Diagnosis of Early Baroclinic NWP Models

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(Manuscript received 23 July 1965, in revised form 4 December 1967)

ABSTRACT

On the basis of the evidence available to date it is concluded that the two most important faults of early baroclinic models, 1) overdevelopment in terms of a general increase in kinetic energy and 2) failure to amplify cyclone-scale, baroclinic wave disturbances which amplify in the atmosphere, were due, respectively, to the absence of a dissipation term to balance the kinetic energy generated in the model and to space truncation which imposed too large a minimum scale for amplification.

The solution to these problems is to reproduce in a model the scale dependence in the atmosphere of net development (development minus dissipation). The three paths available to achieve this goal are: 1) decrease the minimum resolvable scale (grid size), 2) reduce the space truncation of finite difference operators, and 3) increase the scale dependence of the dissipation term so that it removes energy only from the smallest permitted scales.

The current best estimate of the residence time for the total kinetic energy of the atmosphere is 2–4 days. Since about 70% of the total kinetic energy dissipation in the atmosphere occurs above the Ekman layer, it is unlikely that this can be adequately simulated by a surface friction term alone.

A possible source for a characteristically distinct behavior of 2-level models is proposed.

1. Introduction

The current success of operational NWP (Numerical Weather Prediction) grew out of the success of the barotropic model—a model atmosphere incapable of generating additional kinetic energy and therefore having no need for a braking or energy dissipating device. Early experiments with baroclinic models capable of generating additional kinetic energy from the store of available potential energy failed. The baroclinic forecasts were not simply no better—they were *worse* than the barotropic forecasts (Knighting and Hinds, 1960; Bengtsson, 1964). While such candor cannot be found in the literature, the same conclusion can be inferred for the JNWPU/NMC (Joint Numerical Weather Prediction Unit/National Meteorological Center) experiments from the successive dropping of the original Princeton 3-level model, the thermotropic 2-level model, and a subsequent 2-level model in favor of the barotropic (mesh) model. Similar results with early multi-level models seem to follow from the cautious statements of Bushby and Whitelam (1961): "The extra degree of freedom allowed in the new model (3-level) does not give rise to such vigorous over-development as in the two-parameter model . . ."; and by Edelmann and Reiser (1960): "The baroclinic model (5-level) delivers an improvement (over the barotropic) only in the forecast of the North American cyclone; in this area, the usually rather similar horizontal error distributions of the two models are evidently out of phase." The latter indicates that introduction of the primitive equations did not alter the problem.

It is the opinion of the author that the major cause of the failure of early baroclinic models was due to net accumulation of kinetic energy in the models due to the presence of the baroclinicity source and the absence of a dissipative sink of kinetic energy.

2. Atmospheric energy cycle

Let us consider the energy cycle of the atmosphere in terms of the simplified form represented schematically in Fig. 1. In the real atmosphere valves 1, 2 and 3 are open, allowing energy to flow through the system. Viewed on a hemispheric scale the atmosphere operates very nearly as a steady-state system so that the time rate of energy flow may be measured at valve 1 as $G(A)$, generation of available potential energy, at valve 2 as $C(A,K)$, conversion of potential to kinetic energy, or at valve 3 as D, frictional dissipation. The steady state and the existence of the outflow D guarantee a lack of equilibrium in the stores A (available potential energy) and K (kinetic energy) and a positive net flow $C(A,K)$ through valve 2.

