EAS270, "The Atmosphere"   1st Mid-term Exam   5 Oct. 2015

Professor: J.D. Wilson   Time available: 50 mins   Value: 25%

No formula sheets; no use of tablet computers etc. or cell phones. **Formulae and data are provided at the back**. Please record your multichoice answers on the scantron sheet. Respond to calculation questions on the pages provided: **add your name**, tear off and submit. (You may keep other pages of this exam.)

A. Multi-choice (32 x 1/2 → 16 %) [Median score 19/32, range 8–28.]

1. Suppose you are at a height in Earth's atmosphere where pressure $P = 500$ hPa, and let m [$\mathrm{kg\,m^{-2}}$] designate the total atmospheric mass per unit surface area lying above this level. Which is the best estimate for m?
   - (a) 1
   - (b) 5
   - (c) 1000
   - (d) 5000 ✓✓ 33% correct. Q1, Quiz 1 2014. Sec 1.5 (and given equation). $50,000\,[\mathrm{Pa}] = m\,g \approx 5000 * 9.81$
   - (e) 50,000

2. Which statement about the troposphere is **false**?
   - (a) its depth is of order 100 km ✗✗ 87% correct. Fig 1.7
   - (b) it is considered to be well mixed on a time scale of the order of days
   - (c) it is the layer whose variability manifests as (or controls) "weather"
   - (d) its lowest 1 km or so constitutes the atmospheric "boundary layer"
   - (e) on average, its temperature decreases with increasing elevation

3. Which combination lists the most abundant gas in earth's homosphere, and its proportion?
   - (a) O_2, 92%
   - (b) O_2, 78%
   - (c) Ar, 78%
   - (d) N_2, 99%
   - (e) N_2, 78% ✓✓ 97% correct. Table 2.1

4. The Martian atmosphere is composed mostly of CO_2. Neglecting all other gases, and taking the surface pressure on Mars to be 7 hPa, what is the surface density ρ if surface temperature is $T = -53^o$C? (The specific gas constant for CO_2 is $R = 189\,\mathrm{J\,kg^{-1}\,K^{-1}}$.)
   - (a) $0.00017\,\mathrm{kg\,m^{-3}}$
   - (b) $0.017\,\mathrm{kg\,m^{-3}}$ ✓✓ 70% correct. Q2, midterm 2014. $\rho = 700/(189 \times (273.15 - 53))$.
   - (c) $-0.0007\,\mathrm{kg\,m^{-3}}$
   - (d) $0.1\,\mathrm{kg\,m^{-3}}$
   - (e) $1\,\mathrm{kg\,m^{-3}}$

5. Isolines of which "secondary field" are plotted on the CMC isobaric charts for the 500 hPa level?
   - (a) MSLP
   - (b) height of the 500 hPa surface
   - (c) isohumes (isolines of relative humidity)
   - (d) 1000-500 hPa thickness (DZ) ✓✓ 58% correct. Q3 midterm 2014. Table 3.2; Lectures of 11, 21 Sept.
   - (e) temperature-dewpoint spread

6. Which of the statements below in relation to the atmospheric pool is **false**?
   - (a) plant respiration is sink for O_2 while animal respiration is a source ✗✗ 99% correct. Q5, quiz 1 2014.
   - (b) combustion of organic material ($CH_2O + O_2 \rightarrow CO_2 + H_2O$) is a sink for O_2 and a source for CO_2
   - (c) anaerobic decomposition is a source for CH_4
   - (d) volcanic eruptions are a source for CO_2
   - (e) photosynthesis ($CO_2 + H_2O \rightarrow CH_2O + O_2$) is a source for O_2 and a sink for CO_2

7. If the temperature and virtual temperature of an air sample are respectively $T = 298$ K and $T_v = 300$ K, what is the mixing ratio r of the sample in the [kg/kg] unit? [Note: $T_v = T\,(1 + 0.61\,r)$, with temperatures in Kelvin and r in kg/kg].
   - (a) 0.011 ✓✓ 84% correct
   - (b) 0.11
   - (c) 1.1
   - (d) 11

8. Which statement in regard to atmospheric aerosols is **false**?

   (a) gravitational settling rapidly removes aerosols of radius $R > 10\,\mu$m
   (b) precipitation scavenging is an important mechansim for cleaning the air of fine aerosols (radius $R < 10\,\mu$m)
   (c) aerosol "number density" (measured in the unit m^{-3}, i.e. count per unit volume) is larger in marine air than in continental air ✗✗ 51% correct. Sec 2.9. Slide 8 Ch2-C.
   (d) secondary aerosols are formed when precursor gases react in the atmosphere
   (e) aerosols interact with radiation, and serve as surfaces on which water vapour may condense

9. Which statement is **false**?

   (a) "heat energy" is synonymous with (has the same meaning as) the "internal energy" of a system ✗✗ 34% correct. Marginal definition, p69.
   (b) some types of internal energy do not participate (i.e. do not undergo transformation) in natural atmospheric processes
   (c) Einstein's mass-energy equivalence law $E = m\,c^2$ is irrelevant for the budgeting of heat energy in the atmosphere
   (d) temperature is a measure of (i.e. proportional to) the average kinetic energy of air molecules
   (e) added (or subtracted) heat energy that produces no change in a system's temperature is called "latent heat," and is associated with the breakage (or formation) of inter-molecular bonds that accomplishes a phase change in the system's state

10. A parcel of dry air at the surface ($z = 0$ m) has a temperature of 20°C. It is lifted adiabatically to $z = 700$ m then sinks adiabatically to $z = 500$ m. What is its final temperature?

