THE QUANTITATIVE THEORY OF CYCLONE DEVELOPMENT

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Introduction

The title of this article indicates its principal theme but not its full scope, for although we shall explain the method of formulating and solving certain theoretical problems and shall interpret the answers in terms of the initial stages of development of extratropical cyclones and anticyclones, our analysis has also a wider significance. Not only does a fundamentally similar theoretical analysis apply to a wide variety of development problems (including, for example, the development of "long" waves as well as the shorter "frontal" waves and even phenomena due primarily to ordinary convective instability) so that a comprehensive analysis is desirable, but this analysis gives results of primary importance in the theory of development *in general*, that is, from the point of view of the general forecasting problem. We shall infer from our results that there exist, in general, certain ultimate limitations to the possibilities of weather forecasting. Certain apparently sensible questions, such as the question of weather conditions at a given time in the comparatively distant future, say several days ahead, are *in principle* unanswerable and the most we can hope to do is to determine the relative *probabilities* of different outcomes. The full significance of our theoretical problems becomes apparent only when it is clear what *kind* of question we should attempt to answer.

The science of meteorology is a branch of mathematical physics; it can be fully understood only in a quantitative manner. Moreover, all the practical questions we should like to answer are of a quantitative character. Having discovered the relevant equations of motion, we ought to aim at obtaining significant integrals (more precisely, solutions of significant boundary-value problems) which may be applied directly to practical problems. In order to obtain tractable problems, and at the same time to see clearly what we are doing (*i.e.*, "to see the wood for the trees"), we may, for a first analysis, simplify the equations by omitting all factors not vitally affecting the nature of the answer. Later we may refine our solutions by taking into account factors previously omitted (*e.g.*, by the method of successive approximations), thereby testing whether we have in fact included all the vital factors. This is a procedure with which we are familiar and, however laborious it may be in practice, it introduces no new difficulty in principle. The really serious difficulty is to discover what kind of problem ought to be solved, for this difficulty arises as soon as we consider the question of the stability of atmospheric motion. Observation suggests that the motion may, at least sometimes, be unstable, and we shall infer from subsequent analysis that instability (to a greater or less degree) is a *normal* feature of atmospheric motion.

It is important to be quite clear as to the meaning of the term "unstable" when applied to a system of fluid motion. If we suppose the initial field of motion to be given, the final field of motion, after a given interval of time, is determined precisely by the equations of motion, continuity, radiation, etc., together with the appropriate boundary conditions. If we consider a slightly different (perturbed) initial state, the new final state, after the same interval of time, will be determined in a similar manner. The stability or instability of the motion depends on the behaviour of the resulting change (perturbation) in the final state as the time interval is increased. If the final perturbation remains small for all time for *all* possible initial perturbations, the motion is stable. If, on the other hand, the perturbation in some or all regions grows (initially) at an exponential rate for *any* possible initial perturbation, the motion is unstable. There is an intermediate case, conveniently described as *neutral* stability, when the perturbations grow linearly or according to a low-degree power law, but this need not concern us here.

The practical significance of a demonstration that the motion is unstable is clear, for in practice, however good our network of observations may be, the initial state of motion is never given precisely and we never know what small perturbations may exist below a certain margin of error. Since the perturbation may grow at an exponential rate, the margin of error in the forecast (final) state will grow exponentially as the period of the forecast is increased, and this possible error is unavoidable whatever our method of forecasting. After a limited time interval, which, as we shall see, can be roughly estimated, the possible error will become so large as to make the forecast valueless. In other words, the set of all possible future developments consistent with our initial data is a divergent set and any direct computation will simply pick out, arbitrarily, one member of the set. Clearly, if we are to glean any information at all about developments beyond the limited time interval, we must extend our analysis and consider the properties of the set or "ensemble" (corresponding to the Gibbs-ensemble of statistical mechanics) of all possible developments. Thus long-range forecasting is necessarily a branch of statistical physics in its widest sense: both our questions and answers must be expressed in terms of *probabilities*.

