EAS270, "The Atmosphere" 1st Mid-term Exam 3 Oct. 2016

Professor: J.D. Wilson Time available: 50 mins Value: 25%

No formula sheets; no use of tablet computers etc. or cell phones. **Formulae/data at back**.

Respond to part **B** (calculation questions) on the pages provided: **add your name**, tear off and submit. (You may keep other pages of the exam.)

A. Multi-choice (24 x 3/4 → 18 %)

1. Name the ground-based atmospheric layer that is considered well-mixed on a time scale of (roughly) one day (but longer in winter), and whose nominal depth is (about) 10-12 km?
   (a) atmospheric boundary layer
   (b) homosphere
   (c) stratosphere
   (d) tropopause
   (e) troposphere ✓✓ 85% crct. Fig 1.7; slide 4, Ch1; slide 9, Ch2-A; .

2. Which are the two most abundant "constant" (or "permanent") gases in earth's homosphere?
   (a) N_2, O_2 ✓✓ 97% crct.
   (b) H_2O, N_2
   (c) N_2, CO_2
   (d) O_2, CO_2
   (e) Ar, CO_2

3. Which alternative MKS unit is **not** a correct decomposition of the Watt (i.e. MKS unit for power)?
   (a) $\mathrm{J\,s^{-1}}$
   (b) $\mathrm{N\,m\,s^{-1}}$
   (c) $\mathrm{kg\,m^2\,s^{-2}}$ ✗✗ 51% crct. Quiz 1, 2014
   (d) $\mathrm{kg\,m^2\,s^{-3}}$

4. Suppose you are at a height in Earth's atmosphere where the pressure $P = 850$ hPa. Approximately what fraction of the atmosphere's mass lies **beneath** your elevation?
   (a) 85%
   (b) 15% ✓✓ 41% crct. Compare Quiz 1, 2014.
   (c) 8.5%
   (d) 1.5%
   (e) 0.85%

5. Which is the closest estimate for the magnitude of the vertical gradient in atmospheric pressure [$\mathrm{Pa\,m^{-1}}$] at sea level?
   (a) 100
   (b) 10 ✓✓ 80% crct. Slide 8, Ch3-A; given midterm 1, 2015.
   (c) 1
   (d) 0.1
   (e) 0.01

6. Suppose a pool (or reservoir) for "X" contained $M = 10^6$ [kg] of this species, and that there was a balance between the fluxes $Q_{\rm in}$, $Q_{\rm out}$ of X into and out of the pool, with $Q_{\rm in} = Q_{\rm out} = 10^{-4}$ [$\mathrm{kg\,s^{-1}}$]. What is the residence time for X in this pool?
   (a) 10^{10} s ✓✓ 95% crct. Similar to midterm 1, 2015.
   (b) 10^4 s
   (c) 10^2 s
   (d) 1 s
   (e) 10^{-6} s

7. Of the processes listed, which is **not** directly involved in the atmospheric methane budget?
   (a) anaerobic decay
   (b) photosynthesis ✗✗ 97% crct. Slide 2, Ch2-C.
   (c) biomass burning
   (d) rice cultivation
   (e) ruminent digestion

8. What is the main "sink" (removal) mechanism for the pool of atmospheric "secondary aerosols", i.e. those formed in the atmosphere by gas-to-particle conversion?
   (a) gravitational settling
   (b) industrial fixation
   (c) fixation in lightning
   (d) precipitation scavenging ✓✓ 46% crct. Slide 8, Ch2-C; Ross Sec 2.9.2
   (e) anaerobic decomposition

9. Isolines of which "**secondary** field" appear on the CMC analysis for the 850 hPa surface?
   (a) surface pressure reduced to sea-level (MSLP)
   (b) height of the 850 hPa isobaric surface
   (c) isohumes (isolines of relative humidity)
   (d) 1000-500 hPa thickness (DZ)
   (e) temperature ✓✓ 74% crct. Slide 2, Ch2-B; listed as to-be-known

10. Assuming a window whose thickness is 4 mm and whose conductivity is $k = 1\,\mathrm{W\,m^{-1}\,K^{-1}}$, what is the magnitude of the temperature difference across the window if heat is being lost across it at a rate $Q = 500\,\mathrm{W\,m^{-2}}$? (Fourier's Law is given as data.)

