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Turbulent transport within the plant canopy

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Abstract A physically-detailed formulation of canopy evapotranspiration must include a specification of the within-canopy microclimate - the temperature, humidity and velocity of the airstream adjacent to individual leaves or canopy layers - because, in conjunction with the radiant energy supply and plant physiological factors, this local microclimate determines the transpiration rate from each leaf or layer. For predictive purposes the canopy microclimate must be deduced from a small number of remote measurements (including wind, temperature, radiation and humidity well above the canopy), and for this one requires a model of turbulent transport.

In this introductory article for non-specialists the conservation equations governing the microclimatic variables, as well as other necessary tools and concepts, will be described in the simplified context of flow over a bare surface. With this preparation, and having briefly reviewed observations of canopy microclimate, the adequacy of K-theory or "first-order closure" as a model for turbulent transport will be examined from several points of view. It is concluded that the use of K-theory or of the closely related "aerodynamic transfer resistance model" to model turbulent transport within a canopy is generally unacceptable.

The application of closure assumptions within budget equations for the transporting fluxes themselves (rather than in the budget equation for the mean value of the transported entity) results in a more sophisticated (but nevertheless approximate) model of turbulent transport ("higher-order closure"). The nature of the terms in the flux-budget equations will be briefly explored and typical closure approximations will be shown. The success of higher-order closure models in predicting within-canopy wind statistics from an above-canopy reference wind speed gives encouragement that a higher-order model for the transport of heat and water vapour might prove a workable and not too complex component of a canopy evapotranspiration model - replacing the superficial concept of a canopy resistance without many additional (driving) inputs.

Transfert de la turbulence dans le couvert végétal

Résumé On a présenté l'application des statistiques de la vitesse de turbulence à la description des flux à l'intérieur du couvert végétal. C'est à l'aide des statistiques de turbulences observées que l'on a illustré l'importance des rafales ou des coups de vents qui chassent au dehors l'air du couvert végétal. On a présenté les modèles de sortie d'ordre élevé et on a souligné les avantages qu'ils présentent par rapport aux modèles de premier ordre. On a décrit les succès et les problèmes associés aux modèles de sortie d'ordre élevé.

NOTATION

overbars indicate time average

angle brackets indicate average in the horizontal plane

a	leaf area density	$m^2\ m^{-3}$
c_p	specific heat at constant pressure	$J\ kg^{-1}\ K^{-1}$
d	displacement height used in mean wind profile above canopy	m
e	water vapour pressure	$N\ m^{-2}$
$e_*(T)$	saturation vapour pressure at temperature T	$N\ m^{-2}$
g	gravitational acceleration	$m\ s^{-2}$
k	turbulent kinetic energy per unit mass	$m^2\ s^{-2}$
k_v	von Karman's constant	dimensionless
ℓ	length scale	m
p	total atmospheric pressure	$N\ m^{-2}$
r_b	leaf boundary-layer resistance	$s\ m^{-1}$
r_s	leaf stomatal resistance	$s\ m^{-1}$
s	slope of saturation vapour pressure curve	$N\ m^{-2}\ K^{-1}$
t	time	s
$\underset{\sim}{u}$	vector velocity	$m\ s^{-1}$
u,v,w	velocity components along x,y,z axes	$m\ s^{-1}$
u_*	friction velocity	$m\ s^{-1}$

x,y,z	streamwise, cross-stream, vertical axes	m
$\underset{\sim}{x}, \underset{\sim}{x_o}, \underset{\sim}{x_s}$	position vectors	m
z_o	surface roughness length	m
D	*molecular diffusivity*	$m^2\ s^{-1}$
E	vertical vapour flux density (evapotranspiration rate)	$kg\ m^{-2}\ s^{-1}$
$\underset{\sim}{F}$	general vector flux density having components	F_x, F_y, F_z
H	canopy height	m
K	eddy viscosity or diffusivity qualified by subscripts (M,H,V) to specify (momentum, heat, water vapour)	$m^2\ s^{-1}$
K↓	short wave irradiance on a horizontal surface above the canopy	$W\ m^{-2}$
L	distance over which variables are to be averaged along each horizontal axis	m
L_{mo}	Monin-Obukhov length	m
Q_G	soil heat flux density	$W\ m^{-2}$
Q_H	average turbulent flux density of sensible heat directed along the vertical axis	$W\ m^{-2}$
Q_E	average turbulent flux density of latent heat directed along the vertical axis	$W\ m^{-2}$
Q^*	net radiation	$W\ m^{-2}$
Q_i^*	net radiation energy supply to a particular leaf labelled i	$W\ m^{-2}$
R_L	Lagrangian autocorrelation function	dimensionless
S	source/sink term in generalized conservation equation	
T	temperature or time (clear from context)	
T_*	scaling temperature $T_* = -Q_H/\rho c_p u_*$	K
T_o	absolute temperture	K
γ	psychrometric "constant" ($c_p p/0.622\ \lambda$)	$N\ m^{-2}\ K^{-1}$
λ	latent heat of vapourization	$J\ kg^{-1}$
υ	kinematic viscosity (coefficient of molecular friction)	$m^{-2}\ s^{-1}$
φ	volumetric concentration of a general entity "φ". Elsewhere any fluid property.	

$\Phi_{M,H,V}$	Monin-Obukhov universal function for gradients of (M, H, V) mean velocity, temperature, humidity	dimensionless
ρ	moist air density	kg m^{-3}
ρ_v	absolute humidity (vapour density)	kg m^{-3}
$\sigma_{u,v,w}$	standard deviation of velocity fluctuation (u′,v′,w′)	m s^{-1}
τ	time over which variables are to be averaged (~30 min). Also shear stress (clear from context)	N m^{-2}
δ_{ij}	delta function (zero unless i=j)	
δ	depth of the Planetary Boundary-Layer	m
τ_L	Lagrangian integral time scale	s
∇	vector operator for spatial differentiation, $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$	
Γ	adiabatic lapse rate	°C m^{-1}

INTRODUCTION

This article is intended to be a brief and reasonably easy summary of the Eulerian framework[1] for description and prediction of turbulent transport in a canopy for people who are interested in evapotranspiration from canopies but are not specialists in micrometeorology. The fundamental involvement of the canopy airstream properties (velocity, temperature, humidity) in the processes determining canopy evapotranspiration will be well known to many readers. It is logical therefore that a framework for the prediction of areal evapotranspiration should include a determination of the canopy microclimate.

Before addressing the topic of transport within plant canopies, a fairly detailed discussion of flow and transport over a bare surface will be given in order to introduce the reader to key concepts, terminology, and tools required for the description and prediction of turbulent transport. In addition this will serve to emphasize the very marked differences between flow over a bare field and flow through vegetation.

In the context of flow over a bare surface, the "closure problem" will be encountered and discussed. The following section will very briefly review observations of the microclimate of plant canopies and interpret these in the light of the current theoretical framework. The observed occurrence of "counter-gradient transport" in canopy flow will serve as proof that "first order closure" ("K-theory") is an inadequate model for turbulent transport in a canopy.

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[1] A Lagrangian description of scalar diffusion within the canopy has an important advantage over the Eulerian description in that there is no difficulty with the "near field" effects. M.R. Raupach (personal communication) has developed an analytical theory ("Localised near field theory") based on a simplified Lagrangian treatment of scalar diffusion. Note that the Lagrangian approach is unable to predict the wind field, and therefore relies on the wind field being (approximately) known.

The failure of K-theory will be examined from several perspectives, and the notion of second- (and higher-) order closure will be introduced. It will be shown that higher-order closure models can do a good job of predicting the canopy wind and turbulence. Finally, use of higher-order closure models to calculate canopy evapotranspiration will be discussed.

It will herein be assumed that we are concerned with the actual "areal evapotranspiration" over a large area A which is horizontally uniform in the sense that variables averaged both in time and over an area $L^2 << A$ (with L chosen to be much larger than all length scales of canopy irregularity) are independent of the precise horizontal location within A. In addition, although the practical interest may be in the water loss over some large time T (perhaps weeks or months), a proper physical description of canopy evaporation requires specification on a much shorter timescale τ (short compared with the daily cycle, long compared with the longest turbulent fluctuations).

It is also presumed that we wish not to "forecast" evapotranspiration, but to "diagnose" its past rate indirectly (rather than by direct measurement) over a short period τ from a manageable number of related measurements over the same period τ. The ultimate hope is that the number (and type) of measurements required might be small (and simple) enough that finances would permit establishment of an operational network sufficiently fine to enable useful diagnosis of areal evapotranspiration.

Evapotranspiration may be prescribed either by the rate of loss of water from soil storage (strictly, soil and vegetation storage if T is short) or by the net rate of passage of water vapour across a horizontal reference plane at some distance from the ground. The latter prescription is "micrometeorological" in nature and amounts to determination of what is technically called the turbulent convective flux density (loosely, the turbulent flux) of water vapour along the vertical axis.

Fig. 1 shows a slab of air of cross-section ΔA and depth $\Delta z = w\,\Delta t$. This slab is considered to be the air which in a short time interval (Δt) has moved with vertical velocity (w) across the horizontal plane z through area ΔA. The volume of air crossing z through ΔA in time Δt is $w\Delta t\,\Delta A$, and the corresponding mass of water vapour crossing z through ΔA in time Δt is $\rho_v\, w\Delta t\,\Delta A$, where ρ_v is the absolute humidity (vapour density) [kg m^{-3}]. The *rate* of passage of water vapour across plane z per unit area is simply

$$E = \frac{\rho_v\, w\Delta t\Delta A}{\Delta t\Delta A} = w\rho_v \qquad (1)$$

(having dimensions kg m^{-2} s^{-1}, i.e., mass of water per unit area per unit time). This is the instantaneous vertical vapour flux density across the plane at z. The atmosphere is turbulent, so that w and ρ_v fluctuate in time and space. We must therefore form an average. First, we average in time over interval τ and represent the result with an overbar

$$\overline{E} = \overline{w\rho_v} = \frac{1}{\tau}\int_{t}^{t+\tau} w(t')\,\rho_v(t')\,dt' \qquad (2)$$

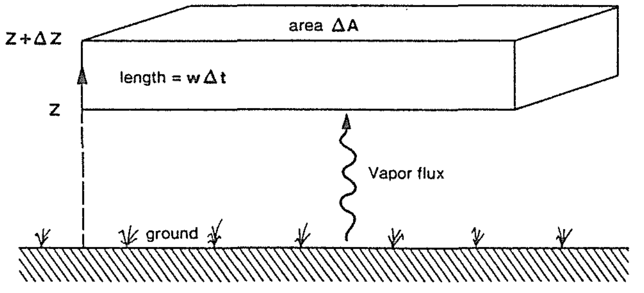

FIG. 1. The slab of moist air drawn is the volume which in time Δt has crossed the plane at z with velocity w through area ΔA.

Secondly, since a canopy is by nature spatially variable, we should average this time average in a horizontal plane

$$\langle \overline{E} \rangle = \langle \overline{w\rho_v} \rangle$$
$$= \frac{1}{L^2} \int_x^{x+L} \int_y^{y+L} \overline{w\rho_v}\,(x',y')\,dx'\,dy' \qquad (3)$$

The "areal evapotranspiration" may then be constructed over any larger time interval T from estimates $\langle \overline{E} \rangle$ over intervals τ. Therefore the challenge is to determine $\langle \overline{E} \rangle$ diagnostically from a small number of related measurements.

The normal micrometeorological terminology will be used. The velocities along the x,y,z axes are denoted u,v,w where z is the vertical and x the streamwise axis (i.e., the average wind direction is aligned with the x axis). It is assumed that the mathematical preparation of the reader extends at least to a familiarity with simple calculus including partial differential equations, vectors, and some of the basic rules of vector analysis.

