Modeling the Effect of Turbulence on the Collision of Cloud Droplets

SHUMING DU AND JOHN D. WILSON

Department of Geography, University of Alberta, Edmonton, Canada

(Manuscript received 14 June 1994, in final form 28 March 1995)

ABSTRACT

From an analysis of scales in the cloud droplet collision problem, the authors infer that a trajectory model that is to be capable of predicting collisions between droplets of *all* possible sizes should be of second-order, that is, should explicitly model particle acceleration. But for collisions between large droplets (radius about 50 μm or larger), which are still much smaller than raindrops, a first-order model is appropriate.

The relative motion of large droplets are studied with a first-order, two-particle trajectory model. Turbulence is found to be unimportant (relative to differential gravitational settling) if the (large) droplet sizes are sufficiently distinct. Zeroth-order two-particle models, of the type hitherto applied to the problem, deteriorate in accuracy as the influence of turbulence on the droplet separation increases, that is, for large σ_v/v', where σ_v is the turbulent velocity scale and v' is the droplet still-air terminal velocity. Under no circumstance is a single-particle model applicable.

1. Introduction

It has been recognized for a long time that turbulence can influence collisions of cloud droplets, possibly spurring the growth of cloud droplet size between the ranges where (initially) effects of condensation and then (finally) gravitational coalescence dominate (Rogers and Yau 1989). Over the last two decades, a number of theoretical investigations have been done on this subject, and in some of them the Lagrangian stochastic (LS: i.e., trajectory or "Random Flight") simulation has been employed (e.g., de Almeida 1976, 1979a, 1979b; Reuter et al. 1988).

Quantitatively the effect of turbulence is expressed in the stochastic collection equation (SCE) for the evolution of the cloud droplet number density distribution function, $N(V, t)$, (m^{-6}). This function is defined such that $N(V, t)dV$ is the average number density (m^{-3}) at time t of cloud droplets of a size lying within droplet volume interval $(V, V + dV)$. The SCE is

$$\frac{\partial}{\partial t}N(V, t) = \frac{1}{2}\int_0^V N(V - v, t)N(v, t)K(V - v, v)dv - N(V, t)\int_0^\infty N(v, t)K(V, v)dv, \quad (1)$$

where $K(V, v)$ is the collection kernel (m^3 s^{-1}), relating to the probability that a droplet of volume V will collect a droplet of volume v (in given time, etc.). Given $K(V, v)$, which is to some extent influenced by turbulence, the SCE determines the evolution of an initial distribution of droplet volume $N(V, t_0)$.

A trajectory simulation is the natural way to study the movement of small particles in turbulent flows, for example, in order to calculate the collection kernel. However, to the authors' knowledge, in LS models applied to date for cloud droplet collisions, the correlation of cloud droplet velocity between consecutive instants was not accounted for; that is, the random displacement of a cloud droplet was assumed to be Markovian. This is incorrect when calculating a cloud droplet trajectory over a time period shorter than the integral timescale of the background turbulent field because the displacement of a particle can be taken as Markovian only when the travel time of interest is much longer than the integral timescale of the droplet velocity (Sawford 1991). Another problem associated with some earlier LS simulations of cloud droplet collisions was the use of independent, single-particle trajectories (single particle models). This in principle is not acceptable because the movements of nearby cloud droplets are highly correlated (in space).

For reasons that will be given in section 3, in this paper we study collisions between large droplets [by large we mean that the radius (r) of droplets $r \geqslant 50$ μm]. For simplicity, we take the coalescence efficiency to be unity (cloud droplets merge upon collision), and hydrodynamic effects when drops are in close proximity are neglected. Although "large–large" collisions are much less frequent than "large–small" or "small–small" collisions, their contribution may be important because overtaking a single large drop (~50

Corresponding author address: Shuming Du, Environmental Engineering, College of Engineering, University of California, Riverside, Riverside, CA 92521-0425.

μm) is equivalent to collecting many ($\sim$100) small ($\sim$10 μm) cloud droplets. On occasion, the majority of cloud droplets may be quite large as a result of flow difluence at the upper parts of the cloud (Kogan 1993).

2. Relevant scales in the cloud droplet collision problem

Before considering the class of Lagrangian stochastic model that might be appropriate in studying cloud droplet collisions, we need to establish some critical timescales of the problem. In doing so, we recognize that turbulence statistics differ in different kinds of clouds, and even within *one* cloud there is spatial variability.

