EAS270, "The Atmosphere" <u>Mid-term Exam</u> 15 Oct. 2014

<u>Professor</u>: J.D. Wilson <u>Time available</u>: 50 mins <u>Value</u>: 20%

No formula sheets or cell phones. **Formulae and data provided at the back**. Please record your multichoice answers on the scantron sheet, choosing the correct or most logical answers (incorrect options *may* feature false or misleading statements or 'facts'). Respond to short answer and calculation questions in the space provided: **add your name**, tear off and submit. (You may keep other pages of this exam.)

A. Multi-choice (15 x 2/3 → 10 %)

1. Molecules of CO_2 combine a carbon atom (atomic number 12) with two oxygen atoms (oxygen having atomic number 16). Given that the universal gas constant is $R^* = 8.3143\,\mathrm{J\,mol^{-1}\,K^{-1}}$, which value below best approximates the specific gas constant R for CO_2?
   (a) $190\,\mathrm{J\,kg^{-1}\,K^{-1}}$ ✓✓[67% correct. Lec 8 Oct., p1]
   (b) $0.044\,\mathrm{kg\,mol^{-1}}$
   (c) $287\,\mathrm{J\,kg^{-1}\,K^{-1}}$
   (d) $0.190\,\mathrm{J\,kg^{-1}\,K^{-1}}$
   (e) $300\,\mathrm{J\,kg^{-1}\,K^{-1}}$

2. Suppose the atmosphere of a certain planet were composed entirely of N_2, for which the specific gas constant is $R = 297\,\mathrm{J\,kg^{-1}\,K^{-1}}$. If the surface pressure and temperature at a certain point on this planet were (80 kPa, 50°C) then what was the surface density ρ?
   (a) $0.1\,\mathrm{kg\,m^{-3}}$
   (b) $0.5\,\mathrm{kg\,m^{-3}}$
   (c) $0.83\,\mathrm{kg\,m^{-3}}$ ✓✓[89% correct;]
   (d) $1\,\mathrm{kg\,m^{-3}}$
   (e) $5.4\,\mathrm{kg\,m^{-3}}$

3. Isolines of which "secondary field" are plotted on the CMC isobaric charts for the 850 hPa level?
   (a) MSLP
   (b) isobaric height
   (c) isohumes (isolines of relative humidity)
   (d) 1000-500 hPa thickness (DZ)
   (e) temperature ✓✓[48% correct; Table 3.2; class map discussions]

4. Suppose 0.5 kg of dry air is contained in a rigid volume of 2 cubic metres. Which value best approximates the change in temperature (final minus initial) if 10^4 J of energy are added to this system?
   (a) -30K
   (b) -15K
   (c) $+15$K
   (d) $+30$K ✓✓[47% correct; Ex 4.5]
   (e) $+60$K

5. At the time of the southern hemisphere summer solstice, what is the noon solar elevation at 23.5°N latitude?
   (a) 0°
   (b) 23.5°
   (c) 43° ✓✓[51% correct; exercises 1 Oct. slide 9]
   (d) 66.5°
   (e) 90°

6. Defining the volumetric sensible heat content of a parcel of air as $h = \rho c_p T\,[\mathrm{J\,m^{-3}}]$ and taking the ρc_p product as $10^3\,[\mathrm{J\,m^{-3}\,K^{-1}}]$, what is the value of the convective heat flux density Q_x if the x-component of the wind is $u = 5\,\mathrm{m\,s^{-1}}$ and the temperature of the parcel is 300 K?
   (a) $1.5 \times 10^6\,\mathrm{W\,m^{-2}}$ ✓✓[55% correct; Lec 26 Sept., p8]
   (b) 3×10^5 K
   (c) $120\,\mathrm{W\,m^{-3}}$
   (d) $120\,\mathrm{W\,m^{-2}\,s^{-1}}$
   (e) $1500\,\mathrm{J\,m^{-3}}$

7. Taking the conductivity of glass as $k = 1\,[\mathrm{W\,m^{-1}\,K^{-1}}]$, what is the rate of conductive heat transfer across a window pane of area $1\,\mathrm{m^2}$ and thickness $\Delta x = 2.5$ mm when the temperature difference across the pane is 4K?
   (a) 1600 J
   (b) 1600 W ✓✓[76% correct; Lec 26 Sept., p3]
   (c) $160\,\mathrm{W\,s^{-1}}$
   (d) 100 W
   (e) 10 W

8. A parcel of dry air at the surface ($z = 0$ m) has a temperature of 20°C. It is lifted adiabatically to $z = 700$ m then sinks adiabatically to $z = 500$ m. Its final temperature is?
   (a) 29°C
   (b) 27°C
   (c) 25°C
   (d) 20°C
   (e) 15°C ✓✓[72% correct]