There are two important connections between these general considerations and subsequent analysis. Firstly, this analysis will show the existence of at least one type of large-scale unstable disturbance in a simplified but typical system, and we shall infer that instability is a normal feature of atmospheric motion. Although the unstable disturbances are continually tending to establish a new stable state, radiative processes are continuously tending to restore the initial system, which therefore remains *permanently* unstable. Secondly, for such a system the study of the ensemble of all possible perturbations is relatively simple. In each system there exists a disturbance of maximum growth-rate (so that we can determine the growth of the margin of error and estimate the limited time interval referred to above) which eventually becomes dominant in subsequent developments by a process analogous to Darwinian natural selection. Almost any initial disturbance tends eventually to resemble the dominant, which is therefore the most probable development. The "ensemble" possesses at least *some* strongly marked statistical properties which may be utilised to extend the range of forecasts. In spite of inaccuracies due to oversimplification this result is practically significant. The disturbances referred to are approximations to nascent cyclones, long waves, etc.; were it not that "natural selection" is a very real process, weather systems would be much more variable in size, structure, and behaviour.

The Basic Equations

We shall regard as basic equations the three dynamical equations, the thermal equation, and the equation of continuity; others, such as the gas laws and the laws of radiation, will be regarded as subsidiary. The number of dependent variables we need, or that we find it convenient to use, depends on the nature of the problem and the degree of accuracy aimed at. In the problems with which we shall be concerned it is possible to express the basic equations in terms of the three components of velocity, pressure, and entropy (or density) alone so that the five basic equations, together with appropriate boundary conditions, form a complete set. Clearly these equations can be appropriate only for a limited range of problems when certain approximations are justified; we shall in fact make further approximations, our aim being to retain only those terms which are of prime importance in the range in which we are interested.

A completely realistic theory of the stability of atmospheric motion should deal with nonsteady initial conditions, but for simplicity we shall confine our attention to the case in which the initial motion is steady, and in fact we shall be concerned mainly with rectilinear horizontal motion. Our analysis will be approximately true even when the very-large-scale distribution is slowly changing.

The relative importance of the terms in our equations depends partly on the scale of the phenomena with which we are concerned. Here we are interested in disturbances of the order of magnitude of nascent cyclones, say 1000 km in horizontal extent and occupying a large part (or the whole depth) of the troposphere. From our point of view ordinary or gravitational convection, originating from static instability (*i.e.*, superadiabatic lapse rate), and ordinary turbulence of frictional or convective origin are small-scale phenomena. The epithet "ordinary" is appropriate in each case because the disturbances whose nascent form we are studying may be regarded as elements of a large-scale convective process and this process, regarded statistically, is a kind of large-scale turbulence. From our point of view the significance of small-scale turbulence, including ordinary convection, lies in its statistical properties, such as ability to transport heat, momentum, etc. Now frictionally induced turbulence is most effective near the earth's surface, and rough calculations (which we have not space to describe) using empirical estimates of skin friction indicate that frictional dissipation of energy usually has a relatively small effect on the development of large-scale disturbances (especially over a sea surface) in their *nascent* stage, provided the unstabilising factors are not too weak. Since we are most interested in those regions where the unstabilising factors are relatively strong, we may obtain a useful first approximation during the nascent stage if we neglect the frictional terms in the equations of motion. It is not possible to neglect frictional terms throughout the whole life-history of a disturbance because in the long run the kinetic energy destroyed by friction must equal that generated as a result of instability.

Surface friction transports heat vertically through a shallow layer, but since we are interested in the behaviour of deep layers this effect will be neglected. Moreover, surface turbulence is partly convective in origin, and we may regard shallow convection as included in this argument. But sometimes (*e.g.*, in strong polar outbreaks) deep and widespread convection transports heat to great heights at a great rate. We shall ignore this possible complication and concentrate our attention on systems in middle and high latitudes which are statically stable in their initial stages.