In mature cumulus clouds, typical values for the standard deviation of the vertical velocity fluctuation and the rate of dissipation of turbulent kinetic energy (TKE) are $\sigma_w = 2.0$ m s^{-1} and $\epsilon = 0.02$ m^2 s^{-3} (Weil et al. 1993). Extreme values in the literature are $\epsilon = 0.0003$ m^2 s^{-3} for small cumuli (Ackerson 1967) and $\epsilon = 0.25$ m^2 s^{-3} in very strong cumulus congestus (Panchev 1971). Assuming for the kinematic viscosity of cloud air $\nu = 1.2 \times 10^{-5}$ m^2 s^{-1}, the corresponding range in the Kolmogorov timescale $t_\eta = (\nu/\epsilon)^{1/2}$, which characterizes the smallest eddies in the flow, is about $10^{-2} \leqslant t_\eta \leqslant 10^{-1}$ s. The corresponding Kolmogorov length scale $\eta = (\nu^3/\epsilon)^{1/4}$ is in the range of $10^{-4} \leqslant \eta \leqslant 10^{-3}$ m.

The Kolmogorov (inner) scales describe the minimum lengths over which changes (in velocity) occur in the airflow. These are to be contrasted with the Lagrangian integral (outer) scales, which measure typical spatial and temporal persistence of the turbulent velocity. In stationary, homogeneous turbulence, the Lagrangian integral timescale can be determined from the velocity variance (σ_V^2) and the TKE dissipation rate (ϵ) as (Tennekes 1979)

$$T_L = \frac{2\sigma_V^2}{C_0\epsilon}, \qquad (2)$$

where C_0 is a (supposedly) universal constant ($C_0 = 3.0$, according to Du et al. 1995). Adopting $\sigma_V \sim 1$ m s^{-1}, then $T_L = 10 \sim 10^3$ s, and the Lagrangian integral length scale is $L = \sigma_V T_L = 10 \sim 10^3$ m. It is our proposition that in some regions of real clouds there exists a wide separation in scale between the dissipation range and the energy-containing range of scales; that is, $t_\eta \ll T_L$ and $\eta \ll L$. The truth of this bears on the validity of the model we later construct for droplet paths.

Now we want to establish a timescale t_c characterizing the collision interval, which will limit the permissable timestep Δt upon which we discretize droplet trajectories. First, consider nonprecipitating cumulus, wherein the liquid water content is of order 1 gm^{-3} and the mean radius of the cloud droplet is about 5 μm (Pruppacher and Klett 1978, pp. 14–16). It follows from simple geometry that, assuming a uniform spatial distribution of droplets, the mean separation (d) between neighboring cloud droplets is about 10^{-3} m; that is, $d \ll L$. It seems d/η could be of the order of unity in small cumuli and about one order larger in deep convective clouds.

For the purpose of discussion, assume (temporarily) that the separation d satisfies the condition $\eta \ll d \ll L$. Using dimensional arguments, $\delta d/\delta t \sim (\epsilon d)^{1/3}$. It follows that for two cloud droplets separated by d, a collision takes place at intervals of order $t_c \sim (d^2/\epsilon)^{1/3} \sim 10^{-2} - 10^{-1}$ s. This is about the same order as the Kolmogorov timescale. If d were of the same order as η, it is very unlikely that the collision time interval t_c could be larger than the Kolmogorov timescale. Hence for the small droplets, the discretization time step Δt is necessarily small with respect to t_η, and droplet acceleration is an autocorrelated time series.

In contrast, for large cloud droplets of radius $r \geqslant 50$ μm, it is estimated that the separation between neighboring drops is of order 10^{-2} m, so if the turbulence in the cloud is strong ($\epsilon \sim 0.1$ m^2 s^{-3}), the collision interval is about $t_c \sim 10^{-1}$ s. This is an order of magnitude larger than the Kolmogorov timescale ($\sim 10^{-2}$), yet much smaller than the Lagrangian timescale ($\sim$10 s), that is, t_c is well within the inertial subrange. In this case it is appropriate to consider droplet velocity and position to (jointly) constitute a Markov process.

3. First-order two-particle LS model for collisions between large droplets

a. Model order

Sawford (1991) summarized the hierarchy of LS models in a study of Reynolds number effects in LS models of turbulent dispersion. According to Sawford, the appropriate order of LS model to be used is determined by the ratios t/t_η and t/T_L, where t is the time interval of interest. We will assume the cloud turbulence can be regarded as homogeneous, isotropic, and stationary: probably a satisfactory assumption, since the separation of initially nearby cloud droplets is caused by the smallest-scale eddies of the field having approximately this simple statistical structure.

If $t \gg T_L$, a "zeroth-order" LS model

$$dX^i = \sqrt{2K}d\xi^i \qquad (3)$$

is sufficient to study the displacement of tracer elements (random walk in position). In (3) and hereafter, $d\xi^i$ is the increment of a Wiener process (i.e., $d\xi^i$ is a Gaussian random number with zero mean and variance dt). The superscript i is the direction index, and K is the eddy diffusivity, $K = \sigma_V^2 T_L$. This very simple model is acceptable because both acceleration and velocity are uncorrelated over discretization intervals Δt satisfying $T_L \ll \Delta t$. However, this model is inapplicable to our cloud droplet problem: we have argued (section 2) that the collision interval (thus t, the duration of simulations) is $t_c \ll T_L$.