    (a) 275.15K
    (b) 20°C
    (c) 0.2K
    (d) 2K ✓✓ 87% crct. Slide 6, Ch4-B.
    (e) 20K

11. If a fraction $S_0/10$ of the incident solar energy the sun's beam were being used to evaporate water from a field of bare soil, estimate the resulting evaporation rate $\widehat{E}$ expressed in the unit $[\mathrm{mm\,day^{-1}}]$. (Here S_0 is the solar constant.)

    (a) 140
    (b) 50
    (c) 5 ✓✓ 36% crct. Slide 10, Ch2-B; Slide 1, Ch2-C.
    (d) 5×10^{-3}
    (e) 6×10^{-5}

For the following two questions let ρ_0 represent nominal air density and P_0 nominal air pressure near sea-level, ρ_w the density of liquid water, L_v the latent heat of vapourization, d the north-south distance (on earth's surface) corresponding to one degree change in latitude, c_p the specific heat of air at constant pressure and Δ the change in 1000-500 hPa thickness per degree change in mean temperature of that layer.

12. Which option correctly identifies the set of numbers $(1, 1000, 10^5, 2.5 \times 10^6)$, interpreted as being in MKS units?

    (a) P_0, c_p, d, L_v
    (b) P_0, c_p, ρ_0, d
    (c) Δ, ρ_w, d, L_v
    (d) ρ_0, c_p, P_0, L_v ✓✓ 69% crct.; list of nos. expected to know
    (e) c_p, d, L_v, P_0

13. Which option correctly interprets the set of numbers $(2, 111, 1000)$ in terms of magnitude and unit?

    (a) $\Delta\,[\mathrm{dam\,K^{-1}}]$, $d\,[\mathrm{km\,deg^{-1}}]$, $\rho_w\,[\mathrm{kg\,m^{-3}}]$ ✓✓ 59% crct.; list of nos. expected to know
    (b) $\rho_w\,[\mathrm{kg\,m^{-3}}]$, $L_v\,[\mathrm{kg\,J^{-1}}]$,$P_0\,[\mathrm{Pa}]$
    (c) $L_v\,[\mathrm{kg\,J^{-1}}]$, $d\,[\mathrm{km\,deg^{-1}}]$,$P_0\,[\mathrm{hPa}]$
    (d) $L_v\,[\mathrm{J\,kg^{-1}}]$, $d\,[\mathrm{km\,deg^{-1}}]$,$P_0\,[\mathrm{hPa}]$
    (e) $\Delta\,[\mathrm{dam\,K^{-1}}]$, $d\,[\mathrm{m\,deg^{-1}}]$, $c_p\,[\mathrm{J\,K^{-1}}]$

14. How high (relative to its starting point) must a 25 kg mass be lifted in earth's gravitational field (taking $g = 10\,\mathrm{m\,s^{-2}}$) in order that its gain in gravitational potential energy be equal to the latent heat energy stored by 1 kg of water vapour?
   (a) 1 m
   (b) 10 m
   (c) 100 m
   (d) 1 km
   (e) 10 km ✓✓ 62% crct; Slide 9, Ch2-B; solution was posted to eClass

15. Suppose 2.5×10^6 J of energy are added to one kilogram of dry air *held at constant volume*. Which option best approximates the resulting changes $(\Delta T, \Delta\rho)$ in temperature and density? (Use the first law of thermodynamics in an appropriate form; the specific heat capacity at constant volume for dry air is $c_v = 718\,\mathrm{J\,kg^{-1}\,K^{-1}}$.)
   (a) $(3500\mathrm{K}, 1\,\mathrm{kg\,m^{-3}})$
   (b) $(3500\mathrm{K}, 0\,\mathrm{kg\,m^{-3}})$ ✓✓ 44% crct. Simil. to midterm 1, 2015.
   (c) $(35\mathrm{K}, 1\,\mathrm{kg\,m^{-3}})$
   (d) $(350\mathrm{K}, 0\,\mathrm{kg\,m^{-3}})$
   (e) $(3.5 \times 10^{-6}\mathrm{K}, 10\,\mathrm{kg\,m^{-3}})$

