EAS 471 Assignment 4 2007

Team NWP project: two-layer quasi-geostrophic model (Update: March 25, 2007)

Team will perform a 12 hour weather forecast, deriving initial conditions from a CMC 12 Zulu analysis and comparing our forecast solution with the following CMC 00 Zulu analysis. Note that since we include neither boundary forcing nor surface forcing, the evolution of any pressure system resolved in the initial state will be highly over-simplified.

Holton (2004, Sec. 8.2) and Haltiner & Williams (1980) both discuss the 'two-layer' baroclinic NWP model based on the quasi-geostrophic vorticity equation and the hydrostatic thermodynamic energy equation. We can regard the dependent variables of the model as being the streamfunction ψ and the vertical velocity ω, because other needed variables can be determined from these, eg. the geopotential, vorticity and Geostrophic velocity components are:

$$\begin{aligned} \Phi &= f_0\,\psi \\ \zeta &= \nabla^2\psi \\ U &= -\,\partial\psi/\partial y \\ V &= +\,\partial\psi/\partial x \end{aligned} \tag{1}$$

where ∇ is the 2D grad operator and f_0 is the Coriolis parameter evaluated at the central latitude of the domain. Note: in the QG model advection is always computed using the Geostrophic wind, and vertical advection is neglected. The only place the 'true wind' explicitly appears is where its divergence appears in the QG vorticity equation, viz. the term $D_p\,f_0$. Now recall that the continuity equation in pressure coordinates reads $D_p + \partial\omega/\partial p = 0$, so that in the equations below the terms involving $\omega_2\,f_0/\Delta p$ are exactly this term. Since ω is non-zero only at level 2, $\partial\omega/\partial p$ is represented by the finite difference $\pm\,(0-\omega_2)/\Delta p$.

Fig. (1) indicates the structure of the two layer model. The governing equations, viz. Holton's eqns.(8.5-8.7) re-written in flux form by exploiting the fact that the Geostrophic velocity is nondivergent (when defined with fixed $f = f_0$, e.g. Holton p148, 4th edition), are

$$\frac{\partial \zeta_1}{\partial t} = -\frac{\partial}{\partial x}\left(U_1\,\zeta_1\right) - \frac{\partial}{\partial y}\left(V_1\,\zeta_1\right) - \beta\frac{\partial \psi_1}{\partial x} + \frac{f_0}{\Delta p}\,\omega_2 \tag{2}$$

$$\frac{\partial \zeta_3}{\partial t} = -\frac{\partial}{\partial x}\left(U_3\,\zeta_3\right) - \frac{\partial}{\partial y}\left(V_3\,\zeta_3\right) - \beta\frac{\partial \psi_3}{\partial x} - \frac{f_0}{\Delta p}\,\omega_2 \tag{3}$$

$$\left(\frac{\sigma\,\Delta p}{f_0}\right)\omega_2 = -\frac{\partial}{\partial t}\left(\psi_1 - \psi_3\right) - \frac{\partial}{\partial x}\left[U_2\left(\psi_1 - \psi_3\right)\right] - \frac{\partial}{\partial y}\left[V_2\left(\psi_1 - \psi_3\right)\right] \tag{4}$$

where the Geostrophic velocity at level 2 can be determined by

$$\vec{U}_2 = \hat{k} \times \nabla\frac{\psi_1 + \psi_3}{2} \tag{5}$$

(linear interpolation of stream function). Note the constant multiplier on the l.h.s. in eqn. (4). The pressure interval $\Delta p \equiv 5 \times 10^4$ Pa, and f_0 is constant. The static stability parameter, in general defined as

$$\sigma = -\alpha\frac{\partial \ln \theta}{\partial p} \tag{6}$$

here takes on a value appropriate to a standard atmosphere under the quasi-geostrophic model. We need σ at the 500 mb level, and Holton quotes $\sigma = 2.5 \times 10^{-6}\ \mathrm{Pa}^{-2}\,\mathrm{s}^{-2}$ as a mid-tropospheric value. However for the sake of the exercise we shall evaluate it from a sounding in our region of interest, at $t = t_0$.

Notice that eqns. (2, 3) are almost entirely uncoupled - the only connection between levels 1,3 of our model atmosphere is through ω_2, the mid-level vertical velocity. If $\omega_2 > 0$ (sink at the LND) there is a tendency for vorticity aloft at 250 mb to be increasing (for at this level $\partial\omega/\partial p > 0$ meaning $D_p < 0$, negative divergence, ie. shrinking column area) — while the opposite occurs below the LND at 750 mb.

Now as regards the nature of the numerical solution procedure, it is significant that each of eqns. (2, 3) is of the same *form* as Holton's (p462) forecast problem based on the barotropic vorticity equation, and differs only because we have a term in ω_2 on the r.h.s. Clearly then, if we regard ω_2 as known, we can treat the entire r.h.s. of each of eqns. (2, 3) as a source function, i.e. $F_1(x, y, t)$ and $F_3(x, y, t)$, and carry over the solution method outlined in Holton's Sec. (13.4).