Just as, in the long run, we cannot ignore skin friction so, in the long run, we cannot ignore radiative processes. Large-scale turbulence (the statistical aspect of our disturbances) appears to be a major factor in transporting heat poleward to compensate the unbalanced radiation flux. But during the nascent stage, development (measured by the time for growth of the disturbance by a given factor) is relatively rapid and it is precisely for this reason that we are able to neglect frictional terms. Hence it is reasonable to suppose that in the nascent stage we may, for a first approximation, neglect the change in the radiation balance caused by the disturbance and use for our thermal equation the adiabatic equation.

Consider first the case of unsaturated air. To a close enough approximation the entropy of dry air is measured, in suitable units, by $\Phi \equiv (1/\gamma) \ln p - \ln \rho$, where p = pressure, ρ = density, γ = specific heat ratio, and Φ is conserved during the motion. In this case we shall define the static stability, which measures the restoring force due to gravity on a particle displaced vertically, as $\partial\Phi/\partial z$ (z = vertical coordinate). Now consider the case of saturated air in contact with a cloud. The static stability is now measured by the difference between the actual entropy lapse and that of the appropriate wet-adiabatic. Thus there is a sharp, and usually a large, reduction in static stability when air becomes saturated. The effective horizontal entropy gradients are also modified as a result of saturation, but to a much smaller extent. Normally a cloud mass behaves, to a sufficiently close approximation, as if the air were unsaturated except for the appropriate modification in static stability.

Our equations are complicated by the fact that air is a compressible fluid, but it is clear that this feature is not, for our purposes, a significant one. We are concerned essentially with a particular type of "vibration" problem though our disturbances have mathematically complex wave velocities. The moduli of these wave velocities are in all cases, as our calculations verify, small compared with the velocity of sound. It is not surprising therefore that the forces associated with compressibility are negligible, that is, that the air behaves *dynamically* as if it were incompressible. The static effect of compressibility, involving a large change of density with height, is a complication which prevents atmospheric motion from being quite the same as that of an incompressible fluid. Nevertheless, even for deep disturbances, the behaviour differs little from that of an incompressible fluid of similar mean density: the modifications are essentially of the nature of distortions of wave structure without much change in more significant features like growth rate. We shall therefore confine our attention to "equivalent" incompressible fluid systems. Our results are, of course, more directly applicable to analogous oceanographic problems.

Lack of space prevents a discussion of these points in mathematical terms. It has been shown elsewhere [3] that the basic equations may be further simplified by the elimination of the pressure field, so that we have finally four equations connecting the four dependent variables V_x, V_y, V_z (velocity components), and Φ (entropy). But our present concern is the physical interpretation and practical significance of certain calculations rather than the calculations themselves.

The General Theory of the Instability of Fluid Motion

The various types of instability occurring in dynamical meteorology merge into one another so that most systems encountered in practice are, to a greater or less degree, hybrid. Nevertheless it is not only simpler but theoretically more instructive to consider certain ideal limiting cases where one or another unstabilising factor acts alone. Four simple types of instability will interest us:

1a. Gravitational instability (ordinary convection or static instability).

1b. Centrifugal instability (dynamic instability).

2a. Baroclinic instability (with thermal wind).

2b. Helmholtz instability (at a velocity discontinuity).

Instability of type *1a* is, in middle and high latitudes, nearly always a small-scale phenomenon, but in low latitudes a modified form, taking into account the rotation of the earth, is intimately concerned with the development of tropical cyclones.

Instability of type *1b* has been the subject of much recent investigation, usually under the heading "dynamic instability," but despite its theoretical importance it is probably rare for large-scale motion. The name "centrifugal" has been preferred to "dynamic" because it is more descriptive and less confusing—other types of instability may reasonably be called "dynamic."

Instability of type *2a* is probably the most important, on a large scale, in middle and high latitudes. It is to this type that our earlier remarks regarding the normality of instability and the existence of "natural selection" directly apply.

Instability of type *2b* was investigated by Helmholtz and Rayleigh for nonrotating barotropic fluids. The Norwegian wave theory of cyclones was a partially successful attempt to extend the theory to rotating barotropic fluids.