    (a) 29°C
    (b) 27°C
    (c) 25°C
    (d) 20°C
    (e) 15°C ✓✓ 81% correct. Slide 9, Ch4-A.

11. Suppose 0.5 kg of dry air is contained in a rigid volume of 2 cubic metres. Based on the 1st law of thermodynamics in the form

$$\Delta q\,[\mathrm{J\,kg^{-1}}] = c_v \Delta T + P\Delta\alpha \tag{1}$$

where $\alpha \equiv 1/\rho$ is the specific volume, which value best approximates the change in temperature (final minus initial) if 10^4 J of energy are added to this system? (Assume $c_v = 718\,\mathrm{J\,kg^{-1}\,K^{-1}}$. The "system" is the 0.5 kg air sample.)

(a) 0 K
(b) −14 K
(c) +14 K
(d) +28 K ✓✓ 48% correct. Q4, midterm 2014. $\Delta\alpha = 0$, thus: $10^4/0.5\,[\mathrm{J\,kg^{-1}}] = c_v\,\Delta T$.
(e) +56 K

12. Defining the volumetric sensible heat content of a parcel of air as $h = \rho c_p T\,[\mathrm{J\,m^{-3}}]$ and taking the ρc_p product as $10^3\,[\mathrm{J\,m^{-3}\,K^{-1}}]$, what is the value of the convective heat flux density Q_x $[\mathrm{J\,s^{-1}\,m^{-2}}]$ if the x-component of the wind is $u = 5\,\mathrm{m\,s^{-1}}$ and the temperature of the parcel is 300 K? (Shortcut: consider units).

(a) $1.5\times10^6\,\mathrm{W\,m^{-2}}$ ✓✓ 63% correct. $Q_x = u\,h$, slide 14, Ch4-B. [or, choose correct units]. Q6, midterm 2014.
(b) 3×10^5 K
(c) $120\,\mathrm{W\,m^{-3}}$
(d) $120\,\mathrm{W\,m^{-2}\,s^{-1}}$
(e) $1500\,\mathrm{J\,m^{-3}}$

13. Taking the conductivity of glass as $k = 1\,[\mathrm{W\,m^{-1}\,K^{-1}}]$, what is the rate of conductive heat transfer across a window pane of area $1\,\mathrm{m^2}$ and thickness $\Delta x = 2.5$ mm when the temperature difference across the pane is 4K?

(a) 1600 J
(b) 1600 W ✓✓ 73% correct. Q7 midterm 2014. Slide 6, Ch4-B
(c) $160\,\mathrm{W\,s^{-1}}$
(d) 100 W
(e) 10 W

14. Suppose a parcel of air at the surface had state $(P_1, T_1) = (930\,\text{hPa}, 285.15\text{K})$. What is the parcel's temperature after it has been lifted adiabatically to the 850 hPa level? (Hint: Poisson's equation).

   (a) 11.7°C
   (b) 12.3°C
   (c) 278 K ✓✓ 87% correct. Slide 3, Ch4-B.
   (d) 293 K

15. A commercial jet is flying at an altitude where the air pressure is 20 kPa and the temperature is 213.15 K (or −60°C). If exterior air is brought inside the aircraft and pressurized (adiabatically) to 80 kPa, what will be its temperature? (Again, Poisson's equation).

   (a) 143.4°K
   (b) 317 K (or 43.7°C) ✓✓ 72% correct. [Only answer with correct sign for temp. change]
   (c) −89.2°C
   (d) −129.8°C

16. The daily temperature cycle is sometimes idealised as a "wave", with maximum surface temperature occurring at (say) time $t = t_{0,\text{max}}$ and with the surface temperature swinging through a daily range ΔT_0 (warmest temperature minus coldest temperature). Which statement regarding soil heat transfer and the wave in temperature at some depth d is **false**?

   (a) the daily temperature range ΔT_d at depth d obeys $\Delta T_d > \Delta T_0$ ✗✗ 67% correct. Quest. C1, midterm 2014.
   (b) the dominant mechanism for soil heat transfer is usually conduction
   (c) the daily temperature maximum at depth d occurs later than $t_{0,\text{max}}$

17. Which is the closest estimate for the vertical gradient in pressure $[\text{Pa}\,\text{m}^{-1}]$ at sea level? Question ought to have specified the **magnitude** of the vertical gradient, for strictly — and according to the conventions we've used — the gradient is negative. That oversight may explain the low success rate.