16. Suppose a parcel of dry air at the surface had state $(P_1, T_1) = (930\,\mathrm{hPa}, 12^{\circ}\mathrm{C})$. What is the parcel's temperature after it has been lifted adiabatically to the 850 hPa level? (Hint: Poisson's equation).
   (a) 11.7°C
   (b) 12.3°C
   (c) 277.9 K ✓✓ 82% crct. Slide 3, Ch4-B. Simil question on midterm 2014.
   (d) 293 K

17. Suppose unsaturated air at the 850 hPa level had temperature $T_{850} = 7^{\circ}\mathrm{C}$, and that the (true) pressure at the ground surface was 930 hPa. Using Poisson's equation, compute the temperature of a parcel of air that descends adiabatically from the 850 hPa level to the surface.
   (a) +14.3°C ✓✓ 64% crct. Slide 3, Ch4-B.
   (b) +7.2°C
   (c) −7.2°C
   (d) −14.3°C

18. A commercial jet is flying at an altitude where the air pressure is 20 kPa and the temperature is 213.15 K (or -60^{o}C). If exterior air is brought inside the aircraft and pressurized (adiabatically) to 80 kPa, what will be its temperature? (Again, Poisson's equation).

    (a) 143.4^{o}K
    (b) 316.9 K (or 43.7^{o}C) ✓✓ 85% crct. Midterm 1 2015. Slide 3, Ch4-B.
    (c) -89.2^{o}C
    (d) -129.8°C

19. A parcel of dry air at the surface ($z = 0$ m) has a temperature of 10^{o}C. It is lifted adiabatically to $z = 700$ m then sinks adiabatically to $z = 500$ m. What is its final temperature?

    (a) 19^{o}C
    (b) 17^{o}C
    (c) 15^{o}C
    (d) 10^{o}C
    (e) 5^{o}C ✓✓ 72% crct. Same as midterm 1, 2015.

20. The instantaneous vertical flux of sensible heat at a point in the atmosphere may be expressed

$$Q = \rho\, c_p\, w\, T \,-\, k\,\frac{\Delta T}{\Delta z}\ ,$$

where w is the vertical wind speed, T is the temperature and $\Delta T/\Delta z$ its vertical gradient, k is the conductivity of air, ρ is air density and c_p is air's specific heat capacity at constant pressure. Which statement is **true**?

    (a) the first term represents the contribution of radiative heat transport
    (b) the second ("Fourier-type") term represents convective heat transport
    (c) the first term represents the contribution of convective heat transport ✓✓ 64% crct. Slides 10-13, Ch4-B.
    (d) except very near ground, the first term is of much smaller magnitude than the second
    (e) adjacent to solid surfaces (the ground, leaves, etc.) the second term is negligible

For the remaining questions, which concern the meteorology of 12Z on 11 September 2016, please refer to Figures (1 – 3).

21. Referring to the surface analysis (Fig. 1), which option is a valid summary of observed surface conditions at the station in NE British Columbia that has been ringed by a dashed circle?

    (a) MSLP = 926.3hPa, $T = 5$°C, $T_d = 4$°C, overcast, snowing, WNW wind at $2.5\,\mathrm{m\,s^{-1}}$
    (b) MSLP = 1026.3hPa, $T = 5$°C, $T_d = 4$°C, overcast, raining, WNW wind at $2.5\,\mathrm{m\,s^{-1}}$ ✓✓ 90% crct.
    (c) MSLP = 1026.3hPa, $T = 5$°C, $T_d = 1$°C, sunny, raining, ESE wind at $5\,\mathrm{m\,s^{-1}}$
    (d) MSLP = 1026.3hPa, $T = 5$°C, $T_d = 4$°C, sunny, drizzle, WNW wind at $25\,\mathrm{m\,s^{-1}}$

22. Referring to Fig. 2, which interpretive statement is wrong or unreasonable?

   (a) over Edmonton, the 850 hPa surface was about $1460 - 700 = 760$ m above ground
   (b) the low centred near the Saskatchewan/Manitoba border has induced a firm northerly wind over N. Alberta (over Edmonton, roughly NNW at $15\,\mathrm{m\,s^{-1}}$)
   (c) based on the Stony Plain wind ($15\,\mathrm{m\,s^{-1}}$ NNW) and the thermal pattern, warming can be expected south of Edmonton ✗✗ 54% crct.
   (d) at the 850 hPa level, north of 60°N and broadly north of the Saskatchewan/Manitoba border, there is a cold pocket of air having $-10 < T \leq -5$°C

23. Referring to the Fig. 2, which option best estimates temperature at the 850 hPa level at the point marked **X** on the Alberta/Saskatchewan border?