At the other extreme, if t_c/t_η is small or order 1, the only permissible choice is a "second order" model, in which the acceleration, velocity, and displacement of the moving tracer particle are taken to be collectively Markovian, and the acceleration is modeled as an autocorrelated stochastic process. From section 2, we conclude that for a general study of cloud droplet collisions, one indeed requires a second-order model. This is a difficulty because such models are not well developed.

However, for collisions between large cloud droplets, and provided the turbulence in the cloud is strong, the required duration of a simulation of the droplet trajectory (estimated in section 2 as "t_c") satisfies $t/t_\eta \gg 1$, and t/T_L is finite. In this case a "first-order" LS model is appropriate (Thomson 1987): we can choose a time step Δt in the range $t_\eta \ll \Delta t \ll T_L$ to resolve the evolution in velocity and position over time period t (Lagrangian acceleration correlation vanishes over time steps $\Delta t \gg t_\eta$). The uniquely correct LS model (for a single nonbuoyant tracer particle in homogeneous, isotropic turbulence) is (Borgas and Sawford 1994)

$$dU^i = -\frac{U^i}{T_L}dt + \sigma_V\sqrt{\frac{2}{T_L}}d\xi^i,$$
$$dX^i = U^i dt. \tag{4}$$

b. Need for a two-particle model

If the separation of two cloud droplets is not much larger than the integral length scale, their movements are spatially correlated because the movements of the fluid elements embedding them are correlated. Turbulent fluctuations at one point can be viewed as the superposition at that point of an ensemble of eddies having different scales and orientations (Townsend 1976). Relative motion (due to turbulent fluctuations in the cloud) of a pair of particles is caused by eddies of sizes smaller than, or of the same order as, the separation. Larger-scale eddies cause only a coordinated displacement of both droplets together. So we conclude, in studying collision problems, it is not appropriate to use a single particle model in which the motion of any particle is assumed independent of all others, and the relative motion of a pair of particles is attributed to *all* eddies of various scales. An example of single-particle model is the work by Reuter et al. (1988), who chose a constant diffusivity, rather than a diffusivity dependent on the separation according to Richardson's law, to study relative movement between two air elements.

c. A heuristic model for trajectories of large droplets

We consider two large cloud droplets of radii r_1 and r_2 moving in a turbulent flow. Since the three components of the relative velocity of air elements that carry the droplets are not independent (being constrained by the incompressibility condition), a three-dimensional, two-particle model must be employed. If we assume a small enough droplet Reynolds number Re $= r\delta U/\nu$ (where δU is the velocity of the droplet relative to the surrounding air, and r is r_1 or r_2), then the air-droplet drag is linear in relative velocity, and we may write a first-order model:

$$\frac{dV_1^i}{dt} = \frac{1}{\tau_{a1}}(U_1^i - V_1^i) - g\delta^{3i},$$
$$\frac{dX_1^i}{dt} = V_1^i,$$
$$\frac{dV_2^i}{dt} = \frac{1}{\tau_{a2}}(U_2^i - V_2^i) - g\delta^{3i},$$
$$\frac{dX_2^i}{dt} = V_2^i, \tag{5}$$

where the superscript (i) is the direction index; the subscript (1 or 2) is the droplet label; V is the droplet velocity; U is the velocity of the air surrounding the droplet; X is the droplet position; g is the gravitational acceleration; and τ_a is the droplet aerodynamic response time (time constant for response to a step change in the velocity of the surrounding air). For large cloud droplets, the following empirical formula for τ_a is appropriate (Pruppacher and Klett 1978, p. 324; Rogers and Yau 1989, p. 126):

$$\tau_a = \frac{8000r}{g}, \tag{6}$$

where r is the radius of the droplet.

Kaplan and Dinar (1988) have given a *heuristic* two-particle model for the evolution of the velocity of a pair of fluid elements in stationary, homogeneous, isotropic turbulence:

$$U_1^i(t+\Delta t) = R_L(\Delta t)U_1^i(t) + [1 - R_L^2(\Delta t)]^{1/2}\theta_1^i(t),$$
$$U_2^i(t+\Delta t) = R_L(\Delta t)U_2^i(t) + [1 - R_L^2(\Delta t)]^{1/2}\theta_2^i(t),$$
$$U_1^i(0) = \theta_1^i(0),$$
$$U_2^i(0) = \theta_2^i(0), \tag{7}$$

where $R_L(\Delta t) = \exp(-\Delta t/T_L)$ is the Lagrangian temporal correlation coefficient, and θ is a random field that is spatially correlated but temporally uncorrelated from t to $t + \Delta t$. By assuming the spatial correlation between the components of $\theta(t)$ is equal to the spatial Eulerian velocity correlation of the turbulent field, and using the conditions of continuity and isotropy, Kaplan and Dinar developed an algorithm to calculate the random field $\theta(t)$. The θ field is strongly dependent upon the separation of the two moving particles: only when the separation is much larger than the Eulerian length scale do θ_1 and θ_2 become uncorrelated. For a detailed description of construction of the θ field, please refer to the original paper.