When the steady-state system represented by the observed data is inserted in a barotropic model, valves 1, 2 and 3 are closed and only the redistribution of energy within K computed. In most barotropic models kinetic energy is lost from the store at a rate of about 5% per day due to truncation errors and smoothing. This is not serious for short-range prediction and

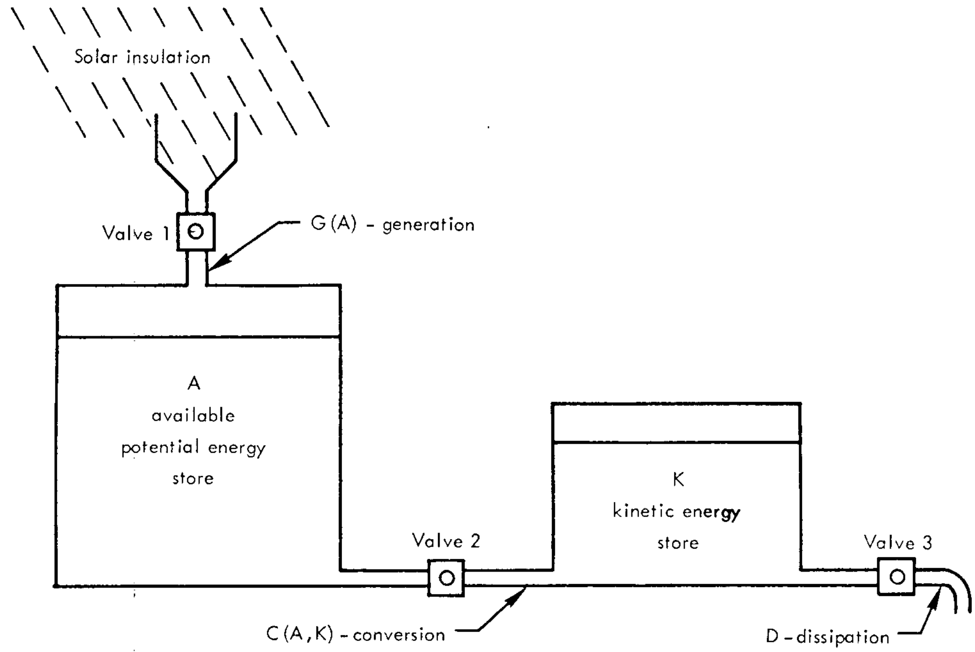

FIG. 1. Simplified schematic of atmospheric energy cycle.

generally improves verification scores (for non-zero phase errors, rms differences can for a time be reduced by decreasing forecast amplitudes).

In early baroclinic models only valves 1 and 3 were closed and because of the lack of equilibrium between the A and K stores, a positive $C(A,K)$ flow was calculated by the model just as was occurring in the atmosphere at the time of observation. Consequently, the kinetic energy store increased as the forecast advanced (commonly referred to in the literature as overdevelopment) and verification scores were worse than for barotropic models.

3. Atmospheric dissipation rate

The argument hinges on the rate of kinetic energy turnover in the atmosphere, i.e., the ratio of the energy flow through the system to that in the kinetic energy store. In a recent review paper Dutton and Johnson (1967) estimated $A=37.5$, $K=15(\pm 6)$ units of 10^5 J m^{-2} and $G(A)=C(A,K)=D=5.53$ in units of 10^5 J m^{-2} day^{-1}. These figures indicate that a sudden closure of valve 3 (by friction proofing the atmosphere or through a baroclinic model without dissipation) should lead to an increase of total kinetic energy at an initial rate of 25–50% per day. They say nothing as to how soon the A and K stores might reach an equilibrium partitioning which would bring the $C(A,K)$ flow to a halt. However, there are many reports in the literature of baroclinic models which displayed net accumulation of kinetic energy throughout forecast periods ranging up to several days. That an equilibrium partitioning exists is clear since the available potential energy cannot vanish unless the kinetic energy does also.