   (a) +5°C
   (b) +2.5°C ✓✓ 77% crct.
   (c) −2.5°C
   (d) −5°C

24. Referring to the Fig. 3, which interpretive statement is correct?

   (a) the height contour running through Edmonton should be labelled 500 dam
   (b) 700 hPa wind over Edmonton should be named a "SE"
   (c) very dry air over Central Alberta
   (d) generally cloudy conditions would be expected over most of Alberta ✓✓ 41% crct.
   (e) the heavy dashed line identifies a ridge on the 700 hPa isobaric surface (it's actually a trough)

Continue to part B, the calculations, on the next TWO pages (4% + 3% → 7 %). Show your working, as you may receive part marks even if your answer is wrong. Round your answers to three significant digits (e.g. 1.23 or 0.0123 or 1.23×10^{-2}), and state their units.

B1 (3.5%). Thermodynamic calculation

Suppose 0.5 kg of dry air is contained in a rigid volume of 2 cubic metres, and that its temperature is $T_0 = 250$ K. Compute the final temperature T_1 and final pressure P_1 of this air sample if 10^4 J of heat energy are added to it. Method: use the 1$^{\text{st}}$ law of thermodynamics in the form

$$\Delta q \,[\mathrm{J\,kg^{-1}}] = c_v\, \Delta T + P \Delta \alpha$$

(where $\alpha \equiv 1/\rho$ is the specific volume), and assume $c_v = 718\,\mathrm{J\,kg^{-1}\,K^{-1}}$ and (specific gas constant) $R_d = 287\,\mathrm{J\,kg^{-1}\,K^{-1}}$. (Hint: you will also need to use the ideal gas law.)

Response:

i) $\Delta q = 10^4/0.5 = 2 \times 10^4\,[\mathrm{J\,kg^{-1}}]$

ii) because both the mass and the volume are fixed, density does not change and so neither does its reciprocal, the specific volume: hence $\Delta\alpha = 0$. (Recognizing this was a key step; those who didn't do so struggled to no eventual gain.)

iii) therefore $\Delta T = \Delta q / c_v = 27.86$ K

iv) the final temperature is $T_f = 250 + 27.86 = 277.86$ K

v) the density, which does not change, is $\rho = 0.25\,[\mathrm{kg\,m^{-3}}]$

vi) thus the final pressure is $P = \rho\, R_d\, T_f \;=\; 0.25 \times 287 \times 277.86 \;=\; 1.994 \times 10^4\,\mathrm{Pa}$, or 199.4 hPa. (This is not an adiabatic process, thus it was not correct to use Poisson's law to get the final pressure from the initial pressure and temperature.)

Irrespectively of other errors, baseline scoring was as follows: (i) if the ideal gas law was used correctly at least once (and with the correct density), 1.5 marks were gained; if it was recognized that $\Delta\alpha = 0$, 1.0 marks were gained.

B2 (3.5%). Single layer soil heat budget

Consider a single layer of soil $d = 10$ mm deep, having volumetric heat capacity $\rho_s\, c_s = 2 \times 10^6\, \mathrm{J\, m^{-3}\, K^{-1}}$ (where ρ_s is soil density and c_s soil specific heat capacity). Define $\overline{T}$ to be the mean temperature of this soil, so that the sensible heat content of the layer per unit of its surface area is given by

$$\Sigma\, [\mathrm{J\, m^{-2}}] = (\rho_s\, c_s\, \overline{T}) \times d\ .$$

Suppose this soil layer were bounded at its base by a perfect insulator, such that the sensible heat flux at the base $Q_b = 0$. Suppose however that there is a steady sensible heat flux $Q_t = 10\, \mathrm{W\, m^{-2}}$ into the top of this soil layer. Draw a schematic diagram (sketch) to represent this scenario, and exploit the principle of energy conservation to compute the rate of increase $\Delta\overline{T}/\Delta t\, [\mathrm{K\, hr^{-1}}]$ in the mean temperature, t being time.