The Kaplan–Dinar model is used to calculate the driving fluid element velocity (i.e., velocity of the fluid element surrounding the cloud droplet). Since the driving fluid velocity time series is not a Lagrangian series (i.e., at different instants the droplet is surrounded by different air elements), we reduced the Lagrangian timescale T_L in the manner suggested by Sawford and Guest (1991):

$$T_{Lg}^{1,2} = T_L\left[1 + \left(\frac{2\beta v'}{\sigma_V}\right)^2\right]^{-1/2},$$

$$T_{Lg}^{3} = T_L\left[1 + \left(\frac{\beta v'}{\sigma_V}\right)^2\right]^{-1/2}. \quad (8)$$

This accounts (heuristically) for the "crossing trajectory" effect (Csanady 1963), that is, the fact that the cloud droplet is not accompanied and driven by the same air parcel at different times. In (8), v' is the terminal velocity of the cloud drop in still air, related to τ_a by $v' = \tau_a g$; β relates Lagrangian and Eulerian length scales (defined as $\beta = \sigma_V T_L/L_f$, L_f is the Eulerian integral length scale in the vertical direction); and following Sawford and Guest we set $\beta = 1.5$. Equation (8) is simply an interpolation between the integral time scales for a passive tracer and for a particle of very large terminal velocity relative to the ambient fluid. For passive tracer $v' = 0$, and the timescale reduces to T_L; for particles of large velocity relative to the surrounding air, the timescale becomes L_E/v' (L_E is Eulerian length scale of the turbulence). Here L_E takes values L_f for the direction parallel to the external force and $L_f/2$ for the direction perpendicular to the external force.

4. The collision probability and the collection kernel

The movement of a cloud droplet can be divided into two parts: movement *with* the ambient air and movement *relative* to the ambient air (Saffman and Turner 1956). For a very small cloud droplet, the former dominates, that is, the turbulent motion of the air controls the movement of the droplet, while for a very large droplet, movement is mainly of the latter type, because in this case the turbulent fluctuation of the cloud air hardly affects the droplet's movement.

When only movement relative to the ambient air is present, the collection kernel in the SCE has the following simple form (Rogers and Yau 1989, p. 130):

$$K(V, v) = \pi(R + r)^2|u(R) - u(r)|E(R, r), \quad (9)$$

where $E(R, r)$ is the collection efficiency, the product of collision efficiency, and coalescence efficiency. The droplets radii R and r are trivially related to V and v by $V = (4/3)\pi R^3$ and $v = (4/3)\pi r^3$. We here set $E = 1$, which assumes that as two cloud droplets approach, one droplet's trajectory is not affected by the presence of the other droplet, and that those two droplets coalesce upon collision.

When movement *with* the ambient air is involved (smaller droplets), the collection kernel becomes far more complicated. For this case, whether or not two cloud droplets can collide depends on the turbulent field in the cloud, in addition to their (initial) relative positions. From the model outlined in the last section, we can calculate the trajectories of a pair of cloud droplets for any given initial separation. When the separation (between the centres of the two droplets) is equal to or less than the summation of the two drops' radii, they collide; otherwise, they do not.

The term $K(V, v)$ is related to the probability $p(R, r, X_1, X_2, T)$ that a pair of droplets of radii r and R, having arbitrary initial separation $(X_1 - X_2)$, will collide within time interval T. According to Reuter et al. (1988),

$$K(V,v) = \frac{2\pi}{T}\int_\Sigma p(R,r,X_1,X_2,T)D_H dD_H dD_Z. \quad (10)$$

Here D_H is the initial horizontal distance between the centers of the two droplets (the projection of $|X_1 - X_2|$ onto the horizontal plane); dD_H and dD_Z are the horizontal and vertical length increments, respectively; and Σ is the initial separation domain over which $p(R, r, X_1, X_2, T)$ is nonzero. Note that the property of symmetry about the vertical axis has been used.

To calculate $p(R, r, X_1, X_2, T)$ numerically, we calculated an ensemble (N members) of trials, in each of which we released a pair of cloud droplets (droplet 1 has radius R and is located at X_1 at $t = 0$; droplet 2 is of radius r and is located at X_2 at $t = 0$) and followed their trajectories to examine whether (or not) they would collide within time $t \leq T$. If the two droplets collide n times in N realizations, then

$$\lim_{N\to\infty} p(R, r, X_1, X_2, T) = \frac{n}{N}.$$

We used our thus-determined collision probability to estimate the function

$$K_r(V, v, D_H) = \frac{2\pi}{T}\int_{-\infty}^{\infty} p(R, r, X_1, X_2, T)D_H dD_Z \quad (11)$$

in terms of which $K(V, v)$ follows by integration with respect to D_H.