4. Model experiments

Fig. 2 shows the redistribution of energy in a simple hemispheric 2-level baroclinic model (see Appendix) using real initial data and a spectral integration scheme (Ellsaesser, 1966a) which preserved the total energy to within 0.4% (dot-dash curve in Fig. 2) for over 22 days. In this integration (without dissipation) the kinetic energy of the mean flow increased initially at a rate of 15% per day. An equilibrium partitioning between A and K appeared to become established after approxi-

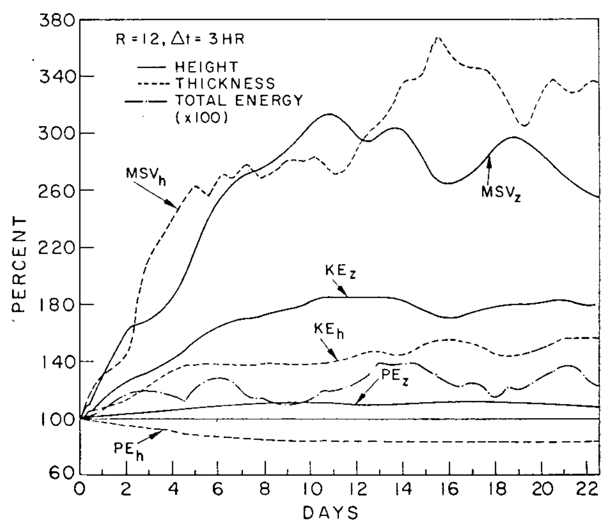

FIG. 2. Evolutions of energy and mean squared vorticity components in a 22-day spectral integration of a 2-level baroclinic model without eddy viscosity using real data. Spectral truncation was at hemispheric wavenumber 12 and time step was 3 hr. Symbols are defined in the Appendix.

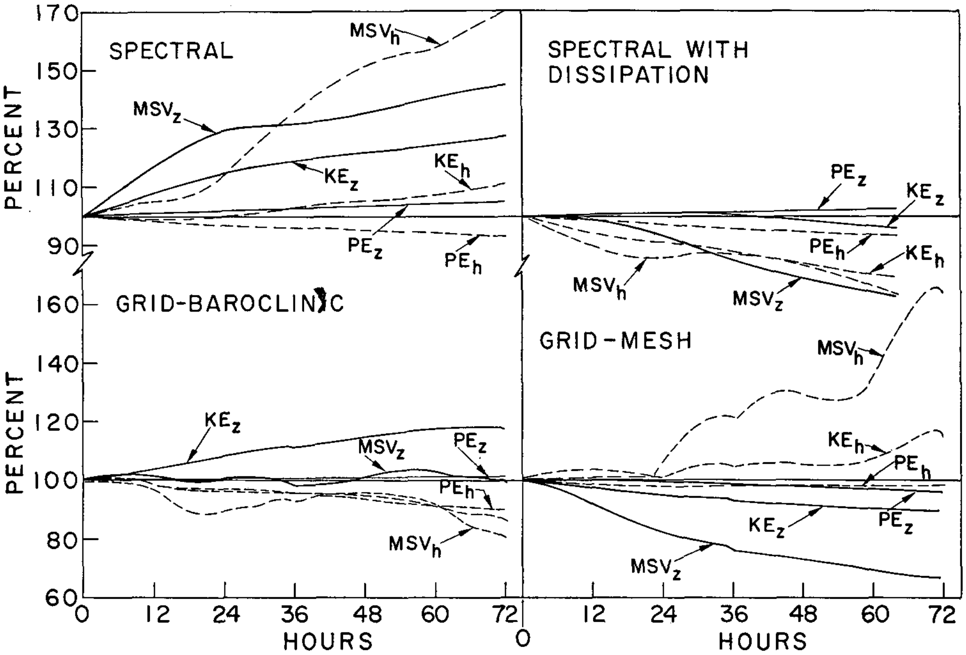

FIG. 3. Evolution of energy and mean square vorticity in 2-level spectral integrations with and without dissipation and 2-level grid integrations in baroclinic and barotropic (mesh) models without an eddy viscosity term. All are from the same real initial data. Symbols are defined in the Appendix.

mately 10 days. From this time on typical values of each energy component as a fraction of their initial values were:

KE_z (kinetic energy of mean flow): 1.70
PE_z (available potential energy of mean flow): 1.10
KE_h (kinetic energy of shear flow): 1.40
PE_h (available potential energy of shear flow): 0.85

The large increase in KE_z was not due to extensive intense cyclogenesis but to a general increase in pressure height gradients over the whole hemisphere. Both the magnitude and character of this increase indicated a serious departure from the behavior of the real atmosphere due in part, presumably, to the closure of valves 1 and 3 of Fig. 1.