Response: As ρ_s, c_s and d are all constants, change in heat content per unit soil surface area $\Delta\Sigma = (\rho_s\, c_s\, d) \times \Delta\overline{T}$, and one must equate this to $Q_t \times \Delta t$ $[\mathrm{J\, m^{-2}}]$. So $(\rho_s\, c_s\, d) \times \Delta\overline{T} = Q_t\, \Delta t$ and

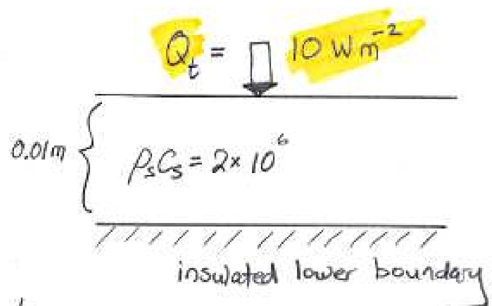
$$\frac{\Delta\overline{T}}{\Delta t} = \frac{Q_t}{\rho_s c_s\, d} = 5 \times 10^{-4}\, \mathrm{K\, s^{-1}}\ .$$

Multiply by 3600 to convert to $1.8\, \mathrm{K\, hr^{-1}}$. The diagram below also works through the problem.

$$\Sigma = \rho_s c_s \overline{T} d \; [J\, m^{-2}]$$

(given)

$$= 2 \times 10^4 \times \overline{T} \; [J\, m^{-2}]$$



In time interval Δt the amount of heat added is:

$$\Delta q = 10\, [W\, m^{-2}]\, \Delta t\, [s] = 10\, \Delta t\, [J\, m^{-2}]$$

This will result in a change in the sensible heat content of the layer, such that

$$10\, \Delta t = 2 \times 10^4 \times \Delta\overline{T}$$

Hence $\dfrac{\Delta\overline{T}}{\Delta t} = 5 \times 10^{-4}\, [K\, s^{-1}] = 1.8\, [K\, hr^{-1}]$

Equations and Data.

- $P = M\ g/A$, the pressure (P, Pa) that results when mass M [kg] overlies area A [m^2], where (for earth) $g = 9.81$ [m s^{-2}]

- N=0 or 360, NNE=22.5, NE=45, ENE=67.5, E=90, ESE=112.5, SE=135, SSE=157.5, S=180, SSW=202.5, SW=225, WSW=247.5, W=270, WNW=292.5, NW=315, NNW=337.5

  The sixteen so-called "cardinal points" of the compass, given alphanumerically and as an angle measured clockwise around the circle. A coarser eight-point subdivision is N, NE, E, SE, S, SW, W, NW; and the four cardinal points are of course N, E, S, W

- one full barb on the wind vector means 5 m s^{-1}, a solid triangle 25 m s^{-1}.

- The "mean residence time" of an atmospheric gas

  $$T^{\text{res}} = \frac{M}{Q}$$

  is the ratio of its total mass M to its mean surface exchange rate Q.

- The hydrostatic equation

  $$\frac{\Delta P}{\Delta z} = -\rho\, g$$

  gives the change ΔP [Pa] in pressure as one ascends a distance Δz [m], where ρ [kg m^{-3}] is the air density and $g = 9.81$ [m s^{-2}] is the gravitational acceleration.

- The ideal gas law

  $$P = \rho\, R_{\text{sp}}\, T_v$$

  inter-relates the total pressure P [Pascals], the total density ρ, [kg m^{-3}] and the virtual temperature T_v [Kelvin]. R_{sp} is the specific gas constant. In the case of earth's atmosphere, within the homosphere $R_{\text{sp}} = R_d = 287$ [J kg^{-1} K^{-1}], the specific gas constant for dry air.

- $T_v = T\ (1 + 0.61\, r)$. Defines the virtual temperature [K] of an air sample whose temperature is T [K] and whose water vapour mixing ratio is r [kg/kg].