5. Results

In addition to the first-order two-particle model defined above (and hereafter referred to as model 1), we examined two simplifications of it: a zeroth-order two-particle model (referred to as model 2—temporal correlation along the driving air parcel trajectory is ignored); and a zeroth-order single-particle model (referred to as model 3—spatial correlation across the two driving parcels is also ignored). These latter trajectory models are fully defined in the appendix. For each initial separation, 5000 pairs of droplets were released. The total flight time for each pair of droplets was 0.1 s.

a. Large droplets, strongly turbulent cloud ($\sigma_V = 2$ m s^{-1}, $\epsilon = 0.1$ m^2 s^{-3})

First we considered droplets of distinct radii 50 and 100 μm. Our results (Fig. 1) for the collection kernel from model 1 and model 2 are probably not significantly different, and as shown in Table 1, the enhancement of K (over purely gravitationally driven coalescence) due to turbulence was quite small (order 20%). The predictions from model 3 were markedly different. Relative to model 1 (which is certainly more rigorous than models 2, 3), model 3 underestimates collision probability when the droplet separation is small but overestimates when the droplets are far apart. The explanation is simple: recall that model 3 assumes that the two cloud droplets move independently, and their relative velocity is simply the difference of two independent velocities. Thus, when two droplets are initially close to each other, if they do not promptly collide, they fly apart rapidly (with the erroneously overestimated relative velocity): the collision probability for later time becomes very small and, as a result, the calculated collision probability for two close droplets is reduced. On the other hand, if the initial separation is large (but still much smaller than the integral length scale), the falsely exaggerated relative velocity gives

TABLE 1. Collection kernels from models 1, 2, 3 for droplets of radii 50 μm and 100 μm in strong turbulence ($\sigma_v = 2.0$ m s^{-1}; $\epsilon = 0.1$ m^2 s^{-3}). The pure gravitational collection kernel for this case is 0.02827. The unit is 10^{-6} m^3 s^{-1}.

Model	Model 1	Model 2	Model 3
Collection kernel	0.03332	0.03464	0.05852
Enhancement factor[a]	1.178	1.225	2.070

[a] Defined as the ratio of collection kernel to the pure gravitational collection kernel.

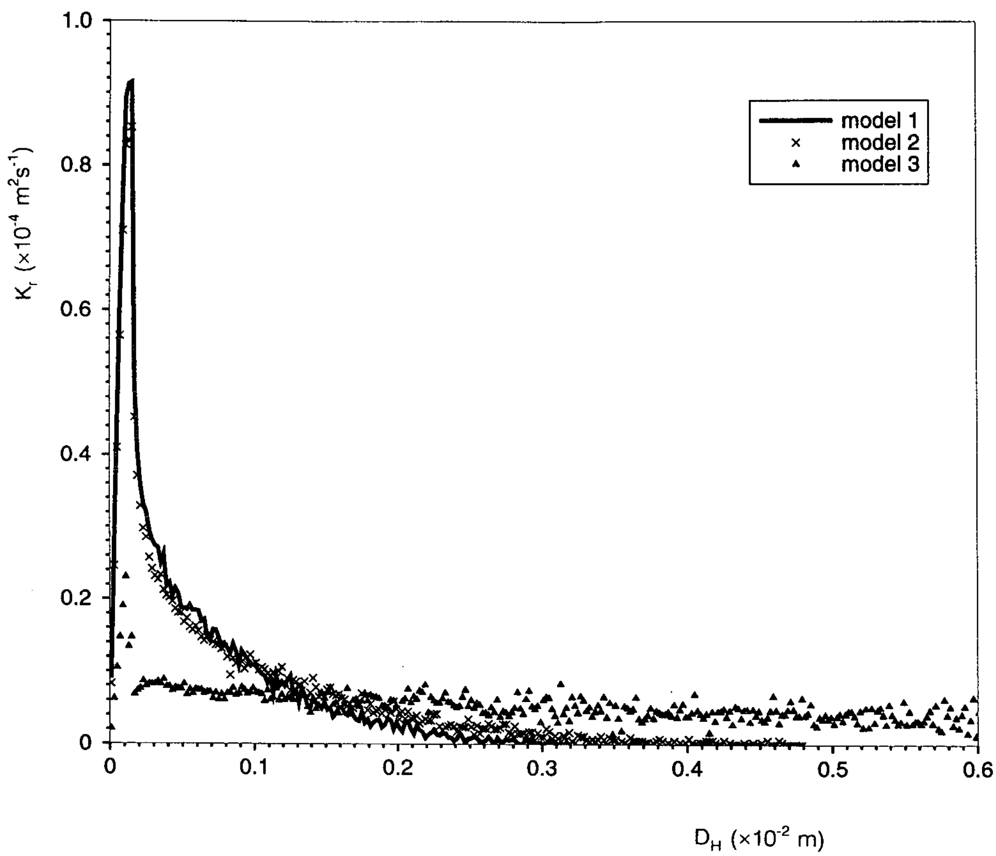

FIG. 1. Distribution of the collection kernel according to models 1, 2, 3 for droplets of radii 50 μm, 100 μm in strong turbulences ($\sigma_V = 2.0$ m s^{-1}; $\epsilon = 0.1$ m^2 s^{-3}).