   (a) 100
   (b) 10 ✓✓ 19% correct. Sec 3.4. RHS of hydrostatic eqn, $\rho\, g$. Slide 7, Ch3-A.
   (c) 1
   (d) 0.1
   (e) 0.01

18. For moist air, the ideal gas law reads $P = \rho\, R_d\, T_v$ where R_d is the gas constant for dry air and T_v is the virtual temperature. Suppose water vapour is added to a sample ("A") of previously dry air whose state was (P, T), and then the resulting volume (sample "B") is adjusted back to the original pressure and temperature. Which statement is correct? "Relative to sample A, sample B has ...

   (a) higher virtual temperature but lower density ρ" ✓✓ 21% correct. Sec 3.3. Initially, $T_v \equiv T$ (dry air). P unchanged, so ditto $\rho \times T_v$. T is unchanged, but $T_v > T$ (vapour added). Hence ρ has decreased.
   (b) higher virtual temperature and higher density ρ"
   (c) lower virtual temperature and lower density ρ"
   (d) lower virtual temperature and higher density ρ"

19. Suppose a certain unsaturated layer of the troposphere were "well mixed". Which would best approximate its temperature stratification?

   (a) temperature unchanging with increase or decrease in height
   (b) temperature increasing by 1 degree Celcius per 100 m increase in height
   (c) temperature decreasing by 1 degree Celcius per 100 m increase in height ✓✓ 76% correct. Slide 10, Ch4-A.
   (d) temperature increasing by 1 degree Celcius per 1000 m increase in height
   (e) temperature decreasing by 1 degree Celcius per 1000 m increase in height

20. Consider a uniform, dry, bare soil illuminated by the summer sun, and suppose that at the upper and lower surfaces of a horizontal layer of the soil the values of the soil heat flux ("Q_z [$\mathrm{W\,m^{-2}}$]", positive for downward heat flow) were respectively

$$Q_z(z_1) = 25\ \mathrm{W\,m^{-2}}\ ,$$
$$Q_z(z_2) = 20\ \mathrm{W\,m^{-2}}\ .$$

   Adopting the 'symmetry assumption' that soil temperature $T = T(z, t)$, which statement regarding the layer $z_1 \leq z \leq z_2$ is correct?

   (a) insufficient information is given to determine whether this soil layer is warming or cooling
   (b) Fourier's law of conduction implies that there must be horizontal heat flows Q_x, Q_y in the layer
   (c) this soil layer must be cooling
   (d) this soil layer must be warming ✓✓ 37% correct. Ch4-B. Lecture of 25/28 Sept. Caption of figure in 'Equations used' file.

21. The instantaneous vertical flux of sensible heat may be expressed as the sum of convective and conductive contributions

$$Q = \rho\, c_p\, w\, T \;-\; k\, \frac{\Delta T}{\Delta z}\ ,$$

where w is the vertical wind speed, T is the temperature and $\Delta T/\Delta z$ its vertical gradient, k is the conductivity of air, and ρ, c_p have their usual meanings (air density and specific heat capacity). Which statement is **false**?

(a) the term involving the $w \times T$ product gives the contribution to the flux due to bulk movement, ie. wind (the "convective" contribution)
(b) the term involving the $\Delta T/\Delta z$ gives the contribution to the flux due to molecular motion (i.e. the "conductive" contribution)
(c) the $w \times T$ term vastly exceeds (in magnitude) the $\Delta T/\Delta z$ term in the bulk of the troposphere
(d) adjacent to solid surfaces (the ground, leaves, etc.) the $w \times T$ term is suppressed, and the $\Delta T/\Delta z$ term dominates (laminar boundary layer)
(e) the quantity $\rho\, c_p\, T$ has the unit [J] ✗✗ 39% correct

22. People hiking or climbing in the wilderness may carry a lightweight aluminized sheet or bag or foil, that can be used to conserve body heat when sleeping in cold conditions. What characteristic of the material makes it useful in this regard?

(a) its thermal conductivity, which is low
(b) its thermal conductivity, which is high
(c) its high infrared emissivity
(d) its high infrared absorbtivity
(e) its low infrared emissivity ✓✓ 8% correct. Table 5.1

23. Referring to Figure (1), select the correct pair of values for the temperature lapse rate in respectively the lower and the upper layers. (Note: a temperature *change* of 1°C is the same as a change of 1 K).

(a) $-0.01\ \mathrm{K\,m^{-1}}$, $-0.005\ \mathrm{K\,m^{-1}}$
(b) $-0.01\ \mathrm{K\,m^{-1}}$, $-0.002\ \mathrm{K\,m^{-1}}$ ✓✓ 40% correct. Read $\Delta T/\Delta z$ off graph.
(c) $-0.02\ \mathrm{K\,m^{-1}}$, $-0.002\ \mathrm{K\,m^{-1}}$
(d) $-0.02\ \mathrm{K\,m^{-1}}$, $-0.005\ \mathrm{K\,m^{-1}}$

24. If a certain body has longwave emissivity $\epsilon = 0.96$ and its temperature is $T = 18^o$C, which answer below most closely states the wavelength λ_{max} of the peak in its thermal emission spectrum and its full-spectrum rate of emission E of longwave radiation?