Although we shall choose our initial systems so that only one type of instability is in question, the same general method of analysis applies in every case. Using the method of small perturbations, we obtain a set of simultaneous, linear, partial differential equations involving the perturbations as dependent variables. By elimination we obtain a partial differential equation with only one dependent variable and look for simple solutions satisfying appropriate boundary conditions. Usually these solutions involve only circular or exponential functions in the horizontal (x and y directions) and all contain the factor $e^{\theta_1 t}$ where t represents time and θ_1 is a constant called the growth rate. For θ_1 to be real we usually have to use a moving coordinate system. Fortunately these solutions for unstable waves are, practically, the most important ones and the disturbance of maximum growth rate, when it exists, is probably dominant relative to one of *arbitrary* initial structure. In any case a study of these particular solutions enables us to *understand* the process of breakdown of the initial system and to estimate the relative importance of various factors.

The method of analysis outlined above is necessary if we require precise results and is the only one which is completely unequivocal. But it is mathematical in form and usually rather involved so that significant physical principles, which give us insight into our problems and immediately suggest generalisations, tend to be obscured. Now, except that our interest is centred in the unstable region, we are concerned with what are essentially vibration problems and we may expect to find that energy considerations are of paramount importance. For, by the law of conservation of energy, the kinetic energy associated with any perturbation must be equal to the decrease in "potential" energy of the system, and a necessary condition for instability is that it should be possible to find displacements which will decrease "potential" energy; the condition will be sufficient only if these displacements are consistent with all the equations of motion and boundary conditions. More precisely, using Rayleigh's method, we may express separately the changes in kinetic and "potential" energy in terms of arbitrary displacements of the form $\delta x = e^{\theta_1 t}\delta x_0(x, y, z)$, etc. The kinetic energy change contains the factor θ_1^2, while the "potential" energy change does not, so that the law of conservation of energy gives an expression for θ_1^2 in terms of the displacements. The possible simultaneous values of the displacements are restricted (or constrained) by the equations of motion and we can to some extent delimit possible values of θ_1^2 by considering only some of the constraints, as in somewhat analogous problems in dynamics. In the present instance we consider only the equation of continuity and one momentum equation (and suitable boundary conditions) and then apply algebraic inequality theory to our undetermined displacements. We thereby obtain an *upper* bound to the possible value of θ_1^2 (corresponding to *negative* square of frequency) just as in dynamics we obtain a lower bound to frequency (squared) by considering a less constrained system.

The value of the foregoing, described in more detail elsewhere [3], derives partly from the fact that we can treat a wider variety of problems than we can by complete solution, while the value of θ_1^2 (maximum) thereby obtained is usually not much greater than the true value of θ_1^2. But its greatest usefulness is that it makes the process of breakdown of unstable systems immediately intelligible. If we take into account only our limited set of constraints, it is immediately evident what kind of displacement field is necessary for a release of "potential" energy. It is of course essential that the term "potential" energy be correctly interpreted. For our purpose it comprises *all* forms of energy other than the kinetic energy of the perturbation and therefore includes, besides gravitational potential energy, the organised kinetic energy of the mean flow (smoothed of harmonic variation). This distinction between two kinds of kinetic energy change is justified by their different roles in the turbulent motion which is the ultimate state in practice: turbulent energy may arise either from a decrease in gravitational potential energy or from a decrease in kinetic energy of the mean motion. We may classify our systems according to whether the "potential" energy source is (*a*) static (gravitational) or (*b*) dynamic (kinetic). Now the displacement field is merely the nascent form of a process of overturning and we shall need to consider only two possibilities: (1) overturning in a *vertical* plane; (2) overturning in a *quasi-horizontal* plane. Thus we may also classify our systems according to the kind of overturning associated with instability. In our list of four simple types of instability we anticipated their classification from both points of view. Let us consider the characteristics of these systems.