In an effort to keep valve 3 open an eddy viscosity term was added to the model and the coefficient adjusted empirically until KE_z remained approximately constant during the early part of the integration. Since valve 1 was closed (PE_h not replenished), the rate of generation of KE_z decreased with PE_h and the dissipation term eventually predominated (see upper right panel of Fig. 3). The value of the eddy viscosity coefficient required to balance the initial growth of KE_z was about 3×10^9 cm^2 sec^{-1}.

In Fig. 3 the behavior of the baroclinic spectral integrations with and without eddy viscosity can be compared with grid point integrations (on NMC 1977 point octagon) of the same 2-level model with (baroclinic) and without (mesh or barotropic) the Sutcliffe development term. Neither grid integration had an eddy diffusion term. To provide comparable scale resolution these spectral integrations were truncated at hemispheric wavenumber 18 and used a 1-hr time step.

From the lower panel of Fig. 3 it is clear that there is substantial energy dissipation in the finite difference processes of the grid integrations, but insufficient to prevent KE_z from rising significantly in the baroclinic model—11% in 36 hr compared to a drop of 6% in 36 hr in the barotropic (mesh) model. The 17% difference in KE_z levels at 36 hr must appear as steeper height gradients and greater amplitudes of major pressure centers in the baroclinic integrations, i.e., overdevelopment.

Table 1 lists standard errors of 500-mb, 36-hr, pressure-height forecasts for the World Meteorological Organization (WMO) NWP Test Data of 29 November through 7 December 1962. Also given is KE_z of the 36-hr forecasts as a function of its value at the initial time. The forecasts were made by spectral integrations truncated at wavenumber 18 with both the barotropic model and the 2-level baroclinic model with a dissipation term. NMC operational mesh model forecasts were included on the data tape and were processed in the same way as the spectral forecasts to obtain the figures in the last two columns of Table 1.

The baroclinic forecasts achieved an average reduction in the 36-hr standard height error of 6% but much of this is due to the lower level of kinetic energy in the baroclinic forecasts, 92.3% of initial compared to

98.7% for the barotropic forecasts.[1] It is instructive to make a log-log plot of the mean forecast errors vs mean KE_z for the three sets of forecasts in Table 1. That the three points fall almost exactly on a straight line does not appear to be entirely fortuitous.

Perhaps we have again demonstrated that the 2-level baroclinic model is no better than the barotropic; we prefer to believe that we have demonstrated that it is *no worse.*

5. 2-level vs multi-level models

Because of the almost universal finding of overdevelopment and inferior verification of 2-level models compared to the barotropic, NWP operational units tended to adopt the barotropic (or mesh) model for operational use and turned their research effort away from 2-level models. The feeling spread that the 2-level model suffered in some unknown way, as Bengtsson (1964) put it, from "... the geometrical constraints on the three-dimensional wind field depending on the lack of vertical resolution." In two separate efforts, however, Wiin-Nielsen (1961, 1962) failed to find any characteristic weakness in the 2-level theory.

That this weakness was not attributed to baroclinic models in general was due to several factors: discounting of earliest limited area multi-level model experiments due to unknown boundary effects; the restricted number of hemispheric multi-level experiments and consequent uncertainty and caution in reporting of results; and the lesser degree of overdevelopment in the British 3-level (Bushby and Whitelam, 1961) and the German 5-level (Edelmann and Reiser, 1960) models. Only the latter of these factors is subject to examination.