THE VITAL ROLE OF THE DETAILED CANOPY MICROCLIMATE IN DETERMINATION OF CANOPY EVAPOTRANSPIRATION

The canopy consists of an assembly of leaves (and the soil surface) evaporating and/or transpiring into an adjacent canopy airstream. The characteristics of this airstream (speed, temperature, humidity) may vary strongly with location in the canopy (in consequence of the localized absorption of radiation and momentum) and with time. From each evaporating surface in the canopy we may formulate the vapour and heat fluxes into the airstream. If we use the combination equation (Brutsaert, 1984), then for the i^{th} leaf we have a latent heat flux density

$$Q_{E_i} = \frac{s}{s + \gamma(1 + r_{s_i} / r_{b_i})} Q_i^* + \frac{\rho c_p (e_*(T) - e)/ r_{b_i}}{s + \gamma(1 + r_{s_i} / r_{b_i})} \tag{4}$$

transferring vapour (latent heat) to the canopy airstream. We need to know, in addition to the radiation absorbed by this leaf and its stomatal resistance (neither of which is independent of canopy microclimate), the airstream temperature, vapour pressure and velocity (the latter determines the leaf boundary-layer resistance r_{b_i}).

The derivation of this equation does not specifically address averaging in a time-varying system. Its validity is restricted to determination of the instantaneous vapour flux across an undisturbed leaf boundary-layer into an airstream with the given instantaneous properties. In reality the boundary-layer on the leaf is disturbed by changes in wind speed, wind direction, and the airstream properties (not to mention the radiation load) which fluctuate on short timescales (seconds and minutes). It is quite possible that the time average values of e, T, u in the airspace beside a leaf may not predict the correct average evaporation rate from the leaf.

No matter how we estimate $\langle\overline{E}\rangle$ the within-canopy properties are important, even if they do not appear explicitly in the formulation used. For example, the popular "big leaf" model requires only a pair of "effective" resistances ("canopy" and "aerodynamic" resistance) and above-canopy reference values of net radiation, temperature, vapour pressure, and wind speed. Finnigan & Raupach (1987) have compared the "big leaf" combination model with a rigorous summation of the vapour contribution from individual leaves and the soil to show the detailed complexity of the effective resistances. While these resistances (in particular the canopy resistance) are often estimated diagnostically (by finding the value which allows the model to agree with observations) they are very difficult to relate to known variables satisfactorily for predictive purposes (see, for example, the article by J.B. Stewart elsewhere in this volume).

We will therefore consider that specification of the in-canopy profiles of vapour pressure, temperature, wind speed, and radiative divergence (energy supply) is necessary for a proper summation of the vapour contribution from distinct canopy layers (and soil) to the overall canopy evapotranspiration.

INTRODUCTION TO TOOLS AND CONCEPTS

Before discussing turbulence in a canopy it is helpful to cover the more simple situation of turbulent flow over an extensive, flat, uniform, bare surface. This will allow the introduction in a fairly simple context of many of the tools useful for examination of canopy flow.

The neutral wind profile over a bare, level surface

Flow near the ground is almost always turbulent. It is therefore helpful to conceive of an average flow, and departures from average ("fluctuations"). The instantaneous streamwise velocity u(t) may be broken into $u(t) = u + u'(t)$, where u is the average value

$$\bar{u} = \frac{1}{\tau} \int_{t}^{t+\tau} u(t')\, dt' \tag{5}$$

and u′(t) the instantaneous fluctuation. This is called "Reynolds decomposition", and the process is carried out for all variables, i.e., $p = \bar{p} + p'$, etc. In the present section we need not average in the horizontal plane since we have assumed a flat, uniform surface.

The average velocity ($\bar{u}$) is observed to increase with distance from the ground z. The "no slip" condition for the flow of real (non-zero viscosity) fluids past a solid surface ensures that $\bar{u}$ vanishes at the ground. In order to understand this height-variation, we need a theory. The only available basis for a theory (apart from the guidance provided by dimensional analysis) is our confidence that mass, momentum, and energy are conserved.

It will be assumed that the reader is familiar with what will be termed the "generalized conservation equation"

$$\frac{\partial \phi}{\partial t} = -\nabla \bullet \underset{\sim}{F} + S \tag{6}$$

This equation states that the concentration (dimensions often, but not necessarily, kg m^{-3}) of some entity changes in time at a fixed point in space only in response to:

(i) a non-zero divergence ($\nabla \equiv \partial/\partial x, \partial/\partial y, \partial/\partial z$) of the vector flux density $\underset{\sim}{F}$, where the components of $\underset{\sim}{F}$ are the flux densities of ϕ along the x,y,z axes (i.e., $F_x(x,y,z,t)$ is the rate of transfer of ϕ through a unit of area of the yz plane at x).

(ii) a local source (or sink) S of the entity ϕ. If ϕ is, for example, a mass concentration [kg m^{-3}] then the dimensions of the source/sink term are [kg m^{-3} s^{-1}], i.e. S is the rate of production/extinction per unit volume.

The generalized conservation equation is easily derived by performing a "box-balance" for the content of ϕ in an imaginary permeable "control volume" fixed in space (and finally shrinking the box to obtain the differential equation valid at a point). The equation expresses formally the elementary notion that if a stream of thieves are stealing vases from the back door of a china shop at the same rate as a stream of suppliers are delivering them through the front door, the number of vases in the shop is constant - except if a bull is obliterating them or a potter manufacturing them in the shop. Specific budget (conservation) equations may be deduced from (6) by specifying the entity ϕ, its flux $\underset{\sim}{F}$, and any creation/destruction term S. For example, setting $\phi = \rho_v$, the absolute humidity [kg m^{-3}], we obtain a budget equation for water vapour. The source term S can in this case only correspond to evaporation/condensation with respect to a suspended liquid phase. To progress further we must specify the flux density which consists of the sum of a convective flux density (due to bulk air motion carrying along the vapour) and a flux density due to molecular diffusion of water vapour with respect to the mixture (Bird *et al.*, 1960).

$$\underset{\sim}{F} = \underset{\sim}{u}\,\rho_v - \rho\, D_v\, \nabla \frac{\rho_v}{\rho} \tag{7}$$

Here D_v is the molecular diffusivity of water vapour in air [m^2 s^{-1}]. Substituting, we obtain

$$\frac{\partial \rho_v}{\partial t} = -\nabla \bullet (\underset{\sim}{u}\rho_v - \rho D_v \nabla \rho_v/\rho) + S \tag{8}$$

expressing conservation of water vapour.

Similarly, if we identify ϕ with the total (moist) density ρ we obtain

$$\frac{\partial \rho}{\partial t} = -\nabla \bullet \underset{\sim}{u}\rho \tag{9}$$

The "continuity equation" contains no molecular diffusion term (the mass fraction of moist air in moist air is 1) and no source/sink term (no creation). It is helpful to expand the continuity equation by making use of the fact that, for any fluid property ϕ, we have the identity (Chorin & Marsden, 1979; Batchelor, 1985)

$$\frac{\partial \phi}{\partial t} + \underset{\sim}{u} \bullet \nabla\phi \equiv \frac{d\phi}{dt} \tag{10}$$

where $d\phi/dt$ denotes the "Lagrangian" or "material" derivative, i.e. the rate of change of the value of ϕ following a particular fluid element. Using the Lagrangian derivative, the continuity equation may be rewritten as

$$\frac{1}{\rho}\frac{d\rho}{dt} = -\nabla \bullet \underset{\sim}{u} \tag{11}$$

If the density is invariant in time and space, or, equivalently, the density is conserved along fluid element trajectories, it follows immediately that

$$\nabla \bullet \underset{\sim}{u} = 0 \tag{12}$$

In reality the density is not constant, but (12) remains a justifiable approximation for most, but not all purposes (Businger, 1982).

In stating the generalized conservation equation and deriving from it equations expressing conservation of water vapour mass and total moist air mass, we have digressed (in the interests of context and along the way obtaining results we will use later) from the object of predicting the mean velocity ($\overline{u}$) above our bare, level surface. For this task we must be concerned with conservation of momentum. Provided one can recognize intuitively the appropriate momentum fluxes, it is possible to deduce the laws of momentum conservation from the generalized conservation equation, at least if the density is constant. For a rigorous derivation one may turn to virtually any textbook on pure or applied fluid mechanics, e.g. Batchelor (1985), Schlicting (1968). Conservation of momentum is expressed by the Navier-Stokes equations, which, under the restriction $\nabla \bullet \underset{\sim}{u} = 0$ and neglecting the Coriolis effect, may be written

$$\frac{\partial u}{\partial t} + \underset{\sim}{u} \bullet \nabla u = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\nabla^2 u \tag{13a}$$

$$\frac{\partial v}{\partial t} + \underset{\sim}{u} \bullet \nabla v = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \upsilon\nabla^2 v \tag{13b}$$

$$\frac{\partial w}{\partial t} + \underset{\sim}{u} \bullet \nabla w = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \upsilon\nabla^2 w - g \tag{13c}$$

It is possible to express these equations in the form of the generalized conservation equation.

The immediate problem is that these equations govern the *instantaneous* velocity rather than the mean velocity we have decided to focus upon. Therefore, one averages the *equations*, integrating each term in time in the same way one would average measurements of u(t) itself in an experiment. The steps will not be shown, but the result for the streamwise momentum equation is

$$\frac{\partial \bar{u}}{\partial t} = -\frac{\partial}{\partial x}\left(\overline{u^2} + \bar{p}/\rho - \upsilon\frac{\partial \bar{u}}{\partial x}\right) - \frac{\partial}{\partial y}\left(\overline{uv} - \upsilon\frac{\partial \bar{u}}{\partial y}\right) - \frac{\partial}{\partial z}\left(\overline{uw} - \upsilon\frac{\partial \bar{u}}{\partial z}\right) \quad (14)$$

Note that (14) has the same form as the generalized conservation equation.

We need to examine some of the unfamiliar terms which have arisen. For example, $\overline{u^2}$ is the mean square streamwise velocity, and may be expressed as

$$\begin{aligned}\overline{u^2} &= \overline{(\bar{u} + u')(\bar{u} + u')} \\ &= \overline{\bar{u}^2 + 2\bar{u}\,u' + u'^2} \\ &= \bar{u}^2 + \overline{u'^2} \end{aligned} \quad (15)$$

The last step follows from the rules of the averaging process (a good discussion of which is given by Monin & Yaglom (1977)). The result may be familiar from elementary statistics. The mean square is composed of the squared mean plus the variance $\overline{u'^2} = \sigma_u^2$ of the fluctuation. It is common to call $\overline{u'^2}$ the "power" in u′ and the dimensionless ratio $i_u = \sigma_u/\bar{u}$ the "turbulence intensity".

In a similar way, we may show that $\overline{uw} = \bar{u}\;\bar{w} + \overline{u'w'}$, where $\overline{u'w'}$ is the "covariance" between fluctuations in u and fluctuations in w. We will see that the term

$$\bar{u}\,\bar{w} + \overline{u'w'} - \upsilon\frac{\partial \bar{u}}{\partial z} \quad (16)$$

whose vertical derivative appears in the $\bar{u}$-momentum equation is (within a factor ρ) a "momentum flux density" composed of convective fluxes due to the mean and fluctuating flow plus a molecular momentum flux. The vertical derivative of this flux at the plane z is the fluid "drag" on the air at z.

The importance of covariance terms like $\rho\overline{u'w'}$ in the budget equations for the mean velocity components was first recognized by Osborne Reynolds, after whom they are commonly called the "Reynolds stresses". They are also often referred to as "momentum fluxes" and as "turbulent (shear) stresses". Covariances like $\overline{u'w'}$ are tangential stresses, while variances like $\overline{u'^2}$, $\overline{w'^2}$ are the "normal stresses". We will examine $\overline{u'w'}$ in more detail later.

Before continuing, let us prove that $\overline{w} = 0$. To do so, note that the time average form of the continuity equation is, in the case ρ = constant,

$$\nabla \bullet \underset{\sim}{\overline{u}} = \frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{w}}{\partial z} = 0 \tag{17}$$

(this follows because the operation of averaging - temporal integration - commutes with the operation of spatial differentiation).