1*a*. *Gravitational Instability.* We consider barotropic conditions, so that initially there is no wind change with height, and for simplicity we suppose that $d\Phi/dz = B$, where B is constant. If, to begin with, we neglect the rotation of the earth, then "potential" energy exists only in gravitational potential form. Suppose that two small parcels of air of equal potential volume were slowly interchanged. Then since potential density would depend only on entropy, we should obtain, if B were negative, a net release of energy for *any* two parcels at different levels, while if B were positive no interchange could release potential energy. Clearly the constraints associated with the continuity equation cannot alter this result—the overturning process is equivalent to a set of such interchanges of different amplitudes. Since horizontal motion does not affect potential energy we need consider only vertical overturning; calculations by the energy method give $\theta_1^2 \leq -gB$, where g = gravitational acceleration. Of course in this simple case it is easy to obtain complete solutions, representing the nascent stage of Bénard cells, and calculations show that θ_1^2 (maximum) is nearly attained for narrow deep cells, where little energy is wasted in horizontal motion. We shall postpone the extension to large-scale convection, where the rotation of the earth is considered, since this is really a combination of types 1*a* and 1*b*.

1*b*. *Centrifugal Instability.* We shall suppose the motion to be barotropic and horizontal with the initial velocity V_x a function of y only. For simplicity we take dV_x/dy constant and, to begin with, we put $B = 0$ (isentropic conditions). Then "potential" energy exists only in the "kinetic" form. Let us consider the change due to overturning in the (vertical) y, z plane. Filaments of air in the x-direction move as a whole and we easily derive from the equations of motion that, during displacement, $\delta V_x = f\delta y$, where f is the Coriolis parameter. If the x-axis is directed toward the east, this corresponds to constancy of absolute angular momentum. But for our purposes, where a mean value of f is used, the orientation of the x-axis is arbitrary. A simple calculation shows that potential energy is released only if $dV_x/dy > f$, corresponding to negative absolute vorticity, and the energy method gives $\theta_1^2 \leq f(dV_x/dy - f)$. Although values of dV_x/dy *near* the critical value are sometimes observed in narrow bands, it is doubtful whether centrifugal instability ever occurs on a large scale except perhaps in low latitudes. The rotation of the earth, normally at least, has a stabilising effect so far as *vertical* overturning is concerned.

Similar results are obtained if, instead of rectilinear motion, we consider a barotropic circular vortex (with no motion relative to the earth as a special case). The condition for instability is again negative absolute vorticity.

We may note that in both the foregoing cases maximum instability occurs for shallow, flat, cells since "potential" energy changes depend only on horizontal motion (no energy is wasted in vertical motion).

1*ab*. *Gravitational-Centrifugal Instability.* It is easy to combine the results of the previous sections for a system in which neither B nor dV_x/dy vanishes. There is instability if either B or $(f - dV_x/dy)$ is negative, for the cells may be either so deep that centrifugal stability is negligible or so shallow that static stability is negligible. The important practical case is that of no motion (special case of circular vortex) with $B < 0$. Instability occurs for disturbances which are sufficiently deep relative to their breadth, the condition being that there should be a *net* release of "potential" energy. In low latitudes not only is $-B$ sometimes (temporarily) relatively large but the stabilising effect of the earth's rotation is small, so that convection cells of relatively enormous diameter (nascent hurricanes) can develop.

In general, maximum growth rate corresponds either to very deep or to very shallow cells but this result is not of great significance because practical systems are very inhomogeneous.

2*a*. *Baroclinic Instability.* We suppose the initial motion to be rectilinear and take the initial velocity V_x to be a function of z only. For equilibrium this implies a horizontal gradient of entropy $A \equiv \partial\Phi/\partial y$, where, approximately ($A$ not too small), $dV_x/dz = -gA/f$. For simplicity we suppose A to be constant. Pure baroclinic instability should correspond to $B = 0$, but it will be convenient to consider directly the more general case $B \neq 0$. In practice we usually have (at least in the mean) $B > 0$, and this is the only case we need examine, for when $B < 0$ the system is obviously unstable. The isentropic surfaces have an angle of slope $\alpha(<\pi/2)$ given by $\tan\alpha = |A/B|$ and in practice we normally have $\tan\alpha \ll 1$.