9. If a certain body has longwave emissivity $\epsilon = 0.96$ and its temperature is $T = 18^{\circ}$C, which answer below most closely states the wavelength $\lambda_{\max}$ of the peak in its thermal emission spectrum and its full-spectrum rate of emission E of longwave radiation?
   (a) $10\,\mu$m, $6 \times 10^{-3}\,\mathrm{W\,m^{-2}}$
   (b) $10\,\mu$m, $390\,\mathrm{W\,m^{-2}}$ ✓✓[96% correct]
   (c) $100\,\mu$m, $390\,\mathrm{W\,m^{-2}}$
   (d) $1\,\mu$m, $410\,\mathrm{W\,m^{-2}}$
   (e) $0.1\,\mu$m, $410\,\mathrm{W\,m^{-2}}$

10. Suppose a parcel of air at the surface had state $(P_1, T_1) = (930\,\mathrm{hPa}, 12^{\circ}\mathrm{C})$. What is the parcel's temperature after it has been lifted adiabatically to the 850 hPa level? (Hint: Poisson's equation).
   (a) 11.7°C
   (b) 12.3°C
   (c) 278 K ✓✓[72% correct; Ex 4.6; Lec 24 Sept., p4]
   (d) 293 K

11. Suppose on a sunny summer afternoon the net radiation over a flat field of bare soil were $Q^* = 500$ W m^{-2}, and the sensible and latent heat fluxes were $Q_H = 180$, $Q_E = 300$ W m^{-2}. What was the soil heat flux Q_G?
   (a) 980 W m^{-2}
   (b) 480 W m^{-2}
   (c) 120 W m^{-2}
   (d) 20 W m^{-2} ✓✓[95% correct]
   (e) -980 W m^{-2}

For the remaining questions, please refer to Figures (1 – 2).

12. What was MSLP and what were the present weather conditions at the station closest to the **X** on Figure 1?
   (a) 915.6 hPa; overcast, rain, light ENE wind
   (b) 915.6 hPa; overcast, snow, light WSW wind
   (c) 1015.6 hPa; overcast, rain, light WSW wind
   (d) 1015.6 hPa; overcast, snow, light WSW wind
   (e) 1015.6 hPa; overcast, snow, light ENE wind ✓✓[87% correct]

13. The wind observations on Figure 1 show anti-clockwise winds about the low pressure system near Churchill (Manitoba), with a component of cross-isobar flow towards low pressure. Which statement is **false**?
    (a) this is the normal pattern of flow about a N. hemisphere low
    (b) the cross-isobar wind implies ascending vertical motion, possibly resulting in cloud and precip
    (c) on the west side of the low, cold air is moving southward to replace milder air (cold advection)
    (d) on the east side of the low, cold air is moving northward to replace milder air (warm advection) ✓✓[46% correct]
    (e) MSLP at the centre of the low was lower than 992 hPa

14. According to Figure 2, the 850 hPa analysis, at which point or points was cold advection occurring?
    (a) A
    (b) A, B
    (c) C
    (d) B, D
    (e) B ✓✓[45% correct]

15. From Figure 2, how would you describe the 850 hPa wind over Edmonton?
    (a) NNW, over $15\,\mathrm{m\,s^{-1}}$, parallel to height contours ✓✓[74% correct]
    (b) SSE, over $15\,\mathrm{m\,s^{-1}}$, parallel to height contours
    (c) NNW, over $30\,\mathrm{m\,s^{-1}}$, perpendicular to height contours
    (d) NNW, over $15\,\mathrm{m\,s^{-1}}$, crossing height contours

Continue to Calculations on next page.

B. Computations (2 x 2 → 4 %)

Round your answers to three significant digits (e.g. 1.23 or 0.0123 or 1.23×10^{-2}), and state the units. These calculations were very well done. A high percentage of students received full marks, and many others *would have* — had they taken the elementary step of rounding the answers, as requested.

The hypsometric equation reads

$$\Delta z = z_2 - z_1 = \left[\frac{R_d\,\overline{T}_v}{g}\right]\,\ln\frac{P_1}{P_2}\,,$$

where $R_d = 287\ \ [\mathrm{J\,kg^{-1}\,K^{-1}}]$ and $g = 9.81\,[\mathrm{m\,s^{-2}}]$. Note that if $z_2 > z_1$, then $P_2 < P_1$.