Since we have assumed horizontal uniformity, the horizontal derivative of any time-averaged variable vanishes. Hence, $\partial\overline{w}/\partial z = 0$ and it follows that $\overline{w}(z)$ = constant. But at $z = 0$, $\overline{w} = 0$ (no flow through a solid surface) so that $\overline{w}(z) = 0$ for all z. In fact, this is an oversimplification. Webb *et al.* (1980) show that if the surface heat flux and/or the surface vapour flux are non-zero, there is a non-zero mean vertical velocity which, while small, must be properly accounted for in order to obtain accurate measurements of the vertical flux of minor atmospheric constituents, e.g., CO_2.

We may now simplify the $\overline{u}$-momentum equation. Assume a steady state ($\partial\overline{u}/\partial t = 0$), neglect the molecular momentum fluxes, note $\overline{w} = 0$, and $\partial/\partial x = \partial/\partial y = 0$. Then

$$0 = \frac{\partial \overline{u'w'}}{\partial z} - \frac{1}{\rho}\frac{\partial \overline{p}}{\partial x} \tag{18}$$

The turbulent vertical momentum flux $\overline{u'w'}$ varies with height at a rate which depends upon the pressure gradient (and in fact, the Coriolis force). However, it is usual to neglect this height variation over a shallow layer near the ground (the atmospheric surface layer, ASL). We define the "friction velocity" $u_* = -\sqrt{(\overline{u'w'})_0}$.

The friction velocity plays a key role in the present theory of ASL flow; in neutral stratification it is found that most velocity statistics "scale with" u_*. For example, the root-mean-square vertical velocity fluctuation is observed in neutral stratification to closely obey

$$\sigma_w = (\overline{w'^2})^{0.5} = 1.3\, u_* \tag{19}$$

provided $z_0 << z << \delta$ where δ is the planetary boundary-layer (PBL) depth and z_0 is the surface roughness length to be defined shortly. The rms values σ_u, σ_v likewise depend primarily on u_* but with a weak dependence on the depth of the PBL.

Fig. 2 attempts to convey an intuitive feeling for the physical mechanism which produces the very significant correlation between u′ and w′. The wind increases with height in some as-yet-undetermined manner. A parcel descending across the plane z_1 will have $w' < 0$, and having come from a region where the flow is on average faster, is likely to have larger streamwise velocity than the average value at z_1, i.e. $u' \approx \Delta z(\partial\overline{u}/\partial z)_{z_1} > 0$ (so $u'w' < 0$). For an upward-moving parcel $w' > 0$ and we would expect a preference for $u' < 0$. Hence, on average $\overline{u'w'} < 0$ (for the type of turbulent flow envisaged). In a time average sense, "slow" layers of air near the ground are "pulled along" by faster layers above and "pulled back" by even more sluggish layers closer to ground. This "pull" is mediated by the vertical motion of air parcels. There is therefore a time average transfer of momentum from aloft towards the ground (this momentum is absorbed by the ground which feels the pull of the air sliding over it).

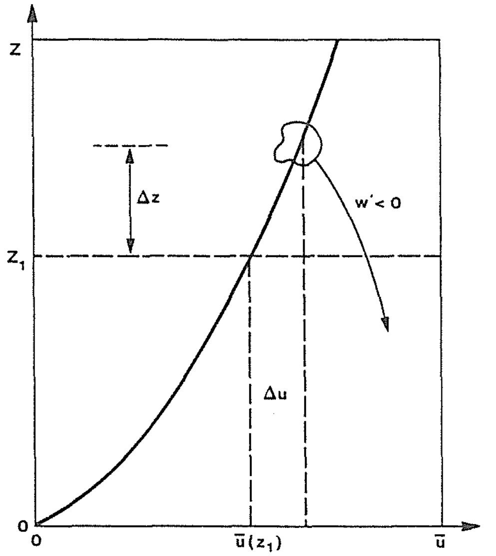

FIG. 2. Illustrating the origin of the correlation between fluctuations in vertical and streamwise velocity in the presence of a vertical gradient in mean streamwise velocity $\bar{u}(z)$. The parcel moving down across z_1 carries a velocity excess $\Delta u \sim \Delta z(\partial\bar{u}/\partial z)_{z_1}$.

We have now made full use of the $\bar{u}$-momentum equation, yet we have not recovered any prediction of the profile $\bar{u}(z)$, and have instead ended up talking about the "turbulent momentum flux" $\overline{u'w'}$. This is a basic problem which always arises when we average the governing equations - we end up with a formulation containing new unknowns. It is possible to derive governing equations for the new unknowns, but these (higher order) equations will contain still more unknowns. This is "the closure problem".

The pioneers of fluid mechanics, aeronautical engineering and meteorology had no means by which to measure these correlations, and were lead to "parameterize" or "model" the unknown fluxes (which are often the flow property of predominant interest). T.V. Boussinesq, L. Prandtl, G.I. Taylor, and T. von Karman reasoned that since turbulence causes mixing, average fluxes should be directed down the gradient in the corresponding average concentration or driving force and should have a magnitude proportional to the driving gradient. Under this assumption the vertical momentum flux in our system would be modelled as

$$\overline{u'w'} = -K_M \frac{\partial \bar{u}}{\partial z} \quad (20a)$$

where K_M is the "eddy viscosity" [$m^2\,s^{-1}$], so called because of the similarity to Newton's law of molecular viscosity

$$\tau/\rho = -\nu \frac{\partial u}{\partial z} \quad (20b)$$

which gives the molecular momentum flux (i.e., the shear or tangential stress) in a fluid of kinematic viscosity υ [m^2 s^{-1}] in a plane parallel motion with the velocity $\underset{\sim}{u} = (u,o,o)$ depending on z alone.

The molecular kinematic viscosity υ is a fluid property depending on temperature and pressure. However, the eddy viscosity (K_M) is a property of the *turbulence* (and, as we will see, the source distribution). In most regions of a turbulent flow its magnitude vastly exceeds υ.

The parameterization (20a) of the turbulent momentum flux is an example of what is called "first order closure". Similar formulations are used for heat transfer; the turbulent vertical heat flux density $Q_H = \rho c_p \overline{w'T'}$ is modelled as

$$Q_H = -\rho c_p \, K_H \left(\frac{\partial \overline{T}}{\partial z} - \Gamma\right) \tag{21}$$

where K_H is the "eddy diffusivity for heat" [m^2 s^{-1}] and Γ is the adiabatic lapse rate, i.e. the vertical gradient in mean temperature which exists under well mixed conditions with zero vertical heat flux as a consequence of the adiabatic expansion and compression of fluid parcels moving along the vertical gradient in atmospheric pressure. Equation (21) states that the heat flux is driven by the departure of the lapse rate from the adiabatic lapse rate Γ. The vapour flux density $\overline{w'\rho_v{}'}$ is modelled as

$$\overline{w'\rho_v{}'} = -K_V \frac{\partial \overline{\rho_v}}{\partial z} \tag{22}$$

where K_V is the "eddy diffusivity for water vapour" [m^2 s^{-1}]. The analogies to Fourier's law of heat conduction and Fick's law of diffusion are obvious.

It was recognized early on that the eddy viscosity and eddy diffusivities must be considered to vary with position in the flow. Schemes to specify the K's were developed, often on the basis of an imposed (externally specified) length scale and a velocity scale derived from the mean velocity gradient (mixing length theory). These will not be mentioned in detail here. To sum up, first order closure or K-theory is the name given to the practice of assuming that turbulent fluxes of heat, mass, and momentum are linearly related to the mean gradient in corresponding driving force.

Now we may return to an attempt to deduce the height-variation of u. Adopting K-theory, and noting our definition of u_*, we have

$$\overline{u'w'} = -u_*^2 = -K_M \frac{\partial \overline{u}}{\partial z} \tag{23}$$

where $\overline{u'w'}$ is regarded as height-independent.

The eddy viscosity is, dimensionally, the product of a length and a velocity. Since its function is to specify the effectiveness of turbulent momentum transfer down a mean velocity gradient, it is reasonable to postulate that the relevant length and velocity would somehow be "characteristic" of the turbulence. G.I. Taylor's (1921) exact theory of turbulent mass diffusion in homogeneous turbulence, which we will discuss later, adds weight to this supposition. Then we may use u_* as a velocity scale characteristic of the turbulence. What about a length scale?

Far from the ground the remoteness of the solid barrier permits the existence of turbulent eddies of large scale (of the order of the PBL depth). However, as we approach the ground the typical eddy size is reduced. It is therefore natural to investigate the usefulness of the distance above ground (z) as the turbulence length scale. Then

$$K_M = u_* \, k_v z \tag{24}$$

where k_v is a proportionality constant named after von Karman, and substituting we obtain

$$\frac{\partial \bar{u}}{\partial z} = \frac{u_*}{k_v z} \tag{25}$$

We may now integrate, defining the roughness length z_o to be the height at which $\bar{u} = 0$. The result is the semi-logarithmic wind profile

$$\bar{u}(z) = \frac{u_*}{k_v} \ln \frac{z}{z_o} \tag{26}$$

There is abundant experimental evidence for the usefulness of (26). There have been numerous laboratory and field experiments to determine von Karman's constant, and present indications are that $k_v = 0.40$ (Hogstrom, 1985) with some controversy. The equation is valid only for $z_o << z << \delta$; as $z \to 0$ assumptions invoked in deriving the equation may be violated, though in practice it is commonly used even when $z >/> z_o$ because there is no simple alternative. The roughness length (z_o) is an empirical parameter related in some way to the detailed character of the surface (Jackson, 1981). Note that, provided all restrictions are met (neutrality, horizontal uniformity), the momentum flux may be estimated directly from the wind profile (whose slope $\Delta\bar{u}/\Delta \ln z$ is u_*/k_v). Since most other statistics of interest (e.g. σ_w) may be accurately estimated from u_*, the wind profile alone provides a wealth of information about neutral surface layer flow and turbulent dispersion within that flow.

Extension to non-neutral stratification

"Monin-Obukhov similarity theory" is central to the presently-accepted concept of the thermally-stratified atmospheric surface layer over uniform terrain.

"Complete similarity" between two physical systems A and B is said to obtain if the governing equations (and boundary and initial conditions) expressed in non-dimensional form with the aid of scale factors can be made identical. In the case of the stratified atmospheric surface layer, the presence of a heat flux and fluctuations in temperature imply that the density is not constant. However, under the "Boussinesq approximation" (Businger, 1982; Plate, 1971) one is able to treat the density as a constant except where it appears multiplied by the acceleration due to gravity (the buoyancy force in the vertical momentum equation) and to retain $\nabla \bullet \underset{\sim}{u} = 0$. The governing momentum equations under the Boussinesq approximation may be cast into a non-dimensional form using velocity, length, and temperature scales, V, L, T_*, respectively. Certain non-dimensional ratios involving the (as yet unspecified) scales will appear. The dimensionless governing equations can be rendered oblivious to the particular ASL conditions by the scaling choice $V = u_*$

$$T_* = -\overline{w'T'} / u_* \tag{27}$$

$$L_{mo} = u_*^2 \,/\, (k_v\,(g/T_o)T_*) \tag{28}$$

(Here we have not considered the details of the lower boundary condition which would introduce further scales into the analysis.) L_{mo} is called the "Monin-Obukhov length" and the ratio z/L_{mo} is used as a stability parameter. The inclusion of k_v is conventional but unnecessary.

According to the Monin-Obukhov similarity theory, if we non-dimensionalize ASL properties using u_*, T_*, L_{mo}, we may expect to form a universal theory (i.e. to find relationships which do not depend on stability, wind speed and location). Had we included the water vapour conservation equation we would have deduced the desirability of scaling absolute humidity using $\rho_{v*} = -\overline{w'\rho_v'}/u_*$ (and so on for other entities).

The dimensionless wind, temperature, and humidity gradients are expected to conform to

$$\frac{k_v z}{u_*}\frac{\partial \bar{u}}{\partial z} = \Phi_M\left(\frac{z}{L_{mo}}\right) \tag{29}$$

$$\frac{k_v z}{T_*}\frac{\partial \bar{T}}{\partial z} = \Phi_H\left(\frac{z}{L_{mo}}\right) \tag{30}$$

$$\frac{k_v z}{\rho_{v*}}\frac{\partial \bar{\rho}_v}{\partial z} = \Phi_V\left(\frac{z}{L_{mo}}\right) \tag{31}$$

where the Φ's are called the Monin-Obukhov universal functions (the gradients rather than the actual properties are dealt with because the absolute values depend on features of the underlying surface not included in the similarity theory).