Once we have updated ψ_1, ψ_3, we use the thermodynamic equation (eqn. 4, which explicitly governs the thickness tendency) to diagnose ω_2.

Numerical Method

The instructor's experience in programming this prog is that an explicit method with a forward time step and central spatial differences is unconditionally unstable, and indeed remains unstable even if artificial diffusion is introduced. This is not too surprising, since the linear advection equation is known to be unstable when discretized this way ('Euler discretization'), and our vorticity equations here entail little else than advection.

The problem of instability can be overcome by using a 'leap-frog scheme' or three time level scheme. Thus, the vorticity and stream function are stored at time levels $n-1, n, n+1$ and

$$\left(\frac{\partial \zeta}{\partial t}\right)^n = \frac{\zeta^{n+1} - \zeta^{n-1}}{2\,\Delta t} = F(\text{forcing at time level n}) \tag{7}$$

The forcing term can be evaluated using central differences.

Initialization

We decide on a domain... let it be square, of sidelength $D = 1000$ km. It will be simplest to use equal gridlengths $\Delta x = \Delta y = \Delta$, and let us suppose the origin of the grid is the SW corner $(I = J = 1)$. Let $I = 1...I_{mx},\ J = 1...J_{mx}$ where $I_{mx} = J_{mx} = 10$.

We discretize the height fields within the domain at 750 mb and 250 mb, to get the initial streamfunction, eg.

$$\psi_1(x_I, y_J, 0) = \frac{g}{f_0}\, h_{250}(x_I, y_J) \tag{8}$$

and we also need to get an initial field of ω_2 off the appropriate 500 mb vertical velocity chart. The initial fields of Geostrophic velocity and vorticity are obtained by differentiating the streamfunction (using the usual finite difference approximations).

Boundary conditions

We will need ψ on the boundary. Of course the gradient $\partial\psi/\partial y$ on the western boundary controls the inflow windspeed, thus ψ will be non-zero on the boundary. Perhaps we will treat the boundary value of ψ as being independent of time.

Time-stepping

We had better choose the timestep to satisfy the CFL condition based on $\Delta = 100$ km. Using a simple Euler step, we integrate forward one timestep, eg.

$$\zeta_1(x_I, y_J, t_0 + \Delta t) = \zeta_1(x_I, y_J, t_0) + \left(\frac{\partial \zeta_1}{\partial t}\right)_{t_0} \Delta t \tag{9}$$

Now, knowing the new vorticity $\zeta_1(x_I, y_J, t_0 + \Delta t)$ we need to solve

$$\nabla^2 \psi_1 = \zeta_1(x, y, t_0 + \Delta t) \tag{10}$$

to get the updated streamfunction... knowing which allows us to differentiate to get the velocity. Eqn. (10) is diagnostic, and constitutes an elliptic (ie. "jury") problem. ζ is a source function. There is a straightforward iterative method to get $\psi_1(x_I, y_J, t_0 + \Delta t)$ (see associated handout).

References

Haltiner, G.J., & Williams, R.T. 1980. *Numerical Prediction and Dynamic Meteorology.* J. Wiley.

Holton, J.R. 2004. *An Introduction to Dynamic Meteorology (Fourth Edition).* Elsevier.

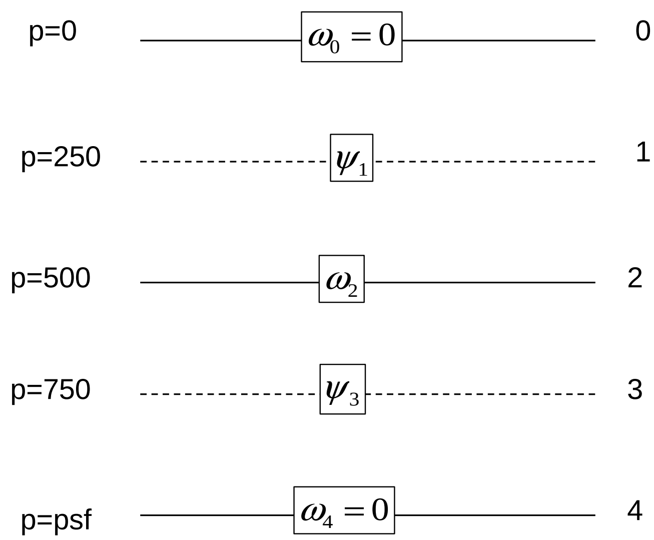

Fig.(8.2) of Holton (2004). Organization of the dependent variables in two-level baroclinic model

Figure 1: On each plane, visualize a discrete (gridded) field of the associated variable, eg. $\psi_1(x_I, y_J)$.