B1. What is the thickness Δz of the 1000 hPa to 500 hPa atmospheric layer if the average virtual temperature of the layer is $\overline{T}_v = 0^{\mathrm{o}}\mathrm{C}$?

This is Ross's Example 3.5.

$$\Delta z = \frac{287\ 273.15}{9.81}\ \ln\frac{1000}{500} = (7.99\times10^3)\times\ln 2 = 5.54\times10^3\ \mathrm{m}\,.$$

Equally acceptable answers are: 554 dam or 5.54 km.

B2. Suppose the distance from ground to the 850 hPa isobaric surface was $\Delta z = 900$ m and that the average virtual temperature of that layer was $\overline{T}_v = 10^{\mathrm{o}}\mathrm{C}$? What is the pressure at the surface?

$$\Delta z = 900\,\mathrm{m} = \frac{287\ (273.15+10)}{9.81}\ \ln\frac{P_{\mathrm{sfc}}}{850} = (8.284\times10^3)\times\ln\frac{P_{\mathrm{sfc}}}{850}\,,$$

thus,

$$\ln\frac{P_{\mathrm{sfc}}}{850} = 0.1086\,,$$

$$\frac{P_{\mathrm{sfc}}}{850} = \exp(0.1086)\equiv e^{0.1086} = 1.115\,,$$

$$P_{\mathrm{sfc}} = 948\,\mathrm{hPa}\,.$$

(If you had the pressure ratio upside down, you'd get 762 hPa).

Continue to Short Answer questions on next page.

C. Short answer questions (2 x 3 $\rightarrow$ 6 %)

C1. Describe (qualitatively) the change in the phase (or, "delay") ϕ and amplitude A of the annual temperature wave with increasing depth z into a soil.

With increasing depth in the soil the amplitude of the annual temperature wave *decreases* while the peaks and troughs are increasingly *delayed* relative to their times at the surface. As an example of what this implies, suppose it is midsummer, so that at the surface temperature has its annual maximum value: at some depth d in the soil the temperature will at its annual *minimum value*, i.e at depth d it is (local) midwinter when (at the surface) it is midsummer. [This answer could be supplemented, or even replaced, by a suitably labelled graphical response showing the temperature wave at several depths, its amplitude diminishing and its phase delay increasing with increasing depth].

Covered Fri. 26 Sept. (slide 6). Comments: solar radiation does not penetrate into the soil; the symbol "ϕ" did ot allude to solar elevation angle; the term "phase" here has nothing to do with latent heat and the change of phase of water; some students stated that on a summer day the soil is cooler at greater depth: this is usually true, but how does it relate to the *annual* cycle?

C2. Consider an isothermal planet of radius R that has no atmosphere, whose solar constant is S_0, and whose albedo is $a = 0.7$. Making reference to a statement of proportionality ($dT/dt \propto ...$), explain the feedback that stabilizes the temperature T of this hypothetical planet.

The energy balance of this planet can be expressed

$$\frac{dT}{dt} \propto \pi R^2 S_0 (1 - a) - 4\pi R^2 \epsilon \sigma T^4 ,$$

where dT/dt is the time rate of change of the planet's temperature (symbols are as defined in the data section of this exam). The first term on the right hand side represents the net rate of gain of shortwave radiative energy (the shortwave absorbtivity being $1 - a$), while the second term is the rate of loss of longwave radiant energy. If T increases, the first term is unchanged but the second increases in magnitude, resulting in $dT/dt < 0$, i.e. a trend back to a lower temperature. This is a stabilizing (negative) feedback.

Covered Fri. 3 Oct. (slide 2).

Equations and Data.

- one full barb on the wind vector corresponds to 5 m s^{-1}, and a solid triangle means 25 m s^{-1}.
- $\frac{\Delta P}{\Delta z} = -\rho\ g$. ΔP [Pascals], the change in pressure as one ascends a distance Δz [m]; ρ [kg m^{-3}] the air density; $g = 9.81$ [m s^{-2}] acceleration due to gravity.
- $R = R^{*}/\mathrm{M}$. Gives the specific gas constant R for a gas composed on molecules having molecular mass M kg mol^{-1}, where $R^{*} = 8.3143$ J mol^{-1} K^{-1} is the universal gas constant. To a first approximation the molecular mass of an *atom* is given by its atomic number, i.e. its place in the periodic table of the elements.
- $P = \rho\ R_d\ T_v$. The ideal gas law. P [Pascals], total pressure; ρ, [kg m^{-3}] the total density; T_v [Kelvin], the virtual temperature; and $R_d = 287$ [J kg^{-1} K^{-1}], the specific gas constant for dry air.
- $\Delta q = c_v\ \Delta T +\ P\ \Delta\alpha$. The first law of thermodynamics, linking changes in the state of a sample of air: Δq [J kg^{-1}] is energy added to the system (zero for an adiabatic process, by definition), c_v is the specific heat capacity of the material at constant volume, P is the pressure and $\alpha \equiv 1/\rho$ is the specific volume (ρ being air density). The $c_v\ \Delta T$ term is the change in the *internal energy* of the system. For an ideal gas $c_v = (5/2)R$, so that for dry air $c_v \approx 720$ J kg^{-1} K^{-1}.
- $$\frac{T}{T_1} = \left(\frac{P}{P_1}\right)^{R/c_p}$$