Similarly, any other dimensionless statistic has its Monin-Obukhov function, e.g. $\sigma_w/u_* = \Phi_1(z/L_{mo})$. Several major experiments have determined that Monin-Obukhov similarity theory is useful and valid in almost all respects, and have determined formulae for the universal functions over a wide range of stability z/L_{mo}. The recent results of Dyer & Bradley (1982) are in line with most observations. A critical review of experimental difficulties is given by Yaglom (1977). There have also been numerical solutions of simplified conservation equations (for the species and its turbulent flux) leading to predictions of the Φ functions in reasonable agreement with observations (Mellor, 1973; Lewellen & Teske, 1973). This should be seen as evidence that the necessary assumptions in the numerical models are reasonable rather than as "proof" of the similarity theory since the latter rests on a solid foundation.

The "flux-gradient relationships" (29-31) are not in any way dependent upon the adoption of K-theory. However, they imply the effective K values

$$K_{M,H,V} = k_v\, u_* z \,/\, \Phi_{M,H,V} \tag{32}$$

We know that $\Phi_M(0) = 1$ (necessary in order to recover the neutral logarithmic wind profile) and it is now believed that $\Phi_H(0) = \Phi_V(0) = 1$ so that in the limit of neutral stability the eddy viscosity and all eddy diffusivities are equal.

It is essential to remember that Monin-Obukhov similarity theory as outlined applies to the horizontally-uniform ASL within which we have noted that the momentum flux is approximately height-independent and for which it can, by similar means, be shown that the heat and vapour fluxes are (more rigorously) constant with height.[2] This certainly precludes use of these relationships within a canopy (where the fluxes are NOT constant with height). Furthermore, no length scales relevant to the roughness structure of the underlying surface have been included, so these results are restricted to heights well above the surface, say $z >> z_o$.

Extension to flow above a tall canopy

The experiments determining the Monin-Obukhov functions have been performed over extensive flat surfaces covered by vegetation whose height is small compared to the depth of the ASL, the heights of measurement, and the magnitude of the Monin-Obukhov length. Therefore, in these experiments the sources (sinks) of heat, water-vapour, and momentum essentially coincide (or at least are separated by a distance which is much smaller than the depth of the ASL, the Monin-Obukhov length, and the height of measurement).

It is common to apply the Monin-Obukhov framework to profiles over tall canopies, most crudely by simply replacing z with $z - d$, where d is called the "displacement height" and is often chosen as an arbitrary fraction of canopy height or by selection of a value which forces the above canopy neutral wind profile to be semi-logarithmic.

There are dangers here. Often the effective height $z - d$ (which is limited by the need to take measurements within the perhaps-shallow constant flux layer over the canopy) will not be large compared to the scales of surface roughness (notably canopy height) and a possible separation in the effective source heights for heat, vapour, and momentum. Consequently, the effective K-values do not necessarily obey (32) (with the Φ values being those obtained over short crops or bare surfaces). An equivalent statement is that fluxes deduced from gradients using Φ's tuned to measurements over short crops may be wrong.

These problems are discussed in detail by Raupach (1979) and Raupach & Legg (1982). We will see that the situation of coincident ground-level sources is a very forgiving one in the context of a search for well-defined and reasonably universal relationships between fluxes and gradients. We will also see that the opposite is true of the tall crop situation; investigation of the mechanisms determining the turbulent flux in and above a canopy shows that K-theory is untenable in and close above a canopy. The Φ functions above a tall canopy can be expected NOT to be universal or equal to those determined over short crops.[3]

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[2] Note that the slow warming and cooling of air layers, which we neglect when assuming a steady state, is a direct result of very small (practically immeasurable) changes in the vertical heat flux across the layer. Likewise for changes in the mean humidity.

[3] The question of the validity of the Bowen Ratio method, specifically the assumed equality of the eddy diffusivities for heat and water vapour above a tall canopy, was raised in discussions. G.W. Thurtell noted that provided the position of measurement is several diffusion length scales above the (perhaps seperated) effective source levels, the diffusivities should take on the "far field" values defined later, and therefore the assumption of equality of the diffusivities should be adequate. Since the diffusion length scale is of the order of 1/3 z to 1/2 z, Bowen Ratio measurements above about $z = 2H$ should be safe (in respect to the assumption of equal K's).

OBSERVATIONS OF CANOPY TURBULENCE AND TURBULENT TRANSPORT

It should first be stated that measurement of canopy flow is not easy. The flow is usually extremely turbulent, in the sense that the typical velocity fluctuations may be much larger than the average flow rate. Anemometry in these conditions requires special care.

There have been a very large number of experiments on canopy flow (including real or model canopies). Rather than attempt to assemble and review these in the space available, the reader is referred to the review by Raupach & Thom (1981) which will provide a reasonably recent point of departure for those seeking out specific experiments. Later experiments may be traced from other references given here.

We can envisage a (reasonably dense) canopy as having, in a qualitative sense, an elevated "active surface", by which is meant a predominant site of radiation exchange (and related heat and vapour production) and momentum absorption. This is not meant to imply that a plane active surface is a legitimate quantitative concept, but to aid the reader in forming an intuition for why wind, temperature, and humidity vary in the way they do in a canopy.

Needless to say, there is an endless variety of possible canopy profiles of wind, temperature, humidity, etc. The following examples have been chosen simply for convenience, to illustrate the discussion of the key processes at work in setting up the turbulent canopy environment. Most of the observations available are, like those given here, single-point time-average values collected at various heights in what is subjectively judged to be a "horizontally uniform" canopy, without any attempt at spatial averaging. It is common to "scale" (make dimensionless) the observations pragmatically in order to minimize sensitivity to varying conditions.

Mean wind and momentum flux

Fig. 3 shows the profile of mean streamwise velocity $\bar{u}$ and Fig. 4 the profile of the vertical flux of streamwise momentum $\overline{u'w'}$ within a mature cornfield of height H = 2.21 m at Elora, Ontario, Canada (Wilson, 1987). These statistics have been normalized by the friction velocity $u_* = \sqrt{-(\overline{u'w'})_H}$. The close connection between the behaviour of $\overline{u'w'}$ and $\bar{u}$ has already been discussed. The key factor rendering the canopy momentum balance (and therefore flow) very different from the simpler flow considered earlier is the drag of the plant parts on the airstream.

By Newton's law of action/reaction, the drag (which fluctuates in time) constitutes an ongoing drain of momentum from the flow. No momentum loss or "sink" term appears in the Navier-Stokes equations (13) which in their given form can be applied only within space occupied by fluid, incorporating appropriate boundary conditions at solid boundaries. A convenient way to formalize conservation in flow through the canopy is to spatially-average the conservation equations (this will be pursued later). The result of such an operation is the appearance of new terms in the equations for the spatially-averaged property which correspond to the net influx-efflux across solid/fluid boundaries.

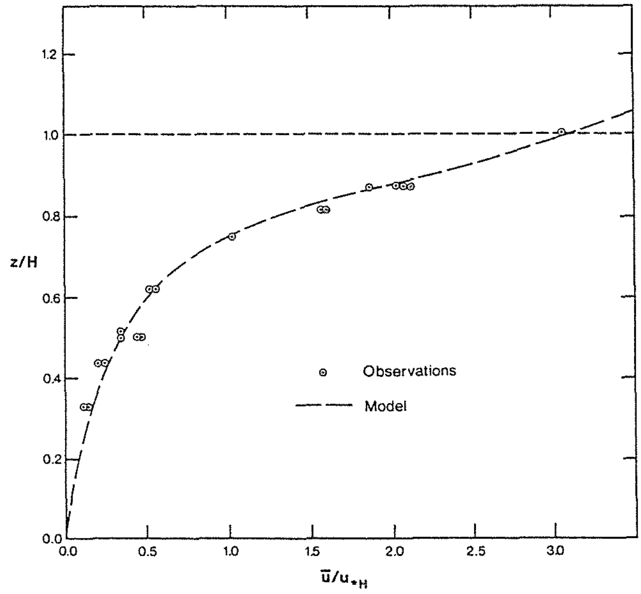

FIG. 3. Observations of the mean streamwise velocity within a corn canopy. The line is the second-order simulation of Wilson (1987).

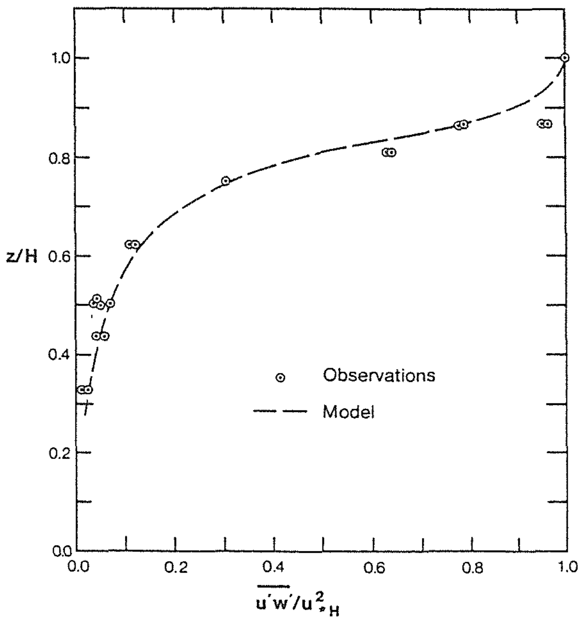

FIG. 4. Observations of the turbulent momentum flux within a corn canopy. The line is the second-order closure simulation of Wilson (1987).

The removal of momentum from the flow implies (see Fig. 5) that less momentum is transferred downwards and out of a given canopy layer than is transferred into the top; a vertical gradient in the momentum flux $\overline{u'w'}$. This is clearly seen in the observations of Fig. 4. We may now understand the very low velocities deep in the canopy. A layer of air deep in the canopy receives (on average) a weak supply of momentum with which to overcome the retarding drag of the plant parts and the even slower-moving air beneath it.

Note that the drag on the plant parts is transferred down the stalk and roots to a soil volume. Thus ultimately the ground absorbs the entire momentum flux fed down from above the canopy, but not, as in the case of a bare soil, on a shallow layer at the surface.

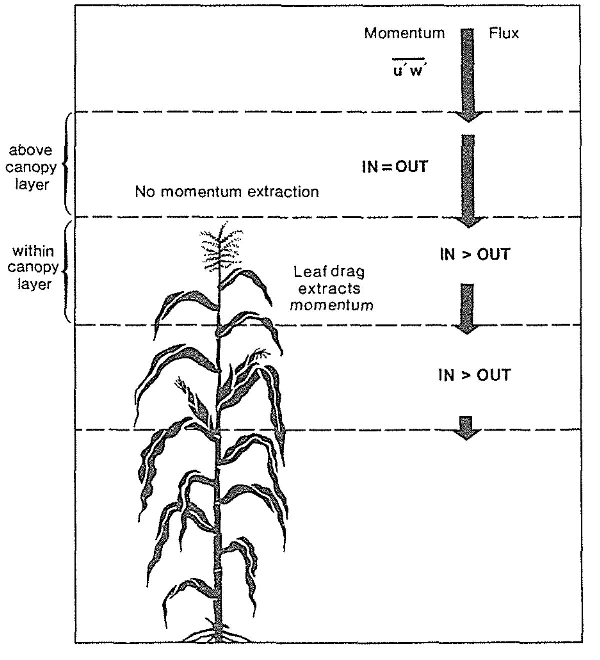

FIG. 5. Illustrating the divergence of the downward momentum flux across canopy layers. Since the drag of the vegetation extracts streamwise momentum from the flow, the downward momentum flux out the bottom of a canopy layer is less than the supply through the top of the layer. Successively lower layers receive a successively smaller supply of momentum.

