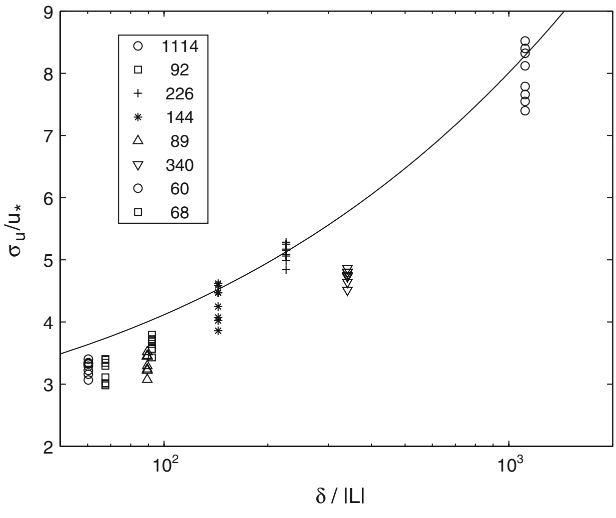

Fig. 5 σ_u/u_* vs. $\delta/|L|$ on afternoon of 24 May (the legend identifying $\delta/|L|$ for each run). The solid line is the Panofsky et al. relation (Eq. 18)

(1991) gave a formula for σ_u/u_* in the neutrally-stratified inertial sublayer due originally to Townsend (1976) and which for present purposes may be adapted as

$$\frac{\sigma_u^2}{u_*^2} = 4 + 0.6\left(\frac{\delta}{-L}\right)^{2/3} - A_1 \ln\left(\frac{z}{\delta}\right). \tag{20}$$

However Eq. 20 does not conform to the present observations.

6 Conclusion

The eight anemometers selected from the Dugway array of 2005 can be assumed to have sampled an ideally uniform atmospheric surface layer. At this site neutral values of σ_w/u_* differ from the figure (about 1.3) that had generally come to be thought normal, as in experiments elsewhere and reported by several other authors. Taken along with the thinking summarized in the above review of MOST, the author concludes that the MOST functions probably are not universal, and that we shall therefore never have their 'perfect' and final incarnation.

On the other hand the everyday utility of a scaling scheme is in inverse proportion to the number of characteristic parameters it demands and the difficulty of measuring them, so that merely patching on additional 'external' governing variables to MOST, while it would result in a scheme able to be tuned more intricately to observations (or errors), would not necessarily be a step forward. Imperfect though it is, MOST equips us rather well, if we are realistic about the achievable accuracy with which our idealized models may represent nature.

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