  Poisson's law linking two states (P, T) and (P_1, T_1) of a sample of ideal gas, assuming the process connecting the two states is adiabatic ($R/c_p = 2/7 = 0.286$).
- $\frac{\Delta T}{\Delta z} = -\frac{g}{c_p}$. The dry adiabatic lapse rate, where g is the gravitational acceleration and c_p [J kg^{-1} K^{-1}] is the specific heat at constant pressure. Our textbook defines the DALR as the *magnitude*, DALR= $|\Delta T/\Delta z|$, often rounded to DALR=1°C/100 m.
- $$Q = -k\,\frac{\Delta T}{\Delta x}$$

  Fourier's law of conduction, giving the conductive heat flux density Q in direction x if there is a temperature gradient $\Delta T/\Delta x$ in a medium whose conductivity is k W m^{-1} K^{-1}. (The sign convention is that Q is positive for a flow of heat towards larger values of x.)

- $E = \epsilon \, \sigma \, T^4$

  Stefan-Boltzmann law for a greybody. E [W m^{-2}], the emitted longwave energy flux density; ϵ, the emissivity of the surface (dimensionless); $\sigma = 5.67 \times 10^{-8}$ [W m^{-2} K^{-4}], the Stefan-Boltzmann constant; T [K], the surface temperature.

- $\lambda_{\mathrm{max}} = \frac{2897}{T}$

  Wien's displacement law. λ_{max} [μm], the wavelength at which the peak in the emission spectrum occurs; T [K], the temperature of the emitting surface.

- $Q^* = Q_H + Q_E + Q_G$. Surface energy balance on a reference plane at the base of the atmosphere. Fluxes normally in [W m^{-2}], but sometimes expressed as totals [J m^{-2}] over (e.g.) a day or a year. Q^* the net radiation, positive if directed towards the surface; Q_H, Q_E the sensible and latent heat fluxes, positive if directed from the surface towards the atmosphere; Q_G the 'soil' heat flux, positive if directed from the surface into ground/lake/ocean.

- $\alpha = 90^{\circ} - |\phi - \delta|$. Solar elevation α at local solar noon time, at a location having latitude ϕ (negative in the S. hemisphere), at the time of year when solar declination is δ (δ is negative during northern hemisphere winter, and its magnitude never exceeds 23.5°).