Turbulent velocity standard deviations and turbulent intensities

The standard deviation σ_u is the symbol given to the root-mean-square value of the streamwise velocity fluctuation u′, i.e., $\sigma_u = (\overline{u'^2})^{0.5}$. It is convenient to use σ_u (and corresponding statistics σ_v, σ_w for the other flow directions) to characterize the degree of turbulence in the canopy, either in absolute terms or in relative terms by forming the "turbulence intensities" $i_u = \sigma_u/\bar{u}$, etc. The velocity standard deviations have a profound influence on turbulent diffusion and, more broadly, all canopy transport processes. The combination

$$k = \frac{1}{2} (\sigma_u^2 + \sigma_v^2 + \sigma_w^2) \tag{33}$$

is called the "turbulent kinetic energy" (TKE; strictly, TKE per unit mass) and is the focus of attention when one considers conservation of mechanical energy in turbulent flow (an especially complex balance in the case of canopy flow).

Figs. 6 and 7 show the vertical variation of σ_u and σ_w within the Elora corn canopy (although not shown, these are expected to be fairly constant above the canopy). It is immediately apparent that in absolute terms the fluctuations are reduced drastically in magnitude below about 0.8H. However, Fig. 8 shows that *relative to the mean wind* the turbulent fluctuations assume an increased importance deep in the canopy, with $\sigma_u/\bar{u}$ taking on values as high as 4.

This implies that the air tends to waft from place to place, back and forth and around, in the bottom of the canopy, with the overall time-average rate of drift being quite low compared to instantaneous velocities. An immediate implication is that leaf boundary-layer resistances should not be expected to scale with the average velocity $\bar{u}$.

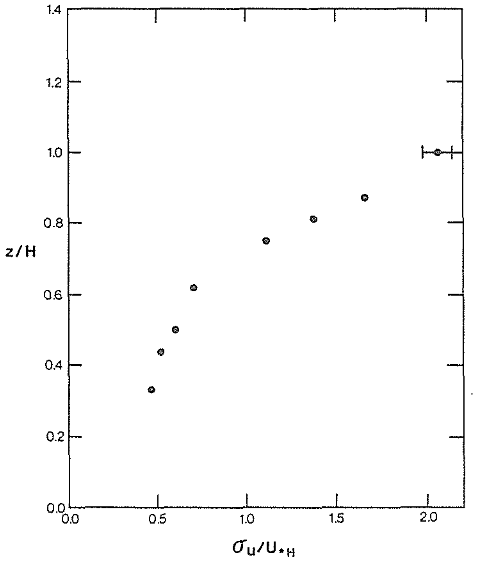

FIG. 6. Observations of the r.m.s. stream-wise velocity fluctuation within a corn canopy. The bar at z = H indicates ±1 sample standard deviation for a sample of size n = 8.

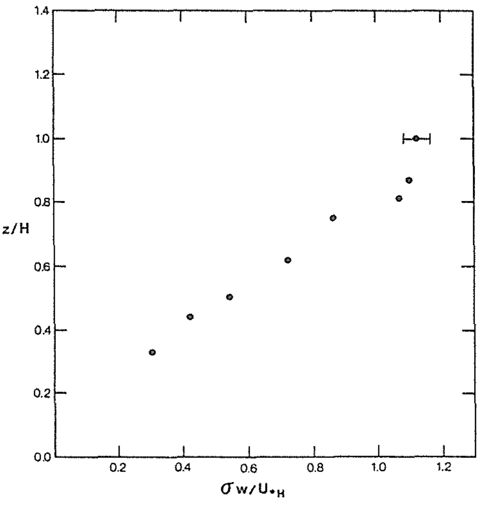

FIG. 7. Observations of the r.m.s. vertical velocity fluctuation within a corn canopy. The bar at z = H indicates ±1 sample standard deviation for a sample of size n = 8.

Temperature and vapour pressure

Fig. 9 shows the mean profile of potential temperature (potential here implies that a small correction has been applied to indicate the temperature which would be observed if the sample was compressed adiabatically to the ground-level pressure), water vapour mixing ratio, and carbon dioxide concentration observed near noon in a horizontally-uniform pine forest near Canberra, Australia (Denmead & Bradley, 1985). One may observe the maximum in temperature and the minimum in CO_2 concentration within the crown layer where much of the incoming radiant energy is absorbed leading to strong heating, transpiration, and photosynthetic rates. Also shown are the (directly measured) turbulent vertical fluxes of heat, water vapour, and CO_2. Evidently, a substantial fraction of the total evapotranspiration was contributed by vapour loss from the ground cover. The startling aspect of Fig. 9 is the clear demonstration of counter-gradient (negative K) transport, an impossibility in the framework of first-order closure. Near ground heat is on average being transported from regions which are cold (on average) to levels which are warmer (on average).

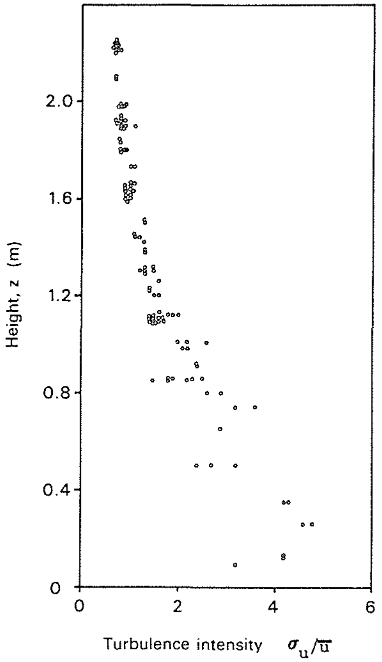

FIG. 8. Observations of the turbulence intensity σ_u/u within a corn canopy of height ~2.2 m.

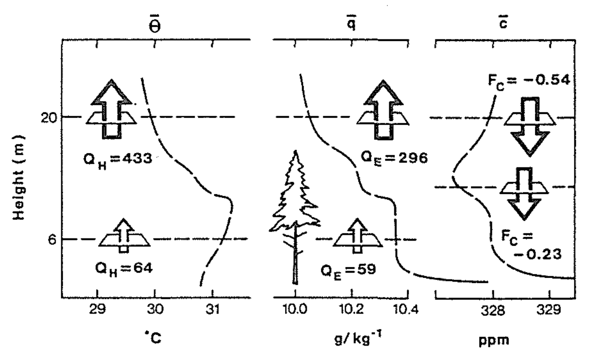

FIG. 9. Observations of the vertical gradients and corresponding fluxes of potential temperature (θ) specific humidity (q) and carbon dioxide concentration (c) for a pine forest. Re-printed with permission from Denmead & Bradley (1985). Note the counter-gradient fluxes (Q_H, Q_E are expressed in W m^{-2}).

Counter-gradient transfer in the lower canopy occurred in almost 70% of the observations. Denmead & Bradley give evidence that their flux measurements were accurate, and these data stand as convincing evidence that one cannot use K-theory in the description of canopy transport.

We could now go on to investigate why K-theory fails in a canopy and to examine alternatives. Before doing so it is valuable to discuss some novel canopy flow observations which will help to explain the inapplicability of K-theory.

The intermittency of turbulent exchange within a canopy

A relatively recent technique giving insight into the mechanisms of turbulent flow and transport is "conditional sampling" in which a measurable "indicator function" is used to classify each measured value of the variable of interest into one of a number of classes. Conditional sampling analyses of flow in and above plant canopies have been reported by Finnigan (1979), Shaw *et al.* (1983), Coppin *et al.* (1986), Raupach *et al.* (1986), Baldocchi & Hutchison (1987), and Baldocchi & Meyers (1988). These studies have revealed the highly intermittent nature of canopy turbulence and turbulent transport and the great importance of occasional penetrations of the canopy by gusts emanating from the boundary-layer above the canopy. From time to time a localized region of the canopy air layer is "flushed out" by a downward gust of wind from above. Between these (erratic) events the canopy is relatively calm, with mixing proceeding on a much smaller scale.

We will later discuss a "budget equation" for the vertical vapour flux $\overline{w'\rho_v'}$ which states that there is a balance between mechanisms creating flux, a mechanism destroying flux, and a "turbulent transport" mechanism which imports or exports flux from other regions of the flow. Analysis of similar equations (for the heat and momentum fluxes) has shown that, in correspondence with the above suggestion of the vital importance of occasional gusts, deep within the canopy the local production rates are negligible relative to the rate of import from higher levels.

We can sum up by saying that much of the activity and exchange deep in a canopy occurs in brief intervals during and immediately following the occurrence of a penetrating gust of large scale (and low repetition rate). Much of the turbulence observed deep in the canopy results from the wholesale advection into the canopy of large volumes of air (by even larger eddies) within which strong gradients have been created prior to dislodgement into the lower canopy.

THE FAILURE OF FIRST-ORDER CLOSURE (K-THEORY)

Intuitive considerations

It must be remembered that the vertical flux density $\overline{w'\phi'}$ of an entity whose instantaneous concentration is ϕ is an *average* of the product of fluctuations in vertical velocity and concentration observed at a fixed point in space x over a span in time $(t, t+\tau)$

$$\overline{w'\phi'} = \frac{1}{\tau} \int_{t}^{t+\tau} w'(\underset{\sim}{x},t')\ \phi'\ (\underset{\sim}{x},t')\ dt' \tag{34}$$

Fig. 10 attempts to direct the reader to think of transport through an area A of a reference plane z_1 from a Lagrangian (parcel-following) perspective. The parcels at z_1 within A at time t are coming from various directions with various velocities and with various concentrations. These parcels are also identified at some earlier time $t - \Delta t$. Referring back to the generalized conservation equation (6), if we identify the total flux of ϕ as $\underset{\sim}{u}\phi - D_\phi \nabla \phi$, and if the entity ϕ is not created or destroyed within the field, we have

$$\frac{d\phi}{dt} = D_\phi \nabla^2 \phi \tag{35}$$

where d/dt, as earlier, refers to the parcel-following derivative. If we neglect molecular diffusion then $d\phi/dt = 0$, i.e. each parcel retains its concentration ϕ. In fact, molecular diffusion enters the picture to slowly change the ϕ of any parcel, but over limited intervals (t_1,t_2) spanning t we may consider a parcel to have been marked with ϕ at t_1, to cross z_1 at t, and to retain its ϕ until t_2.

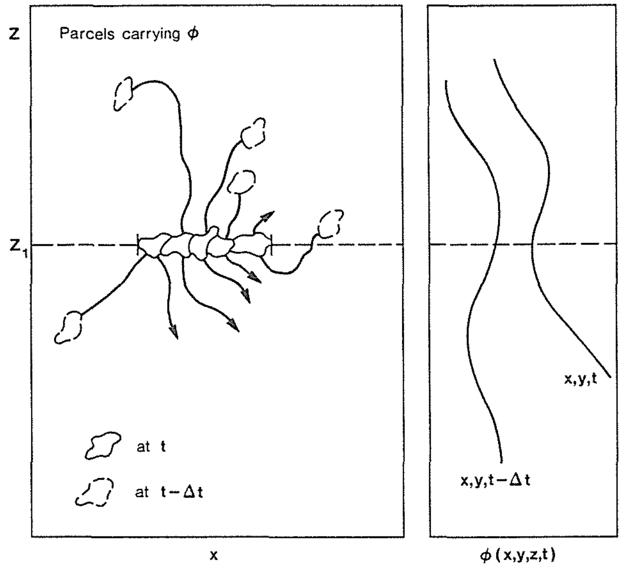

FIG. 10. A schematic snapshot showing the origin at an earlier time $t - \Delta t$ of parcels lying on the plane z_1 within area A at time t. The concentration (ϕ) of each parcel may be considered invariant over short periods of time, and each parcel therefore carries the concentration $\phi(x,y,z,t - \Delta t)$ with which it was "marked" at some earlier time and different location.

Now let us remember that Fig. 10 is a snapshot in time and that our average flux over time τ must be considered to be the superposition of a very large number of snapshots. It follows that the average vertical flux is a property of the full, time-dependent concentration and velocity fields (which are themselves correlated) over a large volume centred about the point where our flux must be measured.