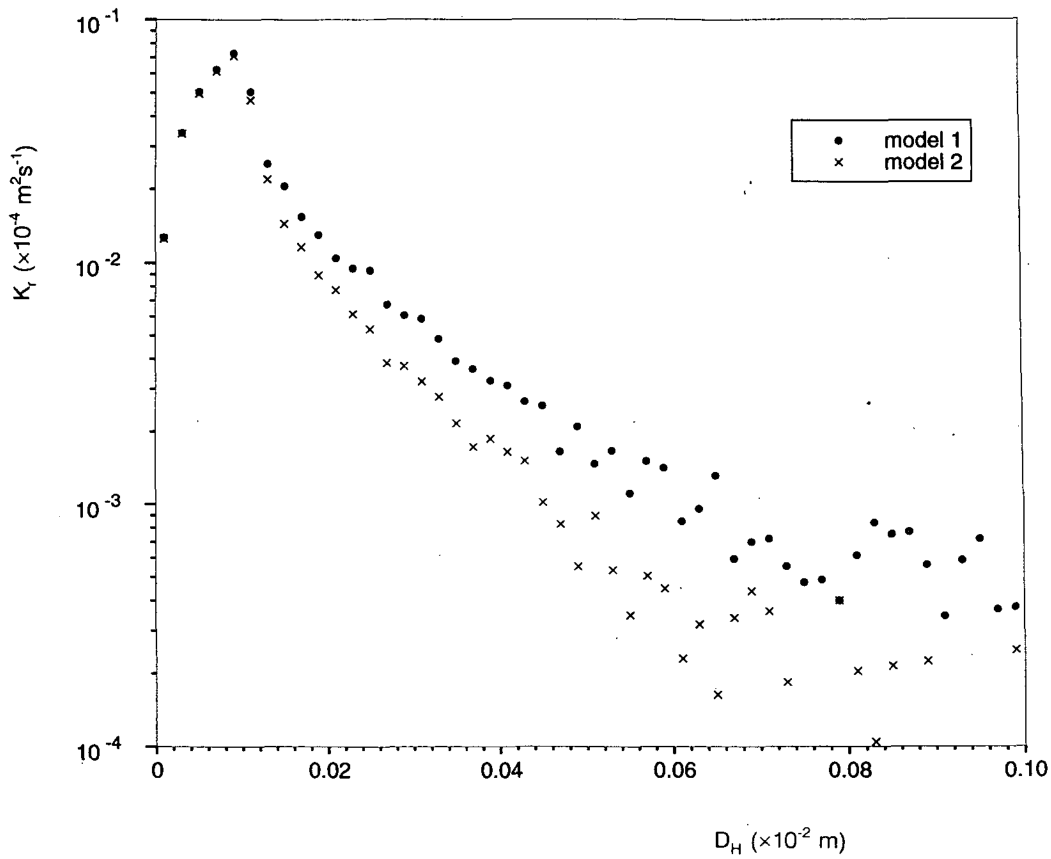

FIG. 2. Distribution of the collection kernel according to models 1, 2 for droplets of equal radii (50 μm) in strong turbulence.