- The hypsometric equation

  $$\Delta z = z_2 - z_1 = \left[\frac{R_d\, \overline{T}_v}{g}\right]\ \ln \frac{P_1}{P_2}$$

  gives the distance Δz [m] between two isobaric surfaces (P_1, P_2), which is controlled by the average virtual temperaure $\overline{T}_v$ of the layer. The specific gas constant for dry air $R_d = 287$ [J kg^{-1} K^{-1}]. Note that if $z_2 > z_1$, then $P_2 < P_1$.

- The first law of thermodynamics may be expressed by either of two equations, viz.

$$\begin{aligned} \Delta q &= c_v\, \Delta T + P\, \Delta\alpha\,, \\ \Delta q &= c_p\, \Delta T - \Delta P/\rho\,, \end{aligned}$$

  and links changes in the state variables of a sample of air. Δq [J kg^{-1}] is energy added to the system (zero for an adiabatic process, by definition), c_v is the specific heat capacity of the material at constant volume, c_p is the specific heat capacity of the material at constant pressure, P is the pressure and $\alpha \equiv 1/\rho$ is the specific volume (ρ being air density). For an ideal gas $c_v = (5/2)R_{\rm sp}$ and $c_p = (7/2)R_{\rm sp}$ (where $R_{\rm sp}$ is the specific gas constant) and for dry air $R_{\rm sp} = R_d = 287\,{\rm J\,kg^{-1}\,K^{-1}}$. Thus for dry air the specific heat capacities are $c_v \approx 718$ J kg^{-1} K^{-1} and $c_p \approx 10^3$ J kg^{-1} K^{-1}.

- Poisson's equation (Poisson's law)

$$\frac{T}{T_1} = \left(\frac{P}{P_1}\right)^{R/c_p}$$

  links two states (P,T) and (P_1,T_1) of a sample of ideal gas, assuming the process connecting the two states is adiabatic ($R/c_p = 2/7 = 0.286$). Temperatures must be expressed in the Kelvin unit.

- The unsaturated (or "dry") adiabatic lapse rate (DALR) is

$$\frac{\Delta T}{\Delta z} = -\,\frac{g}{c_p}\,,$$

  where g is the gravitational acceleration and $c_p \approx 1000$ $[\rm J\,kg^{-1}\,K^{-1}]$ is the specific heat at constant pressure. Our textbook defines the DALR as the *magnitude*, DALR= $|\Delta T/\Delta z|$, often rounded to DALR=1°C/100 m.

- Fourier's law of conduction

$$Q_x = -\,k\,\frac{\Delta T}{\Delta x}\;,\; Q_y = -\,k\,\frac{\Delta T}{\Delta y}\;,\; Q_z = -\,k\,\frac{\Delta T}{\Delta z}\,.$$

  gives the components (Q_x, Q_y, Q_z) of the conductive heat flux density along directions (x, y, z) in response to temperature gradients $\Delta T/\Delta x$ (etc.) along the axes in a medium whose conductivity is $k\,[\rm W\,m^{-1}\,K^{-1}]$. (The sign convention is that Q_x is positive for a flow of heat towards larger values of x, etc.)