K-theory attempts to model the average vertical flux as

$$\overline{w'\phi'} = -K \frac{\partial\bar{\phi}}{\partial z} \tag{36}$$

which involves only two quantities, K and $\partial\bar{\phi}/\partial z$ evaluated (as time averages) at a single location $\underset{\sim}{x}$ on the plane of interest z_1. Instead of retaining information about the time-dependent concentration field in the region we have only the average vertical gradient at one point. The K, which is allowed to vary in space (and with "external" variables such as u_* and L_{mo}) must somehow encapsulate all the relevant information about the range and intensity of length scales involved in the transport process. From this perspective we can see that we might consider ourselves lucky if K-theory was of any use.

Now let us imagine a time-varying concentration field which has relatively weak gradients in the horizontal (at any time) compared to the vertical direction, and, furthermore, whose *instantaneous* vertical gradient at $\underset{\sim}{x}$ ($\partial\phi/\partial z$) is generally not very different from the time average value $\partial\bar{\phi}/\partial z$ and changes by a small proportion of its absolute value over a vertical distance ℓ (the largest active eddies). In this (special) case the mean gradient $\partial\bar{\phi}/\partial z$ tells us essentially all we need to know in order to envisage the net flux brought about by eddies of scale $< \ell$. If we replace $\bar{u}$ in Fig. 2 with $\bar{\phi}$, we may argue (qualitatively) that a parcel crossing z_1 from a higher point of "marking" carries across z_1 an excess which (in view of the assumed similarity of the mean and average concentration profiles) is approximately $\Delta\phi = \Delta z\, \partial\bar{\phi}/\partial z$. K-theory may be reasonable in this case.

A detailed discussion along these lines is given by Corrsin (1974) who gives as an essential (and much earlier recognized) prerequisite to the validity of K-theory the requirement that "the characteristic scale of the transporting mechanism...must be small compared with the distance over which the mean gradient of the transported property changes appreciably". The underlying physical need for this restriction has been briefly outlined here, and we will see that turbulent transport in a canopy does not conform to this restriction.

Rigorous investigation of K-theory for simple flows

The preceding qualitative discussion of the generally weak basis of K-theory may be substantiated by discussing the implications of G.I. Taylor's (1921) rigorous Lagrangian theory of turbulent dispersion in homogeneous turbulence ("homogeneous" implies that all statistics of the flow are independent of position).

Consider an instantaneous source which releases a "puff" of ϕ at a time $t = 0$ and location $\underset{\sim}{x_s}$ in homogeneous turbulence. The vertical "spread" of this puff at later times may be measured by the mean square displacement relative to the initial height z_s ($\sigma_z^2(t) = \overline{(z - z_s)^2}$), where the average is over all molecules of ϕ and over many realisations of the release. Taylor's exact analytical solution for this measure of the spread is

$$\sigma_z^2(t) = 2\sigma_w^2 \int_0^t (t-\xi)\ R_L\ (\xi)\ d\xi \qquad (37)$$

Here $R_L\ (\xi)$ is the Lagrangian autocorrelation coefficient for the (Lagrangian) vertical velocity

$$R_L\ (\xi) = \overline{w(t)\ w(t+\xi)}\ /\ \sigma_w^2 \qquad (38)$$

i.e. the average correlation between values of the vertical velocity of a specific fluid element at times separated by an interval ξ.

Clearly $R_L\ (0) = 1$ (by definition of σ_w and pre-supposing, as may be proven for homogeneous turbulence, that the Eulerian and Lagrangian velocity variances are equal) and we expect $R_L\ (\xi) \to 0$ as $\xi \to \infty$. The Lagrangian integral timescale τ_L is defined by

$$\tau_L = \int_0^\infty R_L\ (\xi)\ d\xi \qquad (39)$$

and may be considered to be an estimate of the typical temporal persistence of the fluid element vertical velocity. These Lagrangian properties are difficult to measure.

Equation (20) has short and long time limits

$$\sigma_z^2 = \sigma_w^2\ t^2 \qquad t/\tau_L << 1 \qquad (40)$$

$$\sigma_z^2 = 2\sigma_w^2\ t\ \tau_L \qquad t/\tau_L >> 1 \qquad (41)$$

where "short" and "long" are judged relative to the Lagrangian time scale τ_L.

If this diffusion problem is instead analyzed by adopting the Eulerian approach, expressing mass conservation as in (6) and assuming fluxes are related to mean gradients via K-theory with a spatially-constant eddy diffusivity $K = K(\sigma_w, \tau_L)$, the result is a prediction that for all t the spread σ_z^2 is proportional to t. Now since Taylor's solution is rigorous, a deficiency in K-theory is revealed. It may be shown (Batchelor, 1949; Csanady, 1973) that the K-theory solution can only be bent to conform to the correct solution by giving up the notion that K depends only on flow properties and accepting that *it depends upon the time t since the puff of* ϕ *was released,*

$$K(t) = \sigma_w^2 \int_0^t R_L\ (\xi)\ d\xi \qquad (42)$$

Noting that $\int_0^t R_L\ (\xi)\ d\xi = \tau_L$, we see that only if $t >> \tau_L$ do we have a constant diffusivity $K = K_\infty = \sigma_w^2\ \tau_L = \sigma_w\ \ell$ depending only on properties of the turbulence (the "far field" limit). At very short times since release of the puff the effective K is much smaller than K_∞, vanishing at the source (in reality the diffusivity does not vanish at the source, but decreases to the molecular diffusivity over a short time $\tau' << \tau_L$ which is a characteristic timescale for the disturbed flow on the boundaries of the physical source).

In passing we should note that (42), a result stemming from the Lagrangian analysis, strengthens and quantifies the earlier assumption that K (where useful) should be interpreted (as is obvious dimensionally) as the product of a turbulent velocity scale and a turbulent length scale.

G.I. Taylor's results may be applied to diffusion from a steady source of ϕ in the presence of a steady streamwise wind of strength u by simply noting that at any distance x from the source, the material has been diffusing for a time $t = x/u$. In real, inhomogeneous turbulence, Taylor's results are not directly applicable, but they provide vital direction. Fig. 11 shows the time-average plumes from an elevated and a ground-level source in surface-layer flow in which (as a result of the presence of a solid wall) the turbulence time and length scales increase linearly with distance z from the wall. Also indicated on the diagram are the mean concentration profiles $\bar{\phi}(z)$ in the plume at various distances from the source. Close to the elevated source there are dramatic changes in $\partial\bar{\phi}/\partial z$ over distances which are much smaller than the length scale $\sigma_w\,\tau_L(z_s)$ at that height: from earlier reasoning we know classical K-theory is invalid in this region, and this is confirmed by (28), (which may still be used for guidance though this is no longer homogeneous turbulence), which states that indeed the effective K in this region is not the convenient flow property $K_\infty = \sigma_w^2\ \tau_L$ which we would have liked it to be.

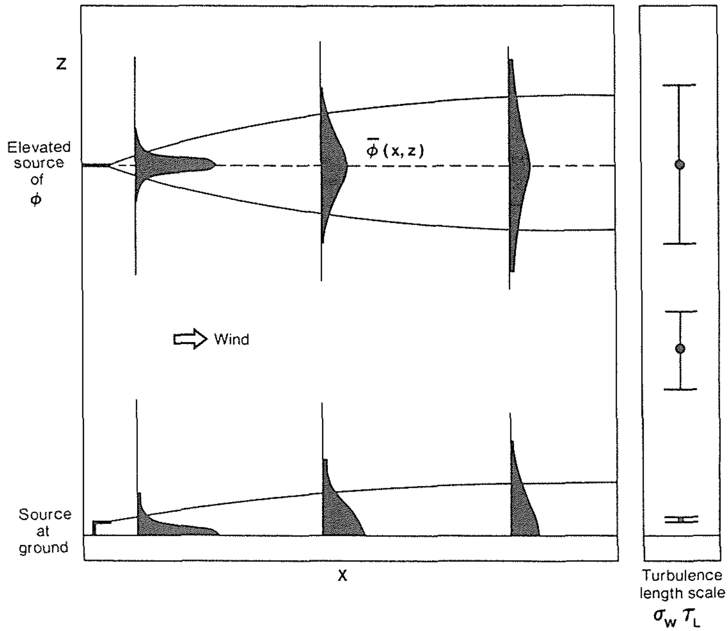

FIG. 11. Schematic ground-level and elevated plumes $\bar{\phi}(x,z)$ from continuous sources. The elevated plume has a cross section which is (near the source) narrow relative to the turbulence length scale (also shown), while the ground-level plume is always thick relative to the (very small) turbulence length scale at ground. K-theory fails for the elevated source.

In contrast, there is no problem with K-theory for ground-level sources and sinks. This is because the turbulence length and time scales become very small at the source (i.e. at the ground) so that by all our earlier considerations we can expect that the effective

K is simply K_∞. Many comparisons of theoretical solutions with experiments on diffusion from ground-level sources have confirmed the usefulness of K-theory for this case.

Why does K-theory fail in canopy flow?

In the light of the preceding discussions of K-theory it is not hard to understand why K-theory is an inadequate model for canopy transport.

Firstly, given the differential absorption of radiant energy we may expect gradients of variables involved in the energy balance to change rather rapidly with height (as is the case, for example, in the pine forest profiles of Fig. 9). But vertical mixing occurs not only on length scales $\ell < H$, but on much larger scales - in fact, mixing is particularly intense during the penetration of gusts from the boundary-layer above the canopy. Hence, we conclude that the restriction stated by Corrsin (1974) for the validity of K-theory is not necessarily obeyed by canopy flow; the length scale for vertical exchange may exceed the length scale for changes in the mean gradients.

Secondly, at a point in mid-canopy the mean concentration $\bar{\phi}$ (heat, water vapour, ...) is a superposition of contributions from fluid elements arriving from outside the canopy and from "marking contact" with leaves at a range of distances upstream and with the underlying surface. It simplifies the discussion if we specify ϕ to be an entity contributed to the airstream only by leaves (say, water vapour in an otherwise dry flow).

Let us consider first the plume from a single leaf and label its contribution to the mean concentration and mean vertical flux at our point of observation $\underset{\sim}{x_0}$ as $\bar{\phi}_i$, $\bar{F}_i$.

From earlier considerations we can expect the effective diffusivity for the material from this source to be independent of the source (leaf) proximity d to the point of observation only if the travel time from the source to $\underset{\sim}{x_0}$ obeys

$$d/u >> \tau_L \qquad (43)$$

The Lagrangian timescale within the canopy is expected to be of the order of $z/\sigma_w(z)$. Looking at Fig. 7 for the Elora corn canopy we have then, $\tau_L(H/2) \sim (H/2) / (0.5\, u_{*H})$ while typically $\bar{u}(H/2) \sim 0.5\, u_{*H}$ (Fig. 3).

Then, for the corn canopy, restriction (26) requires $d >> H/2$. Only the contributions from leaves much further away than about H/2 may be expected to diffuse in the region of $\underset{\sim}{x_0}$ with the far-field diffusivity $\sigma_w^2\ \tau_L$. The plumes from nearby leaves, whose contributions will likely dominate the total concentration field (since each individual plume is rapidly diluted with distance from its origin) will behave with an effective diffusivity $K_e < K_\infty = \sigma_w^2\ \tau_L$, i.e. we are in the "near field" of many leaves.

The overall eddy diffusivity is, by definition, (summation convention not implied by repeated subscript)

$$K = -\frac{\Sigma\ \bar{F}_i}{\frac{\partial}{\partial z}\ \Sigma\ \bar{\phi}_i} = \frac{\Sigma\ K_i \frac{\partial \bar{\phi}_i}{\partial z}}{\Sigma\ \partial \bar{\phi}_i / \partial z} \qquad (44)$$

and is therefore a weighted average of the eddy diffusivities for the individual contributing plumes. Plumes of greater strength (nearby sources) have larger weights in the average.