Consider first the change in gravitational potential energy resulting from interchange of parcels of air in the manner of subsection 1*a*. The result is no longer independent of the y-displacement. If the direction of displacement lies outside the acute angle α, there is an increase of energy, but if it lies inside, energy is released. There is zero change for displacement either along the isentropic surfaces or horizontally, and calculation shows that maximum release of energy occurs (approximately, assuming $\alpha \ll \pi/4$) for displacement in the direction of the bisector of α (in the y, z plane), which we shall call the s-axis. Now consider the change in kinetic "potential" energy. If the overturning were in the vertical plane, this would occur in the manner of subsection 1*b*, with increase of energy. But if overturning occurs in the x, s plane (*i.e.*, quasi-horizontally), with the perturbations varying harmonically in the x-direction, there is no change in the mean motion and no change of energy (correct to the appropriate order of small quantities). Hence our energy method gives instability in *all* cases and on calculation:

$$\theta_1^2 \leq -g_s B_s \approx \frac{1}{4}\frac{f^2}{h^2} \qquad (h^2 >> 1),$$

where g_s and B_s are the components of gravity and entropy gradient, respectively, along the s-axis (note analogy to 1*a*) and h^2 is the Richardson number defined by $h^2 = gB/(dV_x/dz)^2$.

This result is of course provisional but, as in other cases, is verified by complete solution. With artificial (but physically possible) boundary conditions we can obtain nearly the maximum value of θ_1, but a more realistic model gives $\theta_1 = 0.31\, f/h$. The reduction in the coefficient from 0.5 to 0.31 is due to the additional constraints imposed by the boundary conditions, as a result of which displacements cannot everywhere be in the optimum s-direction: for one particular wave length (more precisely, for one ratio of horizontal to vertical "wave length"), the displacements are as near optimum as possible and it is to this dominant wave that the coefficient 0.31 applies. Longer waves grow more slowly while very short waves are stable. Thus there is "natural selection" for one particular wave structure. It has been shown elsewhere how, by considering compound systems containing a region where h^2 is a minimum, realistic models of both nascent wave cyclones and long waves may be constructed. Briefly, the smaller disturbances develop in frontal regions, where cloud masses reduce the effective static stability and therefore also h^2. The long waves occupy the whole troposphere, and secondary modifications, caused by constraints associated with the variability of the Coriolis parameter, are then significant.

1*ab*. *Generalised Vertical-Overturning Instability.* We may consider from the point of view of *vertical* overturning an initial system similar to that of subsection 2*a*, but it will be convenient to generalise by supposing V_x to vary with y as well as with z. Then using the same general method as before, we obtain

$$2\theta_1^2 \leq -\left[gB + f\left(f - \frac{dV_x}{dy}\right)\right] + \sqrt{\left[gB + f\left(f - \frac{dV_x}{dy}\right)\right]^2 + 4\left[(gA)^2 - gBf\left(f - \frac{dV_x}{dy}\right)\right]},$$

where the surd has always to be taken as positive. This is the general formula for vertical overturning, including the examples previously given as special cases. If either $B < 0$ or $dV_x/dy > f$, the system is certainly unstable. If neither condition is satisfied, we require for instability $(gA)^2 > gBf(f - dV_x/dy)$, which is equivalent to $1/h^2 > [1 - (1/f)(dV_x/dy)]$. In the important special case when dV_x/dy vanishes, this condition becomes simply $h^2 < 1$, equivalent to the well-known condition of negative absolute vorticity in the isentropic surfaces.