Fig. 4 shows plots of A, K and $C(A_z,A_E)$ [the latter being a conversion of zonal to eddy available potential energy which must precede $C(A,K)$] from Wiin-Nielsen[2] and Wiin-Nielsen *et al.* (1964), and $C(A,K)$ and D from Kung (1966) as functions of pressure—all evaluated from atmospheric data. Presuming that a baroclinic model without dissipation behaves like that portion of the atmosphere which it knows about (the levels at which observed data are supplied to it), we would expect a 2-level model using 500- and 850-mb data (the most common levels for a 2-level model) to display an anomalously rapid rate of increase in model kinetic energy. The 35% of the atmosphere represented by data contains 46% of A, 19% of K, 45% of $C(A_z,A_E)$ and 19% of $C(A,K)$. While $C(A,K)$ and K are underrepresented by the same amount and should thus have no effect on the percentage rate of increases in K, relative to them A and $C(A_z,A_E)$ are overrepresented by a factor of almost 2.5. In our simplified model of Fig. 1 this corresponds to an equivalent increase in the pressure head behind valve 2. As to the effect in the real atmosphere we need only compare winter and summer and the corresponding values of D and $C(A,K)$, i.e., $C(A,K)$ varies with the ratio of A to K.

From Fig. 4 it is readily apparent that inclusion of any data level above 500 mb and particularly one near 300 or 200 mb would bring the represented quantities more nearly into the correct ratio for the total atmosphere. This may help to explain the much better 2-level model results achieved by Phillips (1958) who used 400- and 900-mb data and Økland (1965) who used 300- and 700-mb data.

Herein appears to lie a possible source for a characteristically distinct behavior of 2-level models (i.e., a more rapid rate of kinetic energy rise) compared to baroclinic models in general. So far as the author is aware no other reason for a characteristically distinct behavior has been proposed.

6. Scale dependence of energy transformations

In addition to the spectral integrations reported above, several (truncated at wavenumber 18) were integrated from initial conditions consisting of a baroclinic zonal current (from real winter data) with a superposed solitary baroclinic wave satisfying the 2-level linear theory instability criteria. In all cases the wave failed to amplify; instead, it dispersed. Simultaneously, net potential-to-kinetic energy conversions occurred at all wavenumbers represented in the flow.

Several conclusions have been drawn from these experiments. The conservation of total energy and the eventual equilibrium in Fig. 2 shows that the instability is not numerical and thus must be an analogue of atmospheric baroclinic instability. Amplification of nonperiodic and non-sine wave disturbances is accomplished through the amplification *and superposition* of the wavenumbers *and their higher harmonics* which make up the disturbance. Amplification with higher harmonics *prohibited* can be accomplished only by placing a severe constraint on the relative amplification rates and phases of the components of the disturbance. Amplification of a wave cyclone of a particular scale size will not appear under Fourier analysis as $C(A,K)$ *solely* at the wavenumber corresponding to this scale. A scale resolution sufficient to represent a particular scale of disturbance is not necessarily sufficient to represent the higher harmonics necessary *to reproduce the evolution* of the disturbance. As shown previously (Ellsaesser, 1966b), an orthogonal expansion and re-

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[1] The decrease of kinetic energy in the barotropic spectral forecasts is not due to dissipation, either physical or numerical. It is a result of the rudimentary type of baroclinicity allowed by the Helmholtz term in the model [see (A3) of Appendix]. In all models with a Helmholtz term tested by the author the net transfer has been from KE_z to PE_z; apparently, another manifestation of the tendency of the models to predict an excess of anticyclogenesis in low latitudes. In only one case was a temporary reverse flow noted. In barotropic models without a Helmholtz term, PE_z was also observed to increase even though KE_z and MSV_z remained constant as anticipated from (A3) with $\kappa^2=0$.

[2] Wiin-Nielsen, A., 1964: Some new observational studies of energy and energy transformations in the atmosphere. Paper presented at the Symposium on Research and Development Aspects of Long Range Forecasting, Boulder, Colo.