    (a) $10\,\mu$m, $6 \times 10^{-3}\,\text{W m}^{-2}$
    (b) $100\,\mu$m, $390\,\text{W m}^{-2}$
    (c) $10\,\mu$m, $390\,\text{W m}^{-2}$ ✓✓ 70% correct. Sec 5.2 & as per slide 6, Ch5-A.
    (d) $1\,\mu$m, $410\,\text{W m}^{-2}$
    (e) $0.1\,\mu$m, $410\,\text{W m}^{-2}$

25. Suppose a pool (or reservoir) for "X" contained $M = 10^8$ [kg] of this species, and that there was a balance between the fluxes Q_{in}, Q_{out} of X into and out of the pool, with $Q_{\text{in}} = Q_{\text{out}} = 10^{-4}$ [kg s^{-1}]. What is the residence time for X in this pool?

    (a) 10^{-4} s
    (b) 10^4 s
    (c) 10^{-4} s $+ 10^8$ s
    (d) 10^{12} s ✓✓ 37% correct. $\tau = M/Q$. Sec 2.2, Eq 2.2

For the remaining questions, please refer to Figures (2 – 3).

26. Referring to Figure 2, this analysis relates to what level of the atmosphere?

    (a) surface
    (b) 850 hPa
    (c) 700 hPa ✓✓ 30% correct. Lec. 25 Sept.
    (d) 500 hPa
    (e) 250 hPa

27. Referring to Figure 2, the flow is visualized at (roughly) which height above sea level?

    (a) 1.5 km
    (b) 3 km ✓✓ 61% correct
    (c) 5.5 km
    (d) 10 km

28. Referring to Figure 2, which is the correct description of the wind over Central Alberta (Stony Plain radiosonde)?
   (a) calm
   (b) SW, about $2\,\mathrm{m\,s^{-1}}$
   (c) NE, about $2\,\mathrm{m\,s^{-1}}$
   (d) SW, about $10\,\mathrm{m\,s^{-1}}$ ✓✓ 79% correct
   (e) NE, about $10\,\mathrm{m\,s^{-1}}$

29. Referring to Figure 2, the solid contours are isolines of which property?
   (a) height of the isobaric surface ✓✓ 76% correct
   (b) temperature
   (c) Greenwich Mean Time
   (d) wind speed
   (e) wind direction

30. Referring to Figure 2, what property of the air is represented by the stippling pattern?
   (a) height of the isobaric surface
   (b) temperature
   (c) thickness of the 1000-500 hPa layer
   (d) small temperature-dewpoint spread (high humidity) ✓✓ 46% correct

31. Referring to Figure 2, which statement in regard to the low at the lower right of the map (south of Manitoba) is **false**?
   (a) it is a closed upper low
   (b) its central pressure is 315 hPa ✓✓ 34% correct
   (c) the wind blows anticlockwise around its centre
   (d) it is liable to be a slow moving feature

32. Referring to the location marked "**X**" on Figure 3, which statement is **false**?
   (a) the heavy dashed line just N of the spot is the 0°C isotherm
   (b) the lighter dashed line just S of the spot is the +5°C isotherm
   (c) it is liable to get warmer at **X** in the following few hours ✗✗ 67% correct
   (d) the solid lines are isolines of the height of the 850 hPa surface (i.e. height contours)
   (e) at the centre of the low (NE of **X**) the isobaric surface drapes down to a local minimum height of 115 dam ASL

Continue to Calculations on the next THREE pages (3 x 3 → 9 %).

B. Computations (3 x 3 → 9 %)

Show your working, as you may receive part marks even if your answer is wrong.

B1. Compute the amount of energy a freezer must remove in order for 1 kg of water at 20°C to be made into a block of ice at −15°C. The specific heat of water is $c_w = 4186\,[\mathrm{J\,kg^{-1}\,K^{-1}}]$; for ice $c_i = 2100\,[\mathrm{J\,kg^{-1}\,K^{-1}}]$; and the latent heat of freezing is $L_f = 3.34 \times 10^5\,[\mathrm{J\,kg^{-1}}]$. Give your answer to three significant digits[1], and state its units.

$$
\begin{aligned}
Q &= (L_f + c_w \times 20 + c_i \times 15) \times 1 \;\; [\mathrm{J/kg} \times \mathrm{kg}] \\
&= 3.34 \times 10^5 + 4186 \times 20 + 2100 \times 15 \\
&= 3.34 \times 10^5 + 0.8372 \times 10^5 + 0.315 \times 10^5 \\
&= 4.4922 \times 10^5\ \mathrm{J} \quad [\text{5 sig. digits, no penalty}] \\
&= 4.49 \times 10^5\ \mathrm{J} \qquad [\text{3 sig. digits}]
\end{aligned}
$$

(Similar to Example 4.4 p72, and to Problem 3 p85, in textbook. Accepted 5 significant digits without penalty, owing to misleading footnote.)