Consequently, the effective bulk diffusivity is dependent on the source distribution, is heavily weighted towards the individual diffusivities for material from nearby leaves, and may be considerably smaller than the far field value $\sigma_w^2\ \tau_L$.

We therefore cannot expect canopy transport to be well-described by first order-closure with effective diffusivity $\sigma_w^2\ \tau_L$ imposed by the flow alone.

How can the flux be directed against the mean gradient?

In order to prove that counter-gradient transport is a physically possible phenomenon, let us consider a very simple one-dimensional diffusion problem in which two parallel continuous plane sources of infinite extent are placed at heights z_{s1}, z_{s2} in an infinite domain (no barriers) occupied by a fluid in homogeneous turbulent motion. Initially we will assume the two sources have equal strength Q [kg m^{-2} s^{-1}]. We will label the average vertical fluxes and average concentrations of the (identical) additives from each source as F_1, F_2, $\overline{\phi_1}$, $\overline{\phi_2}$.

By symmetry, and assuming a steady state, we can easily conclude that half the material emitted from each source goes up, and half down, i.e. the mean vertical flux of ϕ_1 is

$$F_1 = Q/2 \qquad z > z_{s1} \tag{45}$$
$$\quad = -Q/2 \qquad z < z_{s1}$$

Further, according to classical K-theory, we have a constant concentration gradient

$$\frac{\partial \overline{\phi_1}}{\partial z} = -F_1 / K \tag{46}$$

which changes sign discontinuously at the source ($z = z_{s1}$) and the eddy diffusivity K depends only on the turbulent motion. An identical situation prevails for the fluxes and gradients of ϕ_2. Consequently, the average vertical flux of the additive in the region between the two sources is zero (the average upward flux of ϕ_1 cancels the average downward flux of ϕ_2), while above the upper source we have an upward flux Q and below the lower source a downward flux Q.

Fig. 12 shows these individual fluxes and the corresponding individual ($\overline{\phi_1}$, $\overline{\phi_2}$) and combined ($\overline{\phi_1} + \overline{\phi_2}$) concentration profiles according to the classical K-theory argument. In the region between the sources there is no flux and no gradient of $\overline{\phi_1} + \overline{\phi_2}$. (In any real approximation to this ideal one-dimensional situation, the finite upstream extent of source would result in the fluxes falling off in magnitude far from the sources with corresponding reduction in gradients and everywhere positive, or zero, concentration.)

But classical K-theory is wrong, and the effective diffusivities K_1, K_2 depend on proximity to the respective sources, becoming very small at the source (with correspondingly large concentration gradient). Fig. 12b shows the individual and combined concentration profiles which must be expected in the light of a rigorous theory of diffusion (note that there is no alteration of our assignment of the fluxes). Now, between the two sources, we have zero flux, but at only one point, $z = (z_{s1} + z_{s2})/2$, does the vertical gradient of $\overline{\phi_1} + \overline{\phi_2}$ vanish. We have a region of concentration gradient without flux.

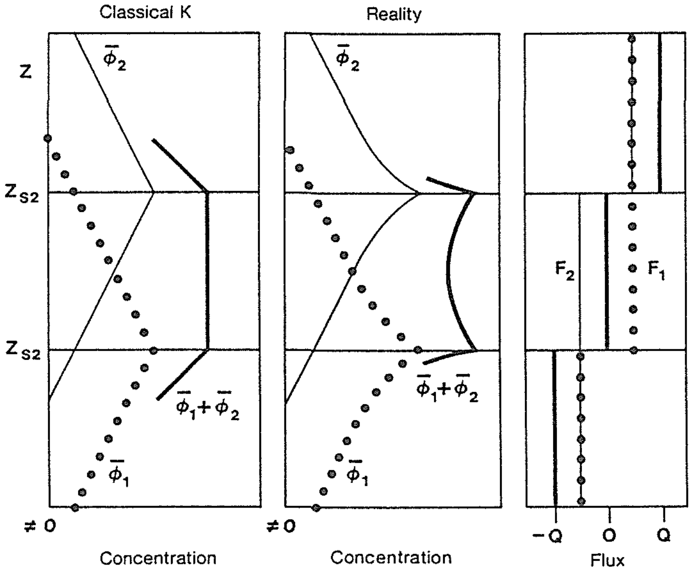

FIG. 12. Infinite plane sources of identical material ϕ_1,ϕ_2 placed at z_{s1},z_{s2} in an infinite unbounded domain of homogeneous turbulence. The source strengths are both Q. The mean vertical fluxes from each source and in total are shown. The left hand and middle diagrams show schematically the profiles of $\bar{\phi}_1,\bar{\phi}_2$, $\bar{\phi}_1 + \bar{\phi}_2$ according to classical K-theory and corrected K-theory, respectively (i.e. in the middle diagram the effect of source proximity, which is to reduce the effective K and therefore increase the concentration gradient, is accounted for).

The final step in this argument (due to G.W. Thurtell) is to allow one of the sources, say the upper source, to be slightly stronger than the other. Now there is a net downward flux in the region between the sources. Since we can make the difference in source strength arbitrarily small, it follows that this flux runs against the mean gradient in the region near, and just above, the lower source.

Raupach (1987) has performed simulations of turbulent diffusion in such a system of distributed sources using the rigorous Lagrangian theory and has shown that, depending on the strength and spacing of the sources, a counter-gradient flux of the nature of those observed in the Urriara pine forest (Fig. 9) is indeed predicted.

As a final comment, it should be noted that the criticisms of K-theory expounded here apply equally to the use of an aerodynamic transfer resistance to parameterize turbulent transport within the canopy.

SECOND-ORDER CLOSURE

We have seen that the microclimatic variables in the canopy airstream adjacent to transpiring leaves fluctuate dramatically in time; the fluctuations in airstream temperature are not necessarily small compared to the mean leaf-air temperature difference, the fluctuations in airstream vapour pressure not necessarily small compared to the mean stomatal cavity - airstream vapour pressure difference. Therefore specification of the mean canopy microclimate is not as satisfactory (from the point of view of integrating leaf transpiration rate) as a knowledge of the full time-dependent properties.

However, for the time being it is enough of a challenge to predict even the time-space average properties, so we will pre-suppose that this is a worthy aim and may lead to a better (if more complex) formulation of canopy evapotranspiration than the simple models in use at present. We have already seen that first-order closure is inadequate. We are therefore led to examine "second-order closure" as a means towards prediction of the time-space average properties $<\bar{e}>$, $<\bar{T}>$, $<\bar{u}>$.

We must average the conservation equations (i.e. integrate the equations in time and space) with recognition of the fact that within our averaging volume there may be "excluded regions" occupied by solid rather than fluid (and that heat, mass, momentum are transferred across the boundaries between solid and fluid). The formal methods for averaging the equations have been developed by Wilson & Shaw (1977), Raupach & Shaw (1982), and Finnigan (1985).

We will assume

(i) that any averaged quantity $<\bar{\phi}>$ is constant in time (steady state) and in horizontal planes (horizontal uniformity),

(ii) that molecular diffusion may be neglected relative to turbulent transport,

(iii) dispersive correlations may be neglected (a dispersive correlation $<\bar{\Theta}''\bar{\phi}''>$ arises when the spatial departures $\bar{\Theta}'' = \bar{\Theta} - <\bar{\Theta}>$, $\bar{\phi}'' = \bar{\phi} - <\bar{\phi}>$ are correlated in space. The complication of dispersive correlations will be neglected, not because this is generally permissible, but because otherwise a multi-dimensional treatment is necessary. Finnigan and Raupach (1987) argue that the dispersive flux is probably not very important if the foliage forms a fairly uniform layer without distinct periodicity.

(iv) that radiative divergence occurs only due to the presence of vegetation where it affects the thermodynamic energy balance through a flow of sensible and latent heat into the airstream,

(v) that there is no suspended liquid water and therefore no vapour/liquid phase change occurring in the airstream.

Under these restrictions and simplifications, conservation of streamwise momentum, water vapour, and thermodynamic energy may be expressed (for details of the averaging procedure, see Finnigan, 1985) as

$$\frac{\partial <\overline{w'u'}>}{\partial z} = S_u \tag{47}$$

$$\frac{\partial <\overline{w'\rho_v'}>}{\partial z} = S_v \tag{48}$$

$$\frac{\partial}{\partial z} <\overline{w'T'}> = S_H \tag{49}$$

Here S_V and S_H are the (suitably averaged) rates of transfer of vapour and heat into the airstream from the vegetation, and in the light of the single leaf combination equation, we know the instantaneous rate depends on the local radiation conditions, temperature, vapour pressure, wind speed, stomatal conductance, and leaf area density. S_u is the average rate of extraction of streamwise momentum from the flow due to viscous and form drag on the plant parts. This is commonly, and for most purposes fairly satisfactorily, parameterized as

$$S_u = -c_d a\langle\bar{u}\rangle\,|\langle\bar{u}\rangle| \qquad (50)$$

This simple formulation states that the mean drag is opposed to the mean flow and is proportional to the square of the velocity, the amount of vegetation present (specified by the leaf area density a(z) [$m^2\ m^{-3}$]), and an effective drag coefficient c_d.

In order to obtain the vapour flux $\langle\overline{w'\rho_v{}'}\rangle$ at z = H (the canopy evapotranspiration) we need the vapour source strength $S_V(z)$ at all levels and the soil contribution $\langle\overline{w'\rho_v{}'}\rangle_{z=0}$ which itself depends on the canopy microclimate. Of the factors determining $S_V(z)$, only the short-wave radiation balance throughout the canopy might be considered "decoupled" from the canopy environment (i.e. to affect but not be affected by the canopy environment; this can only be true for short periods of time). The net long wave radiation balance depends upon the temperature distribution within the canopy, while the stomatal conductance depends on several environmental factors in addition to plant water status (which is linked to soil water status).

Because (47), (48) and (49) (with S_u, S_V, S_H parameterized using average airstream properties) are unclosed and we have rejected first-order closure, we must obtain "budget" ("conservation"/"transport") equations for the turbulent fluxes themselves from the rigorous conservation equation for the instantaneous velocity, temperature, and vapour pressure. The procedure for doing this is straightforward if spatial averaging is not necessary (Busch, 1973; Plate, 1971). When both temporal and spatial averaging are necessary and the flow domain contains plant parts, the derivation of the budget equations is quite complex. The reader interested in details, is referred to the comprehensive derivation given by Finnigan (1985).

An approximate budget equation for the average vertical flux of water vapour is

$$\frac{\partial}{\partial t}\langle\overline{w'\rho_v{}'}\rangle = 0 = \underbrace{-\langle\overline{w'^2}\rangle\frac{\partial\langle\overline{\rho_v}\rangle}{\partial z}}_{GP} + \underbrace{\frac{g}{T_o}\langle\overline{\rho_v{}'T'}\rangle}_{BP} \underbrace{-\langle\overline{\frac{\rho_v{}'}{\rho}\frac{\partial p'}{\partial z}}\rangle}_{D} \underbrace{-\frac{\partial}{\partial z}\langle\overline{w'w'\rho_v{}'}\rangle}_{TT} \qquad (51)$$

Budget equations of a similar form may be derived for a multitude of other covariances and variances, but (51) will serve as a prototype. Here, in addition to the restrictions and simplifications specified earlier, several terms which derive from the molecular diffusion terms in the basic conservation equations have been neglected, as have what Finnigan calls drag and waving source production. Because of these simplifications, (51) is in fact no different from the equation one would obtain without the additional spatial averaging (see, for example, Launder, 1978).

Equation (51) states that the "level" of $<\overline{w' \rho_v'}>$ at any height z in (or above) the canopy is steady (a pre-supposition) as a result of the balance of a number of counteracting mechanisms which control the correlation between w' and ρ_v'. Some of these terms act to increase and some to decrease $<\overline{w'\rho_v'}>$.