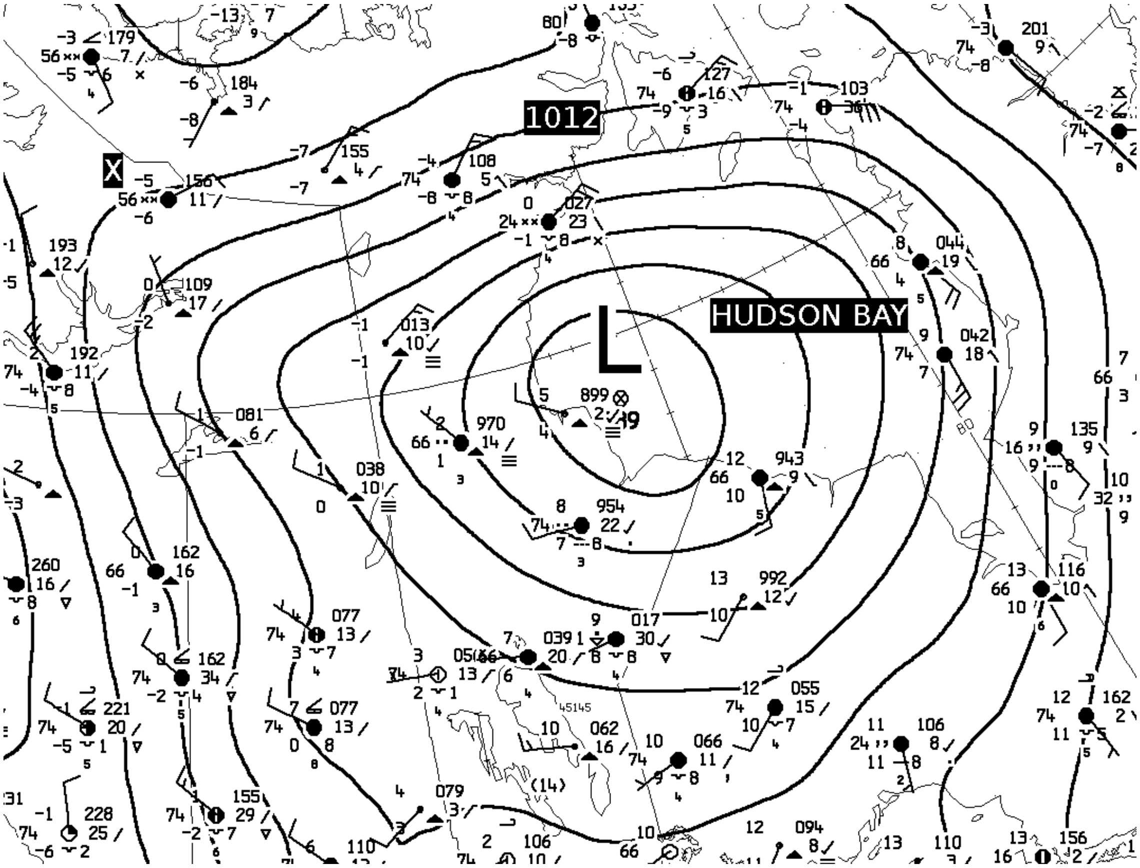

Figure 1: Environment Canada surface analysis (cropped) for 12 UTC Thurs. 2 October 2014.

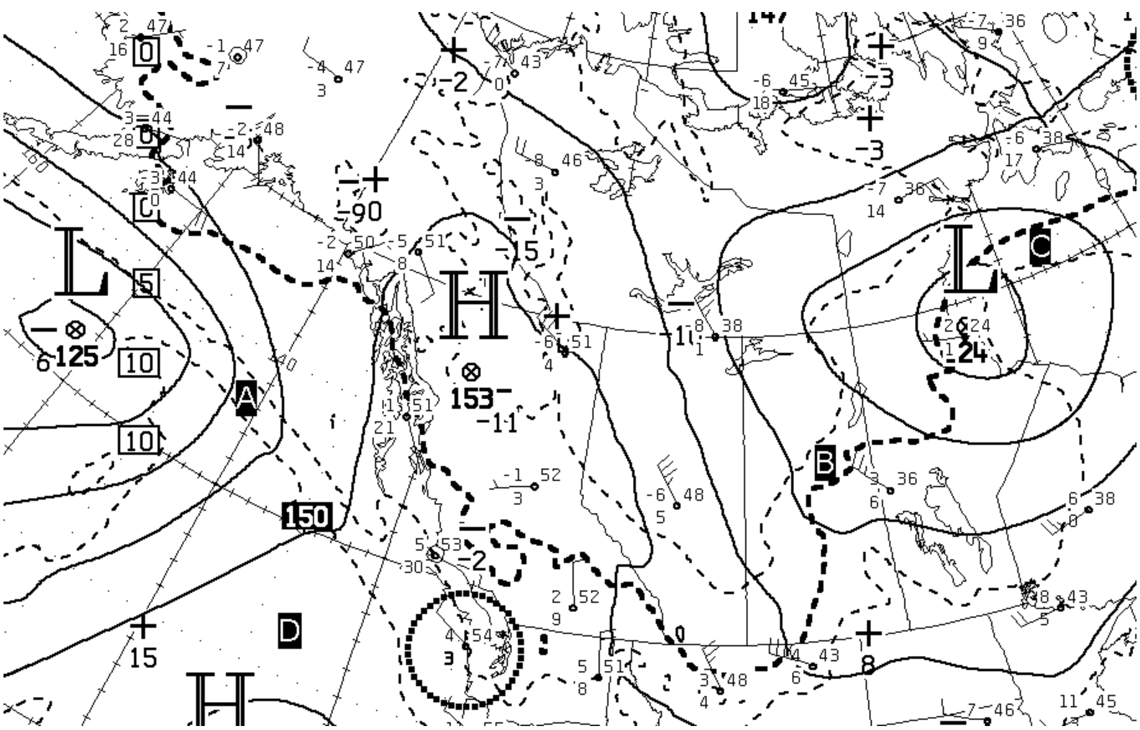

Figure 2: Environment Canada 850 hPa analysis (cropped) for 12 UTC Thurs. 2 October 2014.