Comments: many students added 273.15 K to the temperature *changes*, such that the quantity of energy needing to be removed to cool 1 kg of water from 20°C to the freezing point 0°C was erroneously evaluated as

$$Q_{\text{cool water}}\,[\mathrm{J}] = 1\,[\mathrm{kg}] \times 4186\,[\mathrm{J\,kg^{-1}\,K^{-1}}] \times 293.15\,\mathrm{K}$$

(the 293.15 K should have simply been 20 K— one mark penalty). Others forgot the latent heat term (half mark penalty).

[1] e.g. 1.23 or 123 or 0.0123 or 1.23×10^{-2} (example giving five sig. digits was corrected to three during exam).

B2. Suppose that the ground-level pressure observed in Fort Smith (204 m ASL) was 978 hPa and that the corresponding sea-level corrected value ("MSLP") entered on the surface analysis was 1002.7 hPa (which would be coded as 027). What value had been assumed for the average virtual temperature $\overline{T}_v$ of the (fictitious) 0-204 m layer at the station? Give your answer in Kelvin with five significant digits.

Correct answer: The hypsometric equation reads

$$\Delta z = z_2 - z_1 = \left[\frac{R_d \, \overline{T}_v}{g} \right] \, \ln \frac{P_1}{P_2} \, ,$$

where if $z_2 > z_1$ then $P_2 < P_1$. Therefore we write

$$\begin{aligned} \Delta z = 204 \text{ [m]} \quad &= \quad \frac{R_d}{g} \times \ln \left(\frac{1002.7}{978} \right) \times \overline{T}_v \\ &= \quad \frac{287}{9.81} \times \ln(1.02526) \times \overline{T}_v \\ &= \quad 29.2559 \times \left(2.49420 \times 10^{-2} \right) \times \, \overline{T}_v \\ &= \quad 0.729699 \times \overline{T}_v \, . \end{aligned}$$

Solving for the mean virtual temperature,

$$\overline{T}_v \;=\; 279.57 \text{ K} \, .$$

(See Section 3.5 and Slide 8 in the file Ch3-B.)

Comments: a number of students tried to manipulate Poisson's eqn., but this does not involve the heights (0, 204) m or the height difference 204 m. Other students extracted an effective mean value for density ρ from the given pressure difference (2470 Pa) over the given height interval (204 m), then popped this, along with the *average* pressure (990.35 hPa) into the ideal gas law; this was creative, and gained part marks.

B3. The total flux of sensible heat along the vertical axis z may be written[2]

$$Q\ [\mathrm{W\,m^{-1}}] \;=\; \rho\, c_p\, w\, T \;-\; k\,\frac{\Delta T}{\Delta z}$$

where the first term is the convective contribution (usually by far the dominant part), and the second term is the conductive contribution. Compute a **representative value**[3] for the ratio of the magnitudes of the terms contributing to the heat flux (conductive term divided by convective term), for a layer near sea level ($\rho c_p \approx 10^3\,\mathrm{J\,m^{-3}\,K^{-1}}$) in which the vertical wind $w = +0.05\ [\mathrm{m\,s^{-1}}]$, the temperature $T \approx 300$ K and the temperature lapse rate $\Delta T/\Delta z$ is the dry adiabatic rate (take the molecular conductivity of air to be $k = 0.025\ [\mathrm{W\,m^{-1}\,K^{-1}}]$).

Correct answer:

$$\begin{aligned} R &= \frac{|k \times \Delta T/\Delta z|}{|\rho\, c_p\, w\, T\,|} \\ &= \frac{0.025 \times 0.01}{10^3 \times 0.05 \times 300} \\ &= 1.67 \times 10^{-8} \;\sim\; 2 \times 10^{-8} \;\sim\; 10^{-8} \end{aligned}$$

Comments: It was assumed you would use $\Delta T/\Delta z = -0.01\ [\mathrm{K\,m^{-1}}]$ for the adiabatic lapse rate, this being one of those numbers listed as 'to be known' (and a number which is used elsewhere in the exam). Some students gave the ratio of convection to conduction (instead of conduction-to-convection), viz.

$$R^{-1} \approx 6 \times 10^7.$$

This was accepted without penalty. Other students computed the two contributions individually, and evaluated the total flux:

$$\begin{aligned} Q_{\mathrm{cond}} &= -0.025 \times -0.01 = 2.5 \times 10^{-4}\ [\mathrm{W\,m^{-2}}] \\ Q_{\mathrm{conv}} &= 10^3 \times 0.05 \times 300 = 1.5 \times 10^4\ [\mathrm{W\,m^{-2}}] \\ Q &= 15000.00025\ [\mathrm{W\,m^{-2}}]. \end{aligned}$$

This is not what was asked for, but gained part marks: if either component was evaluated correctly a score of 1.5 was given.

[2] Note added after exam: unit stated for Q ought to have been $\mathrm{W\,m^{-2}}$.