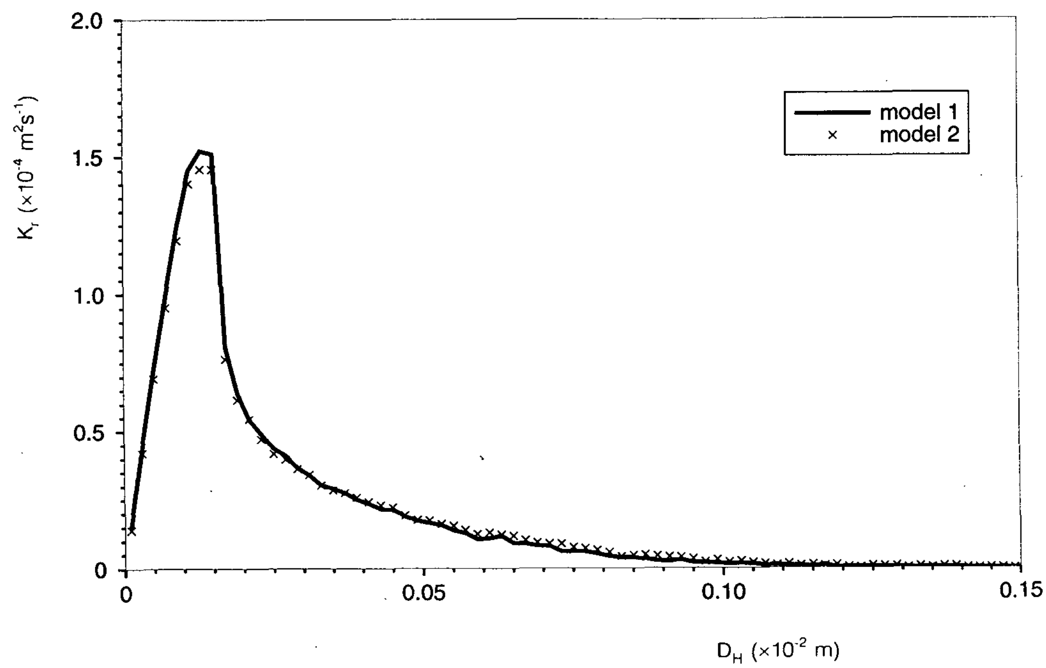

FIG. 3. Distribution of the collection kernel according to models 1, 2 for droplets of radii 50 μm, 100 μm in weak turbulence ($\sigma_V = 0.5$ m s^{-1}, $\epsilon = 0.01$ m^2 s^{-3}).

the droplets more opportunity to collide, and thus model 3 overestimates the collision probability for far more separated droplets. In their comment on the paper by Reuter et al. (1988), Cooper and Baumgardner (1989) argued that model 3 overestimated the turbulence effect. Our calculations confirm this, but model 3 does not overestimate the collision probability everywhere: for small horizontal separation model 3 *underestimates* the collision probability.

As we noted in this example, turbulence has a minor influence on the frequency of collisions between cloud droplets: the relative velocity due to turbulence is much smaller than that which would be caused by gravity alone. But in the case of the collision of two droplets of equal size, the collection kernel due to gravitational collision is zero, so turbulence accounts entirely for collisions (*absolute* movement of each cloud droplet is still strongly affected by gravity). The result of a simulation for the case of large droplets of equal size $r = 50\ \mu$m is given in Fig. 2. Model 2 generated a $K_r(R, r, D_H)$ that is smaller than that from model 1, in the range 0.015 cm $\leqslant D_H \leqslant$ 0.10 cm. For the full-collection kernel K, model 1 yielded $K = 0.94 \times 10^{-9}$ m^3 s^{-1}, whereas model 2 gave $K = 0.77 \times 10^{-9}$; model 2 underestimates the collection kernel by about 20%.

b. Large droplets, weakly turbulent cloud ($\sigma_V = 0.5$ m s^{-1}, $\epsilon = 0.01$ m^2 s^{-3})

It is less defensible to apply the present first-order LS model (model 1) in weak turbulence, so the following result will bear reexamination when better models are developed. As shown in Fig. 3 and Table 2, whether for droplets of different radii (50 and 100 μm) or of equal radii (50 μm), model 1 and model 2 gave (within numerical error) equal results for the collection kernel.

TABLE 2. Collection kernels from models 1, 2 for weak turbulence driving large droplets of (a) different radii (50, 100 μm) and (b) equal radii (50 μm). The unit is 10^{-6} m^3 s^{-1}.

	Model 1	Model 2
(a) Collection kernel (different size)	0.03236	0.03237
(a) Enhancement factor (different size)	1.1447	1.1450
(b) Collection kernel (identical size)	0.000465	0.000455

c. Small droplets, strongly turbulent cloud ($\sigma_V = 2$ m s^{-1}, $\epsilon = 0.1$ m^2 s^{-3})

In the case of small droplets (5, 10 μm), at small (large) horizontal separation, model 2 underestimated (overestimated) the collection kernel. Overall, model 2 substantially overestimated the impact of the turbulence: the collection kernels and enhancement factors (over gravity-driven collection) were

$$\text{model 1:}\quad K = 9.2 \times 10^{-11}\ \text{m}^3\ \text{s}^{-1}, \quad \text{e.f.} = 3.3$$

$$\text{model 2:}\quad K = 1.4 \times 10^{-10}\ \text{m}^3\ \text{s}^{-1}, \quad \text{e.f.} = 4.8.$$

Although model 1 is invalid when separations d between droplets are very small (so that $d \gg \eta$ does not hold), our comparison nevertheless suggests model 2 gives a bad prediction of the collection kernel for small droplets.

6. Conclusions

By considering the scales of motion in a cumulus cloud, in comparison to typical cloud droplet separations, we have suggested that to study the evolution of a full droplet spectrum one will require a second-order, multiparticle trajectory model. Rigorous models of that type are not yet available, and needed turbulence statistics (at the level of fluid element acceleration) are unknown; one must parameterize the spectral region between the dissipation and inertial subranges.

However, for large droplets in a very turbulent cloud, a first-order model will suffice. Using such a model, we have shown the need to account for both the temporal and spatial velocity correlations existing in the cloud over time and space scales relevant to droplet collisions.

Acknowledgments. The authors are grateful to Drs. B. L. Sawford and G. W. Reuter for reviewing this paper. The work was supported in part by the Natural Sciences and Engineering Research Council of Canada (NSERC) and the Atmospheric Environment Service (AES) of Environment Canada.

APPENDIX

Three Droplet Trajectory Models

Equations (5) of section 3 were used to calculate the trajectories of droplets 1, 2 in all three models, but the means to calculate U, the velocity of the air element surrounding the droplet, differs across the three models we have studied.