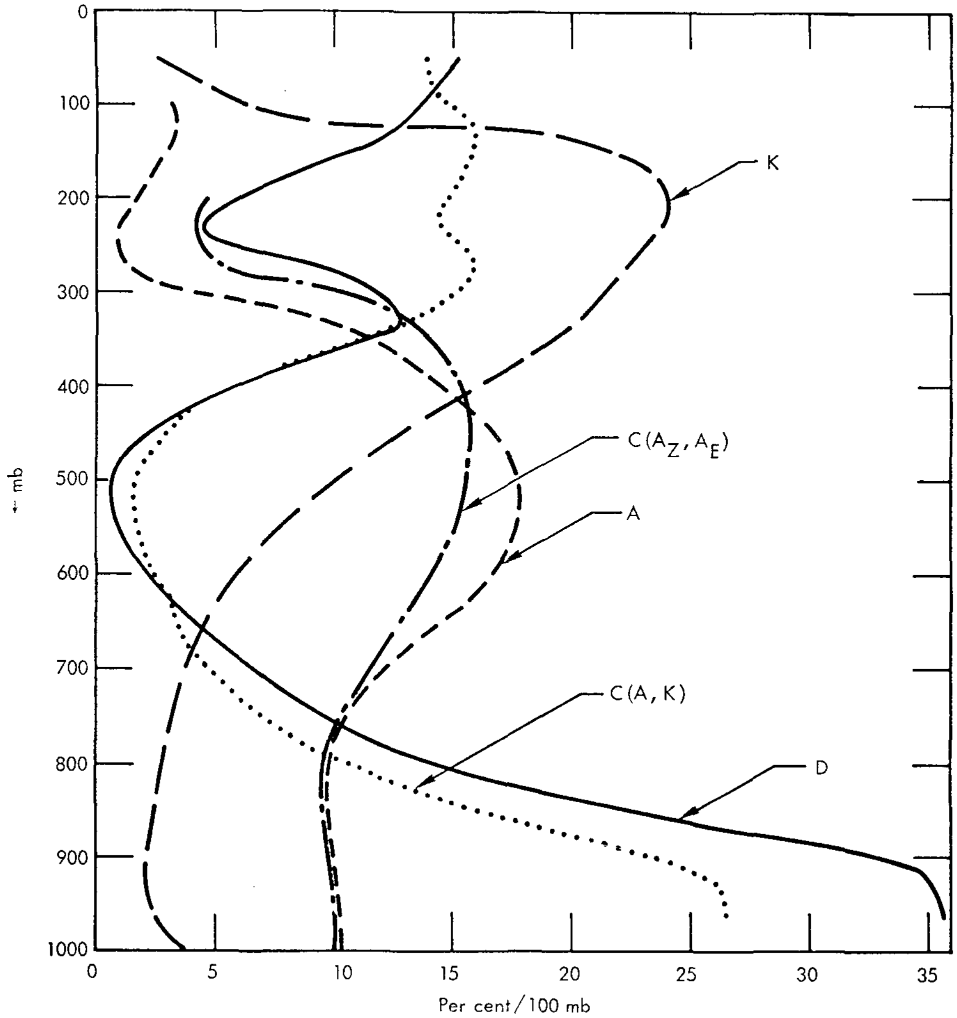

FIG. 4. Available potential energy A and kinetic energy K from Wiin-Nielsen (*loc. cit.*), conversion of zonal to eddy available potential energy $C(A_z,A_E)$ from Wiin-Nielsen *et al.* (1964) and conversion of potential to kinetic energy $C(A,K)$ and dissipation of kinetic energy D from Kung (1966) as a function of pressure.

generation routine was able to regenerate 95.21% of the variance of a stream function field but the regenerated field contained only 63.67% of the vorticity variance of the original.