[3] Note added after exam: the intended interpretation of "representative value" was that you would provide an order-of-magnitude estimate rather than an exact value. This amounts to an invitation to take the magnitude of the adiabatic lapse rate as $0.01\,\mathrm{K\,m^{-1}}$ rather than labour to evaluate g/c_p.

Equations and Data.

- $P = M\,g/A$, the pressure (P, Pa) that results when mass M [kg] overlies area A [m^2], where $g = 9.81$ [m s^{-2}]
- one full barb on the wind vector means 5 m s^{-1}, a solid triangle 25 m s^{-1}.
- $\frac{\Delta P}{\Delta z} = -\,\rho\,g$. ΔP [Pascals], the change in pressure as one ascends a distance Δz [m]; ρ [kg m^{-3}] the air density; $g = 9.81$ [m s^{-2}] grav. acceleration.
- $P = \rho\, R_d\, T_v$. The ideal gas law. P [Pascals], total pressure; ρ, [kg m^{-3}] the total density; T_v [Kelvin], the virtual temperature; and $R_d = 287$ [J kg^{-1} K^{-1}], the specific gas constant for dry air.
- $T_v = T\,(1 + 0.61\,r)$. Defines the virtual temperature [K] of an air sample whose temperature is T [K] and whose water vapour mixing ratio is r [kg/kg].
- $\Delta q = c_v\,\Delta T + P\,\Delta\alpha$. The first law of thermodynamics, linking changes in the state of a sample of air: Δq [J kg^{-1}] is energy added to the system (zero for an adiabatic process, by definition), c_v is the specific heat capacity of the material at constant volume, P is the pressure and $\alpha \equiv 1/\rho$ is the specific volume (ρ being air density). For an ideal gas $c_v = (5/2)R$, so that for dry air $c_v \approx 718$ J kg^{-1} K^{-1}.
- $$\frac{T}{T_1} = \left(\frac{P}{P_1}\right)^{R/c_p}$$

  Poisson's law linking two states (P, T) and (P_1, T_1) of a sample of ideal gas, assuming the process connecting the two states is adiabatic ($R/c_p = 2/7 = 0.286$).
- $\frac{\Delta T}{\Delta z} = -\,g/c_p$. The dry adiabatic lapse rate, where g is the gravitational acceleration and $c_p \approx 1000$ [J kg^{-1} K^{-1}] is the specific heat at constant pressure.
- $$Q_x = -\,k\,\frac{\Delta T}{\Delta x}\ ,\ Q_y = -\,k\,\frac{\Delta T}{\Delta y}\ ,\ Q_z = -\,k\,\frac{\Delta T}{\Delta z}\,.$$

  Fourier's law of conduction, giving the components (Q_x, Q_y, Q_z) of the conductive heat flux density along directions (x, y, z) in response to temperature gradients $\Delta T/\Delta x$ (etc.) along the axes in a medium whose conductivity is k [W m^{-1} K^{-1}]. (The sign convention is that Q_x is positive for a flow of heat towards larger values of x, etc.)

- $E = \epsilon \, \sigma \, T^4$

  Stefan-Boltzmann law for a greybody. E [W m^{-2}], the emitted longwave energy flux density; ϵ, the emissivity of the surface (dimensionless); $\sigma = 5.67 \times 10^{-8}$ [W m^{-2} K^{-4}], the Stefan-Boltzmann constant; T [K], the surface temperature.

- $\lambda_{\text{max}} = \frac{2897}{T}$

  Wien's displacement law. λ_{max} [μm], the wavelength at which the peak in the emission spectrum occurs; T [K], the temperature of the emitting surface.