Model 1: The first-order two-particle model of section 3c. When the temporal correlation of the driving fluid element velocity between consecutive instants is neglected, model 1 reduces to model 2.

Model 2: The zeroth-order two-particle model. Spatial correlation between the two driving-parcel's velocities is taken into account, but the temporal correlation of each air parcel's velocity is not. The trajectory equations for the air parcels containing droplets 1, 2 are

$$U_1^i(t) = \theta_1^i(t)\,,$$
$$U_2^i(t) = \theta_2^i(t)\,, \qquad \text{(A1)}$$

where θ_1 and θ_2 are two spatially correlated random numbers given by the Kaplan–Dinar method. When spatial velocity correlation between the two moving air parcels is neglected, model 2 reduces to model 3.

Model 3: The zeroth-order single-particle model. Neither spatial correlation between the two parcel's velocities, nor temporal correlation of each air parcel's velocity, are accounted. The trajectory equations are

$$U_1^{\prime i}(t) = \xi_1^i(t)\,,$$
$$U_2^{\prime i}(t) = \xi_2^i(t)\,, \qquad \text{(A2)}$$

where ξ_1 and ξ_2 are independent random numbers, each with a zero mean and variance σ_v^2.

REFERENCES

Ackerman, B., 1967: The nature of the meteorological fluctuations in clouds. *J. Appl. Meteor.*, **6**, 61–71.

Borgas, M. S., and B. L. Sawford, 1994: A family of stochastic models for two-particle dispersion in isotropic, homogeneous and stationary turbulence. *J. Fluid Mech.*, **279**, 69–99.

Cooper, W. A., and D. Baumgardner, 1989: Comments on "The collection kernel for two falling cloud drops subjected to random perturbations in a turbulent air flow: A stochastic model." *J. Atmos. Sci.*, **46**, 1165–1167.

Csanady, G. T., 1963: Turbulent diffusion of heavy particles in the atmosphere. *J. Atmos. Sci.*, **20**, 201–208.

de Almeida, F. C., 1976: The collisional problem of cloud droplets moving in a turbulent environment. Part 1: A method of solution. *J. Atmos. Sci.*, **33**, 1571–1578.

——, 1979a: The collisional problem of cloud droplets moving in a turbulent environment. Part 2: Turbulent collision efficiencies. *J. Atmos. Sci.*, **36**, 1564–1576.

——, 1979b: The effect of small-scale turbulent motions on the growth of a cloud droplet spectrum. *J. Atmos. Sci.*, **36**, 1557–1563.

Du, S., B. L. Sawford, J. D. Wilson, and D. J. Wilson, 1995: Estimation of the Kolmogorov constant (C_0) for the Lagrangian structure function, using a second-order Lagrangian model for grid turbulence. *Phys. Fluids*, submitted.

Kaplan, H., and N. Dinar, 1988: A three-dimensional stochastic model for concentration fluctuation statistics in isotropic homogeneous turbulence. *J. Comput. Phys.*, **79**, 317–335.

Kogan, Y. L., 1993: Drop size separation in numerically simulated convective clouds and its effect on warm rain formation. *J. Atmos. Sci.*, **50**, 1238–1253.

Panchev, S., 1971: *Random Functions and Turbulence.* Pergamon, 444 pp.

Pruppacher, H. R., and J. D. Klett, 1978: *Microphysics of Clouds and Precipitation.* D. Reidel, 714 pp.

Rogers, R. R., and Yau, M. K., 1989: *A Short Course in Cloud Physics.* 3d ed. Pergamon, 293 pp.

Reuter, G. W., R. de Villiers, and Y. Yavin, 1988: The collection kernel for two falling cloud drops subjected to random perturbations in a turbulent airflow: A stochastic model. *J. Atmos. Sci.*, **45**, 765–773.

Saffman, P. G., and J. S. Turner, 1956: On the collision of drops in turbulent clouds. *J. Fluid Mech.*, **1**, 16–30.

Sawford, B. L., 1991: Reynolds number effects in Lagrangian stochastic models of turbulent dispersion. *Phys. Fluids*, **A3**, 1577–1586.

——, and F. M. Guest, 1991: Lagrangian statistical simulation of the turbulent motion of heavy particles. *Bound.-Layer Meteor.*, **54**, 147–166.

Tennekes, H., 1979: The exponential Lagrangian correlation function and turbulent diffusion in the inertial subrange. *Atmos. Environ.*, **13**, 1565–1567.

Thomson, D. J., 1987: Criteria for the selection of stochastic models of particle trajectories in turbulent flows. *J. Fluid Mech.*, **180**, 529–556.

Townsend, A. A., 1976: *The Structure of Turbulent Shear Flow.* 2d ed. Cambridge University Press, 429 pp.

Weil, J. C., R. P. Lawson, and A. R. Rodi, 1993: Relative dispersion of ice crystals in seeded cumuli. *J. Appl. Meteor.*, **32**, 1055–1073.