EAS471, Lab 4 "Non-linear advection" Mar, 2007

Task

Use the spectral method outlined in the notes to numerically integrate the non-linear advection equation

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0 \tag{1}$$

on domain $-1 \leq x \leq 1$ subject to the periodic boundary condition $u(-1,t) = u(1,t)$. As initial condition assume

$$u(x,0) = \sin(\pi x) \tag{2}$$

Recall that the basis functions are chosen as

$$\theta_n = e^{j\, k_n\, x} \tag{3}$$

where $k_n = n\pi$ and $n = -N, -N+1, ... -1, 0, 1..., N-1, N$.

Study the time evolution of your solution over $0 \leq t \leq 10$ in relation to the resolution of your discretization, which should cover $\Delta t = (0.05, 0.1, 1.0)$ and $N = 5, 10$.

Note that since contributions from $u^I_{-1}(0)$ and $u^I_{-1}(0)$ are summed to provide the (real) coefficient of $\sin \pi x$:

$$u(x,0) = \left[u^I_{-1}(0) - u^I_1(0)\right] \sin(\pi x) \tag{4}$$

there are many ways to represent the initial state spectrally. The most symmetric choice would be

$$\begin{aligned} u^I_{-1}(0) &= 1/2 \\ u^I_1(0) &= -1/2 \end{aligned} \tag{5}$$

(all other coefficients must vanish at $t = 0$).

Note: Probably you will find the wave "breaks". Zauderer (*Partial Differential Equations*, p65-74) gives a good discussion of the problem.