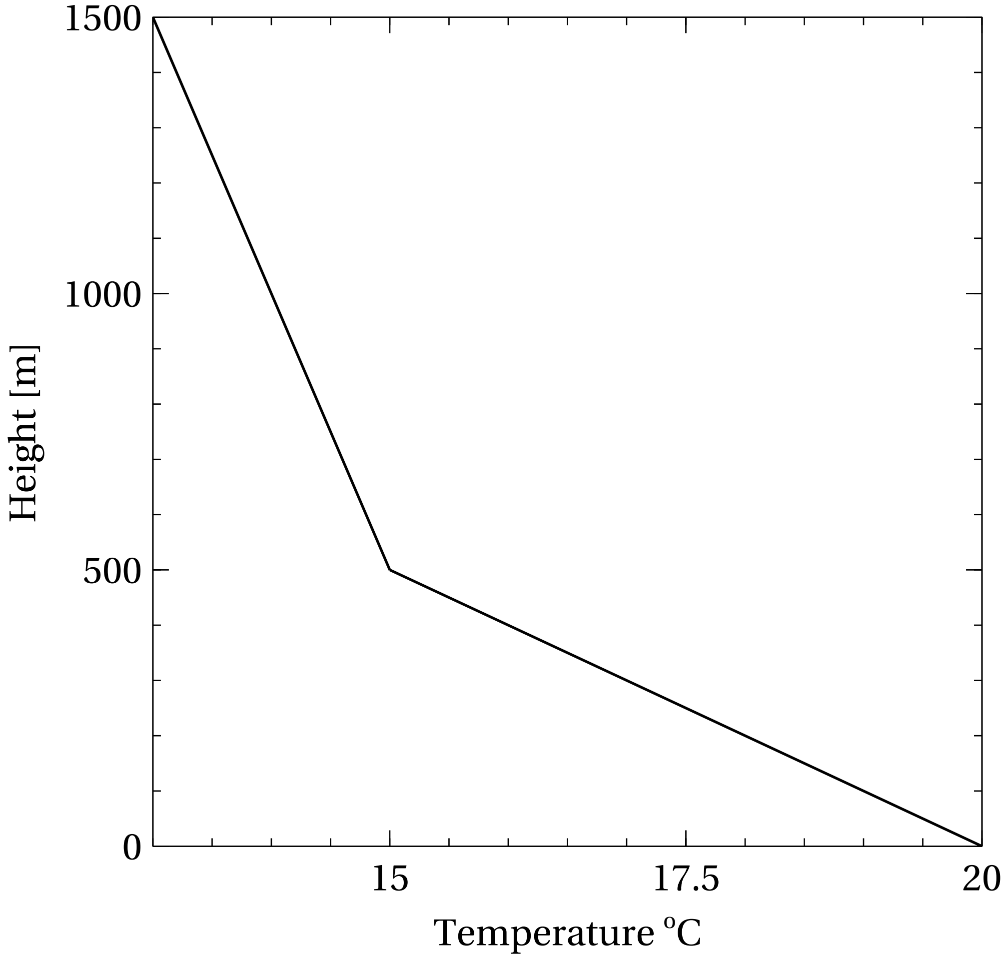

Figure 1: Idealized profile of temperature in the lowest 1500 m of the atmosphere. The "elbow" is at $(z, T) = (500\,\mathrm{m}, 15^{\circ}\mathrm{C})$. Stated during exam and added here with answers: left end of T axis is 13°C.