It is no surprise that new unknowns have arisen from the averaging procedure. We need not consider $<\overline{w'^2}>$ a new unknown, since we may, by steps similar to those leading to (51), derive a conservation equation for each of the variances $<\overline{u'^2}>$, $<\overline{v'^2}>$, $<\overline{w'^2}>$ as well as the other relevant fluxes $<\overline{u'w'}>$, $<\overline{w'T'}>$. The same comment holds for the scalar covariance $<\overline{\rho_v'T'}>$. In each of these additional equations there will, however, arise further new unknowns whose form is typified by those arising in (51).

The terms in any of the conservation equations of fluid mechanics, may be classified as "storage" terms, "source/sink" terms (i.e. "production" or "destruction" terms) or as "transport" terms. Whenever the spatial derivative of a mean quantity appears in a budget equation it is called a "transport term" for the simple reason that when integrated throughout any volume such a term reduces to a difference between influxes and effluxes through the walls of the volume, i.e. transport from a different region of the fluid.

In (51) there are no transport terms due to the mean flow because we have assumed horizontal homogeneity and that there is no mean vertical velocity. The sole transport term is that labelled "TT", "turbulent transport", and, but for the existence of this term, the balance controlling $<\overline{w'\rho_v'}>$ would consist of only "local" production and destruction. The terms in (51) may be interpreted as:

(a) GP, "Gradient Production", $$-<\overline{w'^2}> \frac{\partial <\overline{\rho_v}>}{\partial z}$$

There are no new unknowns here. Because of the explicit appearance of $\overline{w'^2}$ it can be seen that prediction of the turbulent velocity variance(s) is an integral aspect of a physically-sound model of canopy evapotranspiration. The gradient production term is non-zero whenever there exists a mean gradient in the absolute humidity within a turbulent ($<\overline{w'^2}> \neq 0$) flow. For an intuitive feeling for its meaning we can apply the argument used earlier in connection with the momentum flux $\overline{u'w'}$ (replace $\overline{u}$ with $<\overline{\rho_v}>$ in Fig. 2). For example, if $<\overline{\rho_v}>$ decreases with increasing height, upward-moving parcels ($w' > 0$) tend to carry past the reference level an excess of water vapour $\rho_v' > 0$ and vice-versa for downward-moving parcels. This contributes towards $<\overline{w'\rho_v'}> > 0$.

(b) BP, "Buoyant Production", $$\frac{g}{T_o} <\overline{\rho_v'T'}>$$

Temperature fluctuations (T') imply the existence of fluctuating buoyancy forces (as in fact do fluctuations in humidity, since moist air is lighter than dry air; the moisture contribution to buoyancy can often be neglected, but if it must be included, one may formulate the buoyancy term in the vertical momentum equation using the virtual temperature). If warm parcels (which will tend to rise) are also (in a statistical sense) moist, we have a mechanism contributing towards an upward flux of moisture.

(c) D, "Destruction", $$-<\overline{\frac{\rho_v'}{\rho} \frac{\partial p'}{\partial z}}>$$

This term is believed to be the dominant agent destroying correlation between ρ_v' and w'. It is possible and common to split terms like this into a transport term ("pressure transport") and a "pressure-fluctuation x property - fluctuation gradient" term ("pressure destruction"). Here we will simply examine the term in its basic form given above. We know that an instantaneous pressure gradient $\partial p'/\partial z < 0$ will *tend* to induce a $w' > 0$. If the occurrence of such a pressure gradient were correlated with $\rho_v' < 0$, say, then we would expect this to result in an association of $\rho_v' < 0$ events with $w' > 0$ (i.e. $\rho_v' w' < 0$, downward instantaneous flux). But why would $\rho_v' < 0$ be associated with $\partial p'/\partial z < 0$? The instantaneous pressure gradient seems a function of the kinematic state of the system, and why should this be correlated with concentration fluctuations of an (almost) passive additive, water vapour? The answer is probably that there will be no such association (between ρ_v' and $\partial p'/\partial z$) unless there is preferential inhomogeneity in the distribution of water vapour, i.e. unless there is a flux $\overline{w'\rho_v'}$. Let us assume $\overline{w'\rho_v'} > 0$. Each downward-directed gust of air will carry relatively dry air, and its approach to the (blocking) ground may produce a pressure gradient $\partial p'/\partial z < 0$. As argued, such an association will act to drive $\overline{w'\rho_v'}$ in the negative direction.

This (hand-waving) argument leads to the supposition that a simple model for this term might be

$$-\left\langle \overline{\frac{\rho_v'}{\rho} \frac{\partial p'}{\partial z}} \right\rangle = -\left\langle \overline{w' \rho_v'} \right\rangle / \tau_t \tag{52}$$

where τ_t is an unknown timescale. This is a "flux-killer" term and many higher-order closure models do employ such a simplification, though more complex models have been proposed (Launder, 1978).

(d) TT, "Turbulent Transport", $\quad -\dfrac{\partial}{\partial z} \langle \overline{w'w' \rho_v'} \rangle$

This is a new unknown which, in the context of numerical simulation, must be modelled (at the level of second-order closure) or dealt with by adding to the set of equations an approximate budget equation for the triple correlation $\langle \overline{w'w'\rho_v'} \rangle$, (third-order closure). As earlier noted, this transport term arises due to transport of $w'\rho_v'$ by the fluctuating (turbulent) flow w'. The concept of turbulent transport of a *product* of fluctuations (i.e. $w' \times w'\rho_v'$) is not easy to visualize. The author's understanding of the physical meaning of this term is as follows: Not all the correlated fluctuations in ρ_v' and w' which are seen at level z are necessarily *created* at that level. Volumes of air within which correlated fluctuations (ρ_v',w') have been created (by the production mechanisms) are apt to be bodily moved (transported) by occasional very large eddies (of size ℓ >> the volume under consideration) from the "marking" region (where the correlation was produced in response to a vapour-concentration gradient) to another location. For example, in the crown region of a canopy there is strong gradient production of correlation between w' and ρ_v' ($\langle \overline{w'^2} \rangle$ large, $|\partial \langle \overline{\rho_v} \rangle / \partial z|$ large); the large intermittent eddies bring down volumes of air within which there is established correlation so that instruments in the lower canopy (where production is very weak due to small $\langle \overline{w'^2} \rangle$ and $|\partial \langle \overline{\rho_v} \rangle \partial z|$) will measure correlation which was not locally-produced, i.e. the flux seen deep in the canopy may have essentially been created elsewhere.

By simplifying the budget equation for turbulent transport of scalar flux, various authors have suggested models for the turbulent transport of scalar flux (see Launder, 1978). The majority of these include terms having the form of down-gradient transport (of scalar flux) driven by the mean gradient (in scalar flux). For the transport term in our (specialized) vapour-flux budget equation a typical model is

$$<\overline{w'w'\rho_v'}> = -c<\overline{w'^2}> \tau_t \frac{\partial<\overline{w'\rho_v'}>}{\partial z} \tag{53}$$

where $c<\overline{w'^2}>\tau_t$ is dimensionally and effectively an eddy diffusion coefficient.

One of the nice features of the gradient-diffusion parameterization of turbulent transport is that we end up with a "diffusion term" $\partial^2<\overline{w'\rho_v'}> / \partial z^2$ in the budget equation. Such terms have a smoothing effect and tend to ensure the numerical stability of a simulation. However, such a simple model may be criticized on at least two counts

(i) turbulent transport, according to the simple model (53) is non-existent in the constant flux region above the canopy. Experimental measurements above model and real canopies have indicated that turbulent transport does not vanish immediately above the canopy.[4]

(ii) It may be shown (Deardorff, 1978) that if one is to describe scalar diffusion from sources in homogeneous turbulence rigorously using higher-order closure, any effective diffusion coefficient appearing at higher-order must retain a dependence on time since release (or source proximity).

It is instructive to examine under what assumptions the rigorous flux-budget equation will reduce to K-theory. In the case of (51), if we

(i) neglect turbulent transport

(ii) neglect buoyant production

(iii) adopt the simple flux-killer model for pressure destruction

then we have

$$0 = -<\overline{w'^2}> \frac{\partial<\overline{\rho_v}>}{\partial z} - <\overline{w'\rho_v'}> /\tau_t \tag{54}$$

which is in effect a flux-gradient relationship,

$$<\overline{w'\rho_v'}> = -<\overline{w'^2}> \tau_t \frac{\partial<\overline{\rho_v}>}{\partial z} \tag{55}$$

We therefore see once again how far from generality is first-order closure. We must make sweeping and generally unsatisfactory simplifications to the rigorous laws of turbulent diffusion in order to draw K-theory into being. Measurements of the various terms in a scalar flux budget equation in a canopy have been reported by Coppin *et al.* (1986). These and other observations have proven that turbulent transport terms are very significant deep in the canopy.

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4 Some of the models which have been proposed do allow turbulent transport above the canopy.

SECOND-ORDER CLOSURE MODELLING OF CANOPY WIND AND TURBULENCE

Higher-order closure models of canopy wind and turbulence have been described by Wilson & Shaw (1977), Lewellen *et al.* (1980), Meyers & Paw U (1986), and Wilson (1987). The agreement with observed wind profiles $\bar{u}(z)$ demonstrated has usually been good (see, for example, Fig. 3 for predicted and measured wind profiles in a corn canopy) while the prediction of the turbulent velocity variances has been not quite so good but adequate in the present context. The reported simulations have dealt with a variety of fairly dense and uniform canopies. The simulations may be criticized in that the agreement with observation has been procured through an optimal specification of the drag coefficient c_d by either direct inference (from measured velocity and shear stress profiles) or by trial and error. Overlooking this criticism, it seems fair to say that higher-order closure models can predict wind and turbulence in a dense canopy with an accuracy that should be more than adequate for the purposes of modelling canopy evapotranspiration.

MODELLING CANOPY EVAPOTRANSPIRATION USING HIGHER-ORDER CLOSURE

Simulations of the canopy environment using higher-order closure have been reported by Hwang & Shaw (1985), Paw U *et al.* (1985), and Meyers & Paw U (1987). Attention is restricted to dense, uniform canopies of large extent on level terrain with uniform soil and soil moisture conditions, in order to minimize the importance of horizontal transport and dispersive vertical fluxes.

The model of Meyers & Paw U illustrates the general pattern one would expect of a higher-order closure model of the canopy environment. A canopy radiation model was used to compute the radiant energy supply for each canopy layer. This energy supply was then partitioned into contributions to the airstream sensible and latent heat using a single-leaf energy balance formulation. The energy balance at the soil surface was evaluated to calculate the heat and vapour fluxes from the ground. A set of 22 equations in 22 unknowns were solved at 40 grid-points lying below $z = 2H$. This required only about 3 min on a microcomputer to yield rather good predictions of the evapotranspiration from a soybean canopy. The experimental data-set included measured above-canopy values of global incoming shortwave radiation $K\downarrow$, net radiation Q^*, wind speed, temperature, and specific humidity, as well as the canopy leaf area density profile, the soil heat flux, and measured stomatal resistance values. If additional sub-models were added for the stomatal resistance, for soil heat and moisture transport, and for the estimation of the net radiation, the required input would be reduced to the above-canopy values of $K\downarrow$, $\langle u \rangle$, $\langle e \rangle$, $\langle T \rangle$ and a knowledge of the leaf area profile. It is noteworthy that in spite of the added complexity of a higher-order closure model for canopy evapotranspiration (relative to, say, the big leaf combination equation) the inputs required to operate in a diagnostic sense (i.e. to estimate actual evapotranspiration from related measurements) do not exceed the inputs required by much simpler models of actual canopy evapotranspiration. By extension of the computational domain to the top of the PBL it is in principle possible to further reduce the "local" (i.e. immediately above-canopy) input.

CONCLUSION

It should be clear that a comprehensive and well-founded evapotranspiration model must of necessity include a model of turbulent transport within the canopy in order to estimate the microclimate of the airstream into which the leaves are transpiring. K-theory or "first-order closure" does not provide a suitable model for this purpose, and we have briefly examined the alternative of higher-order closure. Although there are theoretical difficulties with the higher-order closure approach, there is reason to be optimistic that a higher-order model, in conjunction with models of other aspects of the canopy energy balance (radiation, soil and plant status), will be both a conceptual and a practical improvement over the more superficial models in use at present.

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