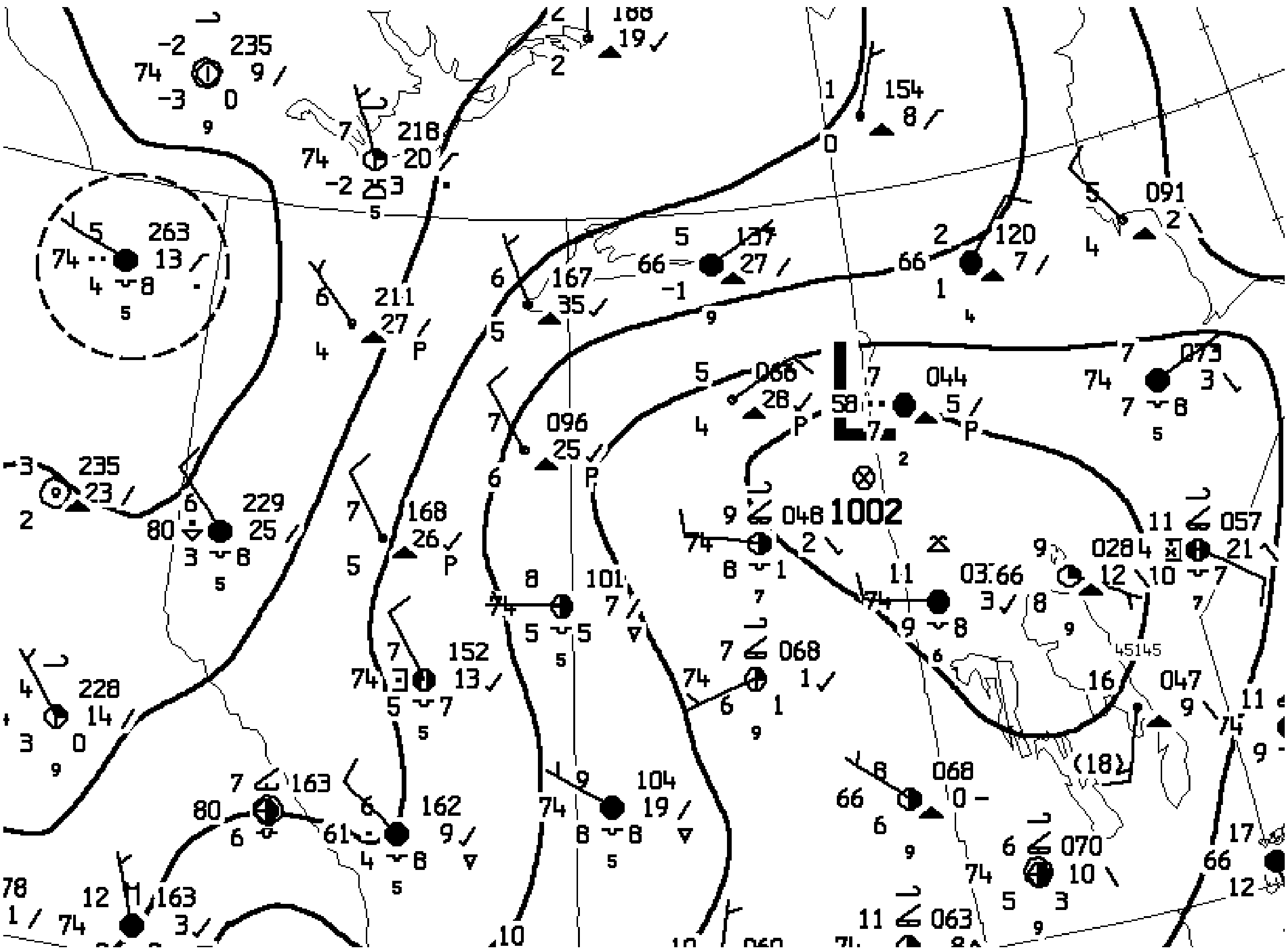


Figure 1: CMC surface analysis (cropped) for 12Z on 11 Sept. 2016.