2*b*. *Helmholtz Instability.* Consider the system of two barotropic air masses with uniform horizontal motion $V_x = U_1$ in one, and $V_x = U_2$ in the other, separated by a vertical "front" at the x, z plane. If the earth were not rotating, we could apply the well-known results of Helmholtz (complete solutions) which show this system to be unstable for *all* perturbation wave lengths, growth-rate being inversely proportional to wave length, but it is instructive to apply the energy method. Helmholtz's solutions involve only horizontal motion, associated with corrugation of the "front," so we need consider only *horizontal* overturning. The "potential" energy is entirely "kinetic" and the manner of its release is clear from the flow pattern, obtained by considerations of continuity and boundary conditions alone. Outside the y-limits to which the corrugations of the "front" extend, the mean motion is unaltered, but inside these limits there is a change in the mean flow of each air mass (in opposite directions) resulting in a decrease in organised kinetic energy (see Fig. 1).

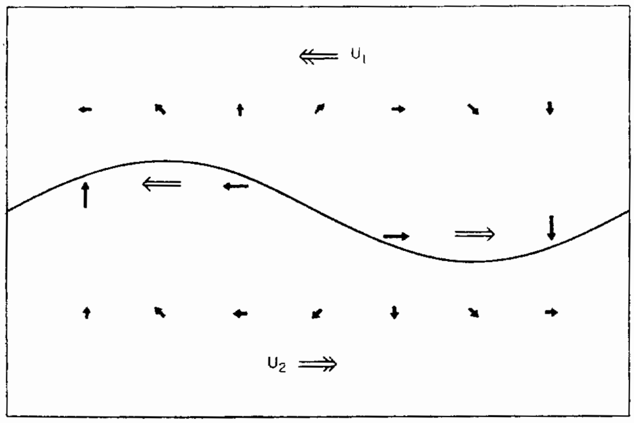

FIG. 1.—Helmholtz waves. Arrows with a single head and a single shaft denote perturbation velocities; those with a single head and double shaft, perturbation mean velocities; those with a double head and a double shaft, initial velocities.

Now, as we have seen, the earth's rotation has a stabilising effect only when there is *vertical* overturning. Hence we should expect results similar to those mentioned above when the earth's rotation is taken into account. Complete solution of the problem confirms this, and $\theta_1^2 = (\lambda^2/4)(U_1 - U_2)^2$, where $2\pi/\lambda$ is the wave length for a rotating, as well as for a nonrotating, system.

In practice, B is usually strongly positive so that any vertical motion would decrease the net release of "potential" energy. Moreover, frontal surfaces are not usually vertical and in general the boundary conditions cannot be satisfied by purely horizontal motion. Hence a sloping front is less unstable than a vertical one. Since the unstabilising effect of a velocity discontinuity is inversely proportional to the wave length, whereas the stabilising effect of static stability is independent of the scale of motion, it follows that only waves shorter than a critical wave length are unstable. Very short waves are always unstable because static stability may be neglected.

Future Developments

The discussion above is merely an outline of elementary principles. Although our analysis shows that the equations of dynamical meteorology are by no means intractable from the point of view of computing future developments, the results so far obtained are only of limited applicability. Our calculations give only the initial form of the most probable (dominant) new development and we need to compute the further development when the perturbations are no longer small. As the period of the forecast is extended, analytical methods become increasingly involved and clumsy and sooner or later we have to resort, at least partly, to numerical methods. An adequate degree of accuracy is practically attainable only with the use of computing machines, and electronic large-memory computers will play an important part in extending and generalising the elementary theory.

The development of numerical methods, even to the extent of a direct attack using observed data, does not absolve us from the necessity of understanding the precise significance of our solutions. Not only do we have to know how and where to approximate, but the reliability of our solutions varies with time, place, and forecast period. In fact for long forecast periods what is significant is not the detail, which is usually partially, perhaps entirely, accidental (*i.e.*, dependent on minutiae below the margin of error), but the general nature (*e.g.*, persistently settled or unsettled) of the majority of possible solutions. We need to develop the statistical theory referred to earlier. Not all the questions about future weather we should like to answer are in fact answerable, but it may well be that the growth of uncertainty in some directions is compensated by statistical regularity in others.

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