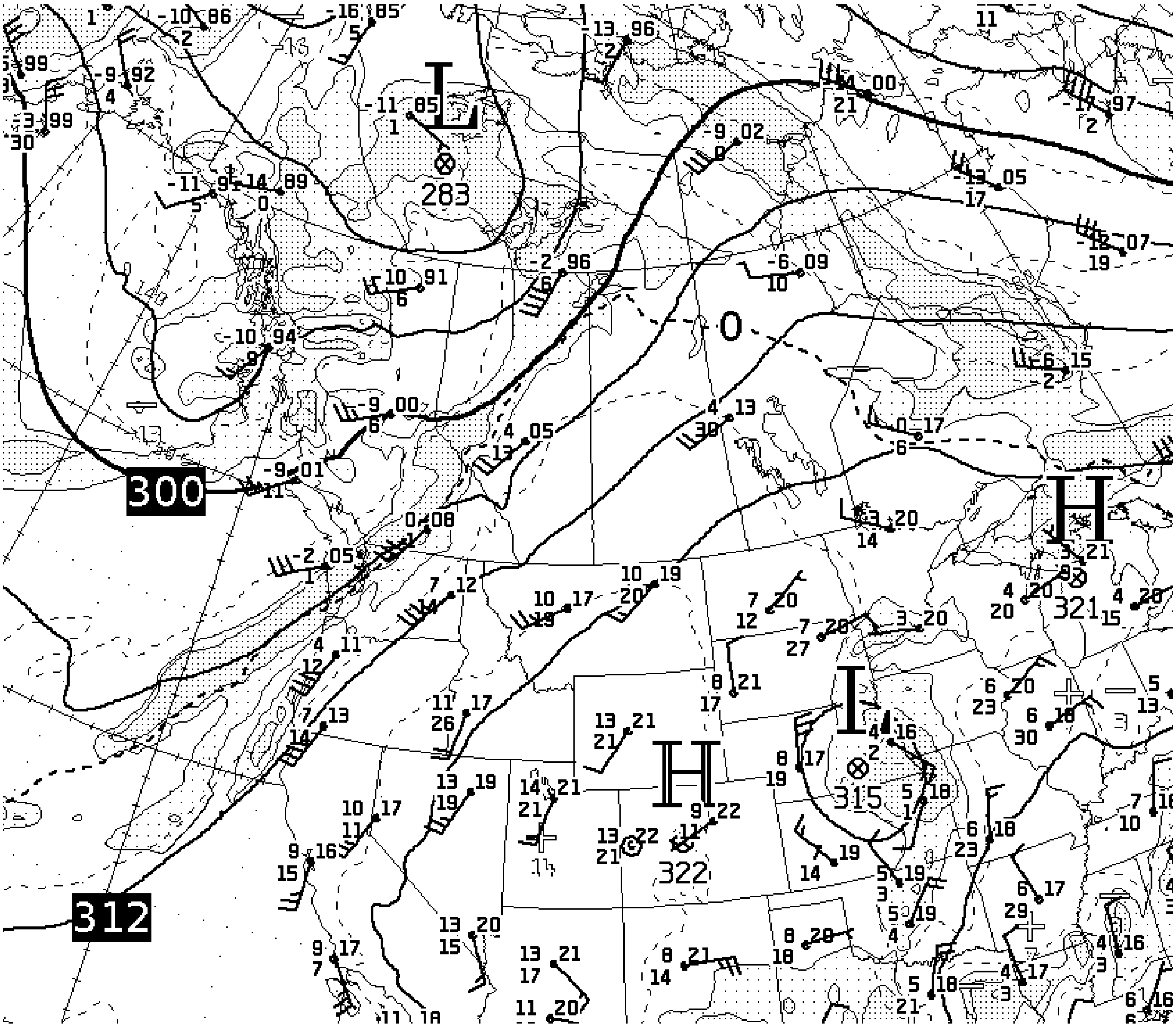

Figure 2: Environment Canada analysis (cropped) for 12 UTC 25 September 2015. Dashed lines are isotherms, with 5°C interval.

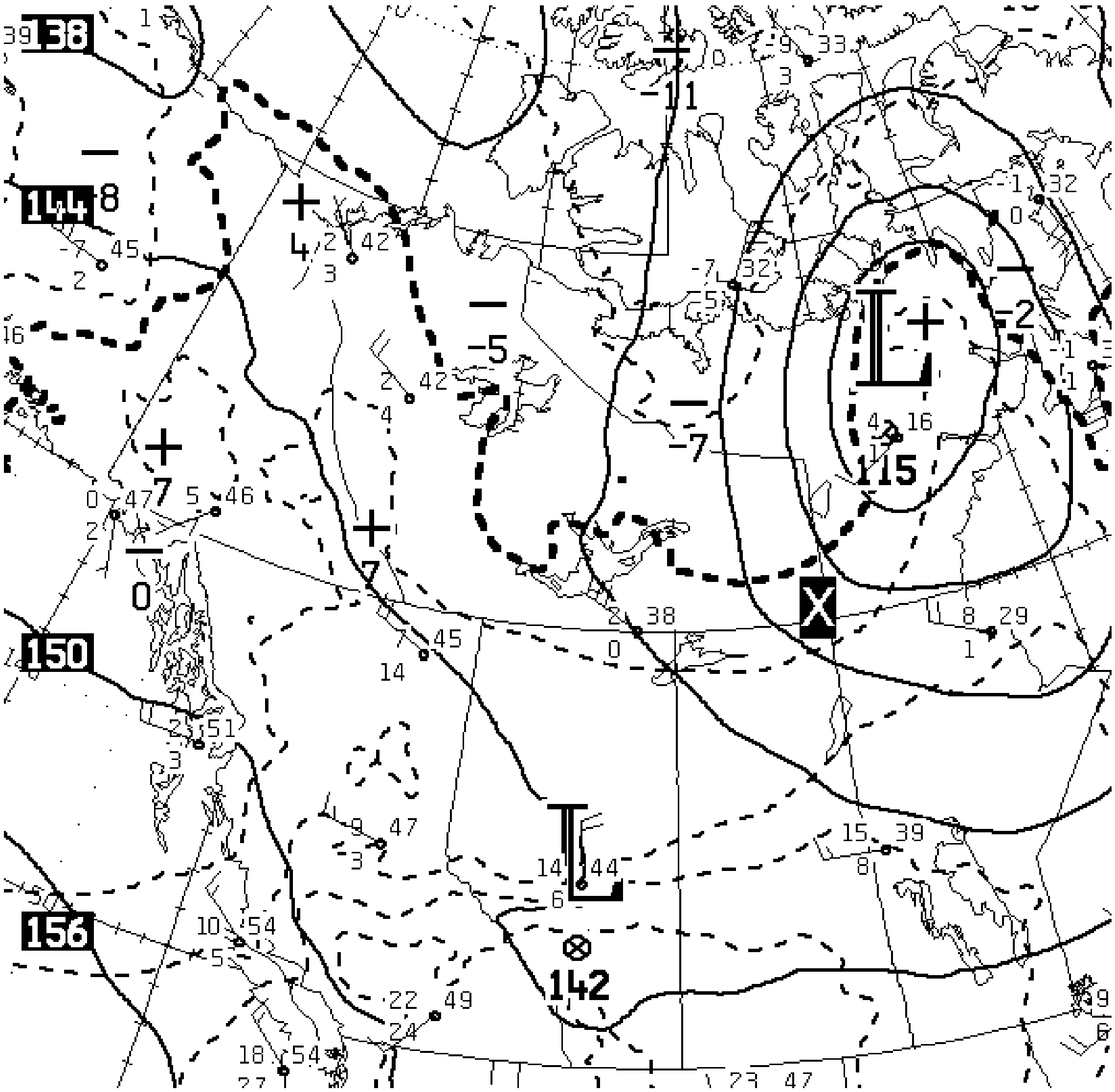

Figure 3: Environment Canada 850 hPa analysis (cropped) for 00 UTC 13 September 2015. Height contours are at 6 dam interval.