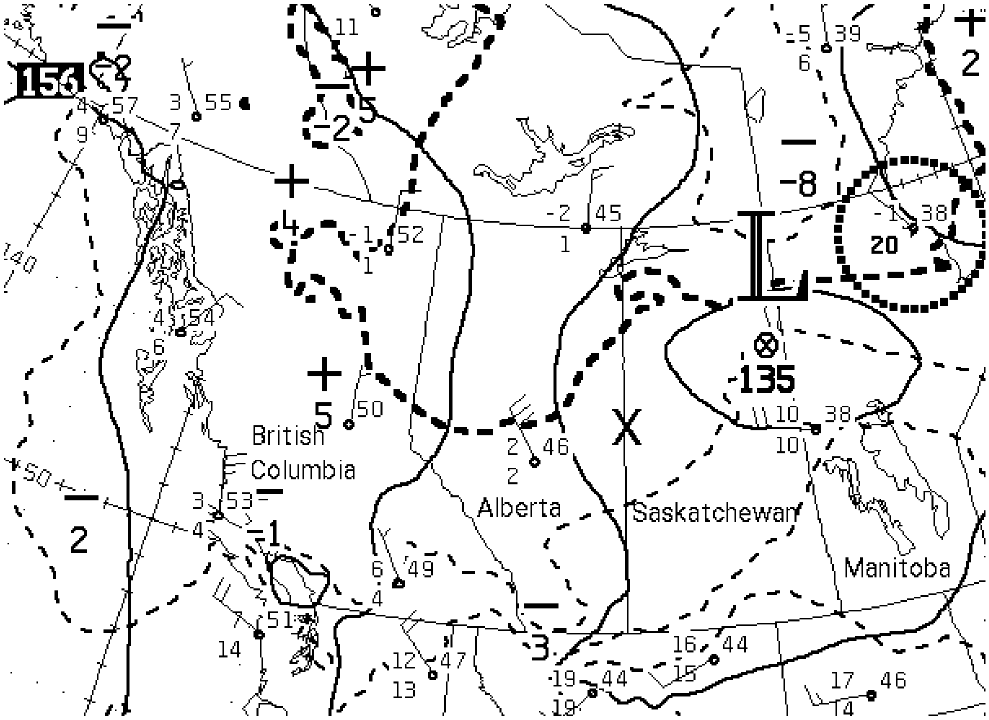


Figure 2: CMC 850 hPa analysis (cropped) for 12Z on 11 Sept. 2016. The bold dashed line is the 0°C isotherm, and the isotherm contour interval is 5°C .

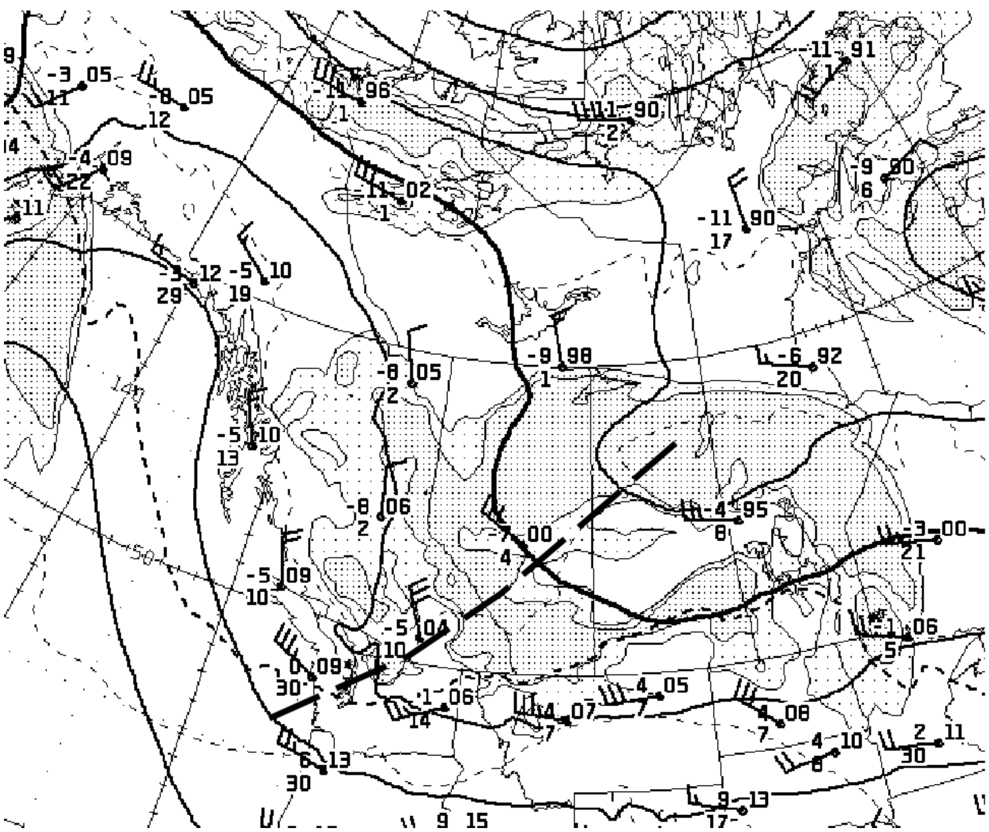


Figure 3: CMC 700 hPa analysis (cropped) for 12Z on 11 Sept. 2016. Heavy stippling identifies areas where $T - T_d \leq 2$°C, and lighter stippling denotes $T - T_d \leq 5$°C. (Heavy dashed line added by the instructor.)