In the preceding sections we merely proposed a diagnosis of the major problem of early baroclinic NWP models. Addition of an eddy viscosity term of the type shown in the Appendix can stop the accumulation of kinetic energy in the model but does not guarantee a better forecast of atmospheric behavior, the reason being that the scale dependence of energy transformations (development, dissipation and the net difference) of the model is not the same as that of the atmosphere. This brings us to the second most important weakness of early baroclinic models, the failure to reproduce the cyclone-scale wave cyclogenesis observed in the atmosphere.

The major source of the difference in scale dependence is the finite number of degrees of freedom in the models determined by the grid size or the truncation parameter. The minimum resolvable scale of a model imposes a somewhat *larger* minimum amplifiable scale both because of space truncation in computations of derivatives and because of inability to represent the higher harmonics contributing to amplification. This leads to the apparent contradiction in characteristics of the 2-level model (reported by Gates 1961), "to overestimate the growth of relatively mature disturbances . . ." and ". . . to underestimate the growth of relatively new large-scale disturbances." A recent study by Brown (1967) is very relevant to this contradiction—it showed the wavelength of maximum *baroclinic* instability to be substantially shorter (and therefore more subject to damping by truncation error and viscosity) than the wavelength of maximum *barotropic* instability.

Cressman (1963) reported amplification rates varying directly with scale size and a minimum scale for amplification of his 3-level model. Scale dependence of model

TABLE 1. Standard errors in feet and values of KE_z as a fraction of its initial value, for 36-hr predictions of the 500-mb pressure height for the WMO NWP Test Data, 29 November through 7 December 1962. Forecasts were made by a barotropic model and a 2-level baroclinic model with dissipation both integrated by the spectral method and truncated at wavenumber 18.

Valid times (GMT)	Barotropic		Baroclinic (with dissipation)		NMC Operational	
	KE_z	Std. error	KE_z	Std. error	KE_z	Std. error
1200 1 Dec.	0.985	189	0.911	183	0.853	185
0000 2 Dec.	0.986	178	0.900	170	Missing	
1200 2 Dec.	0.982	187	0.896	169	0.846	165
0000 3 Dec.	0.983	187	0.930	169	0.860	175
1200 3 Dec.	0.982	200	0.942	186	0.823	182
0000 4 Dec.	0.980	205	0.943	193	0.872	171
1200 4 Dec.	0.996	227	0.930	210	0.893	186
0000 5 Dec.	0.998	204	0.912	191	0.814	166
1200 5 Dec.	0.994	201	0.929	189	0.854	162
0000 6 Dec.	0.985	193	0.929	178	0.855	170
1200 6 Dec.	0.984	214	0.941	209	0.925	175
0000 7 Dec.	0.991	205	0.921	193	0.835	166
1200 7 Dec.	0.989	219	0.909	200	0.814	180
Average	0.987	200.7	0.923	187.7	0.854	173.6

behavior has also been reported by Smagorinsky,[3] Howcroft (1967), and in the original tests at Princeton by Charney and Phillips (1953).

The minimum resolvable scale imposed by the grid size also affects the scale dependence of the dissipation term. In the spectral results presented in Table 1 the effect was not so apparent but in comparable grid integrations a dissipation term just sufficient to prevent an increase in KE_z noticeably damped some large-scale disturbances which were not damped by the atmosphere. The scale dependence of the diffusion term used to control nonlinear instability in general circulation and convection experiments is well known. Integration of Leith's model from real initial data (unpublished) rapidly reduced the mean squared meridional velocity. Similar, but less marked, results have been reported for Smagorinsky's model (Miyakoda *et al.*, 1967). Differences in model and atmospheric scale dependence is also indicated by the greatly reduced ratio of eddy to zonal kinetic energy in general circulation experiments as compared to the atmosphere.

Three paths are open to bring the scale dependence of net energy transformations (development minus dissipation) in numerical baroclinic models nearer to that in the atmosphere: 1) reduce the minimum resolvable scale, 2) reduce the space truncation of finite difference operators, and 3) increase the scale dependence of the dissipation term.

The first has been tried by Smagorinsky (*loc. cit.*) and Howcroft (1967) with considerable success. The second was tried by the author with limited success in the spectral investigations reported above. This did not improve the small-scale cyclogenesis of the model but did improve the scale dependence of the dissipation term. Due to the reduction in degrees of freedom, the minimum resolvable scale may have been larger in the spectral integrations. Cressman (1963), Shuman (1962), and Økland (1965) appear to have achieved considerable success by local space averaging of redundant finite difference operators which appears to both reduce space truncation and remove energy from disturbances of a scale comparable to the grid size.[4] Making the dissipation term more sensitive to scale size was achieved by Smagorinsky (1963) by making the eddy diffusion coefficient a function of the local deformation. From the ratio of eddy to zonal kinetic energy in his general circulation experiments and the time decay of amplitudes in his integrations from atmospheric initial data, it is clear that more remains to be done. Either his grid size is too large to permit normal growth of atmospheric scale eddies, or his diffusion term is not sufficiently selective in removing energy only from the smallest scales, or both.

Acknowledgments. This work was performed under the auspices of the United States Atomic Energy Commission.

APPENDIX

The equations of the NWP model used by the author are

$$\left.\begin{aligned}(\nabla^2-\kappa^2)\partial\bar{\psi}/\partial t&=-J(\bar{\psi},\nabla^2\bar{\psi}+f)\\&\quad-J(\psi',\nabla^2\psi')-K\nabla^2(\nabla^2\bar{\psi})\\(\nabla^2-\nu^2)\partial\psi'/\partial t&=-J(\bar{\psi},\nabla^2\psi')-J(\psi',\nabla^2\bar{\psi}+f)\\&\quad+\nu^2J(\bar{\psi},\psi')-K\nabla^2(\nabla^2\psi')\end{aligned}\right\}, \quad \text{(A1)}$$

where

$\nabla\bar{\psi}\equiv 0.85(g/f_0)\mathbf{k}\times\nabla Z_{500}$,

$\nabla\psi'\equiv 0.636(g/f_0)\mathbf{k}\times\nabla(Z_{500}-Z_{850})$,

$\kappa^2\equiv 4f_0^2/(gz)$, $z=0.85\times 500$-mb standard height,

$\nu^2\equiv 2f_0^2g/\sigma^2$, $\sigma^2\equiv$ stability factor,

$f_0\equiv$ Coriolis parameter at 45N,

$K=3\times 10^9$ cm^2 sec^{-1}.

[3] Smagorinsky, J., 1967: Paper presented before the New York Academy of Sciences at the 47th Annual Meeting of the Amer. Meteor. Soc.

[4] *Note added in proof:* a factor considered to be of greater importance in this regard was Økland's (1965) use of Lagrangian integration, a fact which he neglected to report.

When K is set to zero this model has the integral invariant

$$KE_z+KE_h+PE_z+PE_h=\text{constant}, \qquad \text{(A2)}$$

where

$$KE_z\equiv\frac{1}{A}\int(\nabla\bar{\psi})^2dA,$$

$$KE_h\equiv\frac{1}{A}\int(\nabla\psi')^2dA,$$

$$PE_z\equiv\kappa^2\times\text{variance of }\bar{\psi},$$

$$PE_h\equiv\nu^2\times\text{variance of }\psi'.$$

With ψ' also set to zero (barotropic case) the integral invariants become

$$\left.\begin{array}{l} KE_z+PE_z=\text{constant} \\ MSV_z+\kappa^2KE_z=\text{constant} \\ MSV_z\equiv\text{mean squared vorticity of mean flow}\end{array}\right\}. \qquad \text{(A3)}$$

With κ^2 set to zero (nondivergent barotropic case) the invariants are clear from (A3) and the definition